1状態および2状態1ビット通信セルラ・オートマトンの数列生成能力について

上川 直紀 梅尾 博司

大阪電気通信大学大学院 工学研究科 情報工学専攻

概 要

セルラ・オートマトン (CA) は非線形モデルの１つとして考えられている．CA はセルと呼ばれる有限状態オートマトンにより構成される．セルは自らと，隣接するセルの内部状態という局所的な情報を元に，自らを遷移させる機能しか持たない．この局所的な相互作用がモデル全体に影響を及ぼし，巨大で複雑な事象をシミュレートすることができるという特徴を持つ．本稿では，セル間の通信量を１ビットに制限したセルラ・オートマトンのモデル $CA_{1-\mathrm{bit}}$ 上での実時間数列問題について考察を行なう．$CA_{1-\mathrm{bit}}$ は最も計算能力が小さいモデルであると考えられているが，高々2 状態しか持たない $CA_{1-\mathrm{bit}}$ で複雑な非正則数列が生成可能であることを示す.

キーワード

１ビット通信セルラ・オートマトン，セル間通信量，実時間数列生成問題，並列アルゴリズム

A Note on Sequence Generation Power of One-Bit Cellular Automata with One and Two Internal States

Naoki Kamikawa and Hiroshi Umeo

Osaka Electro-Communication Univ., Graduate School of Engineering

Abstract

Cellular automaton (CA) are considered to be a non-linear model of complex systems in which an infinite one-dimensional array of finite state machines (cells) updates itself in a synchronous manner according to a uniform local rule. We study a sequence generation problem on a special restricted class of cellular automata having 1-bit inter-cell communications ($CA_{1-\mathrm{bit}}$). The 1-bit CA can be thought to be one of the most powerless and simplest models in a variety of CAs. However, we show that a rich variety of non-regular sequences can be generated in real-time by the $CA_{1-\mathrm{bit}}$ with small internal states.

key words

cellular automata, 1-bit inter-cell-communication, real-time sequence generation problem, parallel algorithm, computational complexity

1 Introduction

A model of cellular automata (CA) was devised for studying self-reproduction by John von Neumann. It is studied in many fields such as complex systems. Mazoyer [1] and Umeo [2] devised a model of 1-bit inter-cell-communication cellular automata ($CA_{1-\mathrm{bit}}$). The $CA_{1-\mathrm{bit}}$ can be thought to be one of the most powerless and simplest models in a variety of CAs. We study a sequence generation problem on the $CA_{1-\mathrm{bit}}$. It has been shown that infinite non-regular sequences such as Fibonacci sequence and sequence $\{2^n \mid n = 1, 2, 3 \ldots\}$ can be generated in real-time by a $CA_{1-\mathrm{bit}}$ in Kamikawa and Umeo [3]. In this paper, we study the sequence generation power of $CA_{1-\mathrm{bit}}$ with 2 internal states. First, we give a characterization of sequences generated by the $CA_{1-\mathrm{bit}}$ with 1 internal state in real-time. Then, we consider the class of sequences generated by $CA_{1-\mathrm{bit}}$ with 2 internal states and give partial classifications. It is shown that a sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$ can be generated in real-time by a $CA_{1-\mathrm{bit}}$ with 2 internal states, but not generated by any $CA_{1-\mathrm{bit}}$ with 1 internal state. State-efficient generation algorithms are also presented.

2 Sequence Generation Problem

2.1 1-bit inter-cell-communication cellular automata

$CA_{1-\mathrm{bit}}$ consists of an infinite array of identical finite state automata, each located at a positive integer point (See Fig. 1).

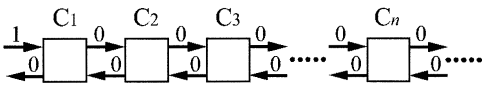

Figure 1: 1-bit inter-cell communication cellular automaton.

