# 正則構成子からなる分散協調文法の多段階導出制御について

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各構成子が左線形な導出規則のみからなる型の分散協調文法の導出規則の適用を別の分散協調文法で制御した場合の言語生成能力について考察をする．本稿においては，各構成子の能力が同一の場合，単独の分散協調文法の言語生成能力より導出規則の適用を別の分散協調文法で制御した文法システムが真に言語生成能力が高いことを示す．また，単独の分散協調文法と多段階の分散協調文法，そのときの構成子の能力との相関などについても示す．これらの結果は階層的なマルチエージェントシステムを設計する上で効果的な手法を提供する．

# Iterative Controls on Cooperating Distributed Grammar Systems with Regular Components

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We consider the language generation of a grammar system under the control of another grammar system for cooperating distributed grammar systems with regular components. We show that the language generation by a grammar system under iterative controls is more powerful than the generation by one grammar system. We also show a case where a grammar system can be decomposed to two grammar systems with keeping the power of the language generation. These lead us to an effective method of designing multi agent systems in a hierarchical way.

## 1 Introduction

As computers become more powerful and computer networks become well organized, we have to deal more and more with complex tasks distributed among a set of "processors", which are working together in a well defined way. The concept of *multi agent systems* is introduced to understand problems with this kind of nature. Although the concept of multi agent systems are widely used to explain practical systems, formal aspects of the concept are not investigated so well. In particular, an effective method of designing multi agent systems is strongly required in practice.

A *cooperating distributed grammar system* [1] is introduced to analyze a formal aspect of multi agent systems. This system is proposed as a formal model of *blackboard ar-*

*chitectures* from the formal language theory's point of view; blackboard architectures are developed for distributed problem solving, especially for opportunistic problem solving, in the Artificial Intelligence research area [5]. Results on these systems lead us to methods to design practical multi agent systems in a more effective and more systematic way. This motivates our work in this