Each automaton is referred to as a cell. A cell at point i is denoted by C_i, where $i \geq 1$. Each C_i, except for C_1, is connected to its left- and right-neighbor cells via a left or right one-way communication link. These communication links are indicated by right- and left-pointing arrows in Fig. 1, respectively. Each one-way communication link can transmit only one bit at each step in each direction. One distinguished leftmost cell C_1, the communication cell, is connected to the outside world. A cellular automaton with 1-bit inter-cell communication (abbreviated by $CA_{1-\mathrm{bit}}$) consists of an infinite array of finite state automata $A = (Q, \delta)$, where

1. Q is a finite set of internal states.
2. δ is a function, defining the next state of any cell and its binary outputs to its left- and right-neighbor cells, such that δ: $Q \times \{0,1\} \times \{0,1\} \to Q \times \{0,1\} \times \{0,1\}$, where $\delta(\mathtt{p}, x, y) = (\mathtt{q}, x', y')$, $\mathtt{p}$, $\mathtt{q} \in Q$, $x, x', y, y' \in \{0,1\}$, has the following meaning. We assume that at step t the cell C_i is in state $\mathtt{p}$ and is receiving binary inputs x and y from its left and right communication links, respectively. Then, at the next step, $t+1$, C_i assumes state $\mathtt{q}$ and outputs x' and y' to its left and right communication links, respectively. Note that binary inputs to C_i at step t are also outputs of C_{i-1} and C_{i+1} at step t. A quiescent state $\mathtt{q} \in Q$ has a property such that $\delta(q, 0, 0) = (q, 0, 0)$.

Thus, the $CA_{1-\mathrm{bit}}$ is a special subclass of *normal* (i.e., *conventional*) cellular automata.

2.2 Sequence generation problem on $CA_{1-\mathrm{bit}}$

We now define the **sequence generation problem** on $CA_{1-\mathrm{bit}}$. Let M be a $CA_{1-\mathrm{bit}}$, and let $\{t_n | n = 1, 2, 3, ...\}$ be an infinite monotonically increasing positive integer sequence defined for natural numbers, such that $t_n \geq n$ for any $n \geq 1$. We then have a semi-infinite array of cells, as shown in Fig. 1, and all cells are in the quiescent state at time $t = 0$. The communication cell C_1 is input the 1-bit signal '1' from the outside world at time $t = 0$ for initiation of the sequence generator. We say that M generates a sequence $\{t_n | n = 1, 2, 3, ...\}$ in *real-time* if and only if the leftmost end cell of M outputs 1 via its leftmost communication link at time $t = t_n$.

We study sequences which can be generated on $CA_{1-\mathrm{bit}}$ with 1 and 2 states, respectively, by using a personal computer. Let k be any natural number such that $k \geq 1$. We enumerate all of the transition rules of the k-states $CA_{1-\mathrm{bit}}$. Each cell takes a state out of k internal states and its input is a 1-bit signal from left- and right-neighbor cells. The quiescent state $q \in Q$ has a property such that $\delta(q, 0, 0) = (q, 0, 0)$. Thus, there are $4 \cdot k - 1$ kinds of combinations of input of transition rule. When the cell C_i changes internal state, the cell C_i outputs the 1-bit signal to its left and right communication links. There are $4 \cdot k$ kinds of combinations of output of transition rule. Therefore, $CA_{1-\mathrm{bit}}$ with k internal states has at most $4 \cdot k^{4 \cdot k - 1}$ transition rules.

3 Characterization of sequences generated on $CA_{1-\mathrm{bit}}$

3.1 $CA_{1-\mathrm{bit}}$ with 1 internal state

We study sequences generated on $CA_{1-\mathrm{bit}}$ with 1 internal state, where it has 64 transition rules. We examine the 64 transition rules with the personal computer. Table 1 shows sequences which can be generated on $CA_{1-\mathrm{bit}}$ with 1 internal state.

Transition rule no. in the Table 1 shows the transition rule number shown as follows:

Let M be a 1-state $CA_{1-\mathrm{bit}}$. M is formulated as $M = (Q, \delta(\mathtt{Q}, 0, 0) = (\mathtt{Q}, 0, 0), \delta(\mathtt{Q}, 1, 0) = (\mathtt{Q}, a, b), \delta(\mathtt{Q}, 0, 1) = (\mathtt{Q}, c, d), \delta(\mathtt{Q}, 1, 1) = (\mathtt{Q}, e, f))$, such that $\mathtt{Q} \in Q$, $a, b, c, d, e, f \in \{0, 1\}$.

Transition rule no. $= f \cdot 2^5 + e \cdot 2^4 + d \cdot 2^3 + c \cdot 2^2 + b \cdot 2^1 + a \cdot 2^0 + 1$.

The symbol "-" in the Table 1 shows that the leftmost end cell C_1 always outputs '0' at any time. Sequences generated on $CA_{1-\mathrm{bit}}$with 1 internal state is classified as a union of 25 finite sequences, 6 linear sequences and a non-regular sequence. The class of finite sequence consist of 24 $\{1\}$ and $\{1, 3\}$. The class of linear sequence consist of 7 $\{2n - 1 \,|\, n = 1, 2, 3, ...\}$. The class of non-regular sequence consist of $\{2^n - 1 \,|\, n = 1, 2, 3, ...\}$.

Table 1: Sequences generated on $CA_{1-\mathrm{bit}}$ with 1 internal state

Transition rule no.	Sequence	Transition rule no.	Sequence
1	-	33	-
2	-	34	-
3	-	35	-
4	-	36	-
5	$\{1\}$	37	$\{1\}$
6	$\{1\}$	38	$\{1\}$
7	$\{1\}$	39	$\{1\}$
8	$\{1\}$	40	$\{1\}$
9	-	41	-
10	-	42	-
11	-	43	-
12	-	44	-
13	$\{1\}$	45	$\{1\}$
14	$\{2n-1 \mid n=1,2,3,\ldots\}$	46	$\{2n-1 \mid n=1,2,3,\ldots\}$
15	$\{1\}$	47	$\{1\}$
16	$\{2^n-1 \mid n=1,2,3,\ldots\}$	48	$\{1,3\}$
17	-	49	-
18	-	50	-
19	-	51	-
20	-	52	-
21	$\{1\}$	53	$\{1\}$
22	$\{1\}$	54	$\{1\}$
23	$\{1\}$	55	$\{1\}$
24	$\{1\}$	56	$\{1\}$
25	-	57	-
26	-	58	-
27	-	59	-
28	-	60	-
29	$\{1\}$	61	$\{1\}$
30	$\{2n-1 \mid n=1,2,3,\ldots\}$	62	$\{2n-1 \mid n=1,2,3,\ldots\}$
31	$\{1\}$	63	$\{1\}$
32	$\{2n-1 \mid n=1,2,3,\ldots\}$	64	$\{2n-1 \mid n=1,2,3,\ldots\}$

3.2 $CA_{1-\mathrm{bit}}$ with 2 internal states

In this section, we study sequences which can be generated on $CA_{1-\mathrm{bit}}$ with 2 internal states. It simulates by 2097152 transition rules with a personal computer and the generated sequences are examined. Table 3.2 shows the class of sequences generated on $CA_{1-\mathrm{bit}}$ with 2 internal states. The class 7 consist of union of more sequence and random sequences. Union of more sequence generated by $CA_{1-\mathrm{bit}}$ with 2 internal states is $[\{1\} \cup \{6 \cdot n - 4 | n = 1, 2, 3, \ldots\}]$, $[\{1\} \cup \{4 \cdot n - 2 | n = 1, 2, 3, \ldots\}]$, etc. Next, we show a generation algorithm of sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$ on $CA_{1-\mathrm{bit}}$ with 2 internal states.

3.2.1 Generation algorithm for sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$

Sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$ can be generated in real-time by a $CA_{1-\mathrm{bit}}$ with 2 internal states that is given in Table 3. In Fig. 2, we show a time-space diagram for real-time generation of sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$.

Real-time generation of sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$ is described in terms of 3 waves: *a-wave*, *b-wave*, *e-wave*. The a- and e-waves propagate in the right direction at 1/1 speed. The b-wave propagates in the left direction at 1/1 speed. At time $t = 0$, the communication cell C_1 is input the 1-bit signal

Table 2: The class of sequences generated on $CA_{1-\mathrm{bit}}$ with 2 internal states.

Class	Type	Sequences	Number of $CA_{1-\mathrm{bit}}$
1	Finite sequence	Ex. $\{1\}$, $\{1,3\}$	66698
2	$a \cdot n + b$	Ex. $\{n \mid n=1,2,3,\ldots\}$ $\{3 \cdot n - 1 \mid n=1,2,3,\ldots\}$	20784
3	$a \cdot 2^n + b$	$\{2^n - 1 \mid n=1,2,3,\ldots\}$ $\{2^n + 1 \mid n=1,2,3,\ldots\}$ $\{2 \cdot 2^n - 2 \mid n=1,2,3,\ldots\}$ $\{2 \cdot 2^n - 3 \mid n=1,2,3,\ldots\}$ $\{4 \cdot 2^n - 3 \mid n=1,2,3,\ldots\}$ $\{\frac{3}{2} \cdot 2^n - 1 \mid n=1,2,3,\ldots\}$ $\{\frac{3}{2} \cdot 2^n - 2 \mid n=1,2,3,\ldots\}$ $\{\frac{5}{2} \cdot 2^n - 4 \mid n=1,2,3,\ldots\}$ $\{\frac{9}{2} \cdot 2^n - 4 \mid n=1,2,3,\ldots\}$	92
4	$a \cdot 2^n + b \cdot n + c$	$\{7 \cdot 2^n - 4 \cdot n - 8 \mid n=1,2,3,\ldots\}$ $\{\frac{7}{2} \cdot 2^n - 4 \cdot n - 2 \mid n=1,2,3,\ldots\}$	2
5	$n^2 - n + 1$	$\{n^2 - n + 1 \mid n=1,2,3,\ldots\}$	2
6	$\frac{6+2\sqrt{5}}{3\sqrt{5}-5}(\frac{1+\sqrt{5}}{2})^{n-1} - \frac{6-2\sqrt{5}}{3\sqrt{5}+5}(\frac{1-\sqrt{5}}{2})^{n-1} - 2 \cdot n - 3$	$\{\frac{6+2\sqrt{5}}{3\sqrt{5}-5}(\frac{1+\sqrt{5}}{2})^{n-1} - \frac{6-2\sqrt{5}}{3\sqrt{5}+5}(\frac{1-\sqrt{5}}{2})^{n-1} - 2 \cdot n - 3 \mid n=1,2,3,\ldots\}$	1
7	Other sequences	union of more sequence random sequence	204639

a, b, c : rational number.

Table 3: Transition rules for real-time generation of sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$.

1

Q	R=0	R=1
L=0	(Q,0,0)	(A,0,0)
L=1	(A,1,1)	(Q,1,1)

2

A	R=0	R=1
L=0	(A,0,0)	(Q,1,1)
L=1	(Q,0,0)	(A,1,1)

'1' from the outside world. As a result, the a-wave is generated. When the a-wave propagates in the right direction, the a-wave generates "partition" to every cell at every step. The e-wave is generated on the cell C_1. The e-wave propagates in the right direction. When the e-wave reaches the leftmost partition, the e-wave and the leftmost partition are eliminated and the b-wave is generated. The b-wave propagates in the left direction. When the b-wave reaches the communication cell C_1, the C_1 outputs to the outside world. At after 2 steps, the e-wave is generated. These waves are continuously generated. Therefore, sequence $\{n^2 - n + 1 \mid n = 1, 2, 3, \ldots\}$ is generated in real-time by a $CA_{1-\mathrm{bit}}$.

Let i be any positive integer and let j, l, m be any positive natural number. At time $t = i$, it is assumed that a partition is arranged on all cells since cell $C_l (l \geq j)$ and the cell C_1 outputs the 1-bit signal '1' to outside world. At time $t = i+2$, the e-wave is generated on the cell C_1. The e-wave propagates in the left direction at 1/1 speed. When the e-wave reaches C_j at time $t = i+2+j-1$, the e-wave and the leftmost partition on cell C_j are eliminated and the b-wave is generated. The b-wave, generated by C_j at time $t = i+2+j-1$, propagates in the left direction at 1/1 speed. The b-wave reaches C_1 and the cell C_1 outputs the 1-bit signal '1' to outside world at time $t = i+2+2(j-1)$. The e-wave is generated at time $t = i+2+2(j-1)+2$.

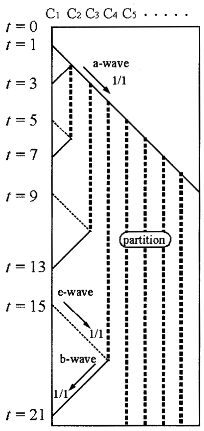

Figure 2: Time-space diagram for real-time generation of sequence $\{n^2 - n + 1 \,|\, n = 1, 2, 3, ...\}$.

Because the leftmost partition moves to the cell C_{j+1}, the cell C_1 outputs the 1-bit signal '1' to outside world at time $t = i + 2 + 2(j-1) + 2 + 2\{(j+1) - 1\}$. Therefore, the cell C_1 outputs the 1-bit signal '1' to its left communication link at time $t = i + \sum_{k=1}^{m} 2 + \sum_{k=1}^{m} 2\{j + (k-1) - 1\} = m^2 + (2j-1)m + i \, (m = 1, 2, 3, \ldots)$.

The initial configuration is all cell takes state Q and the leftmost cell C_1 is input the 1-bit signal '1' from the outside world. At time $t = 1$, the cell C_1 takes state A, and outputs the 1-bit signal '1' to left- and right-neighbor cells. it is approved that $j = 1, i = 1$. C_1 outputs the 1-bit signal '1' to its left communication link at time $t = 1$ and $t = m^2 + m + 1 \, (m = 1, 2, 3, \ldots)$. Therefore, C_1 outputs the 1-bit signal '1' to its left communication link at time $t = n^2 - n + 1 \, (n = 1, 2, 3, \ldots)$. It is seen that the scheme given above can exactly generate sequence $\{n^2 - n + 1 \,|\, n = 1, 2, 3, ...\}$ in real-time. We have implemented the algorithm on a computer. We have tested the validity of the rule set from $t = 0$ to $t = 20000$ steps. We obtain the following theorem.

[Theorem 1] Sequence $\{n^2 - n + 1 \,|\, n = 1, 2, 3, ...\}$ can be generated by a $CA_{1-\mathrm{bit}}$ with 2 internal states in real-time.

In Fig. 3, we show a number of snapshots of the configuration from $t = 0$ to 31.

4 Conclusions

We have studied a sequence generation problem on $CA_{1-\mathrm{bit}}$ with small number of internal states. Characterization of sequences generated by the $CA_{1-\mathrm{bit}}$ with 1 and 2 internal states has been given. It has been shown that sequence $\{n^2 - n + 1 \,|\, n = 1, 2, 3, \ldots\}$ can be generated in real-time by a $CA_{1-\mathrm{bit}}$ with 2 internal states. This algorithms for generation sequence on $CA_{1-\mathrm{bit}}$ are optimal in the number of states. A future study in sequence generation problem on $CA_{1-\mathrm{bit}}$ is to compare sequence generation power of $CA_{1-\mathrm{bit}}$ with sequence generation power of conventional CA.

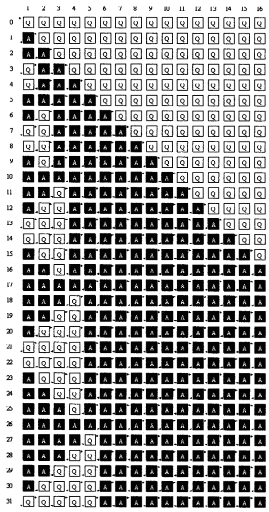

Figure 3: A configuration of real-time generation of sequence $\{n^2 - n + 1 \,|\, n = 1, 2, 3, ...\}$.

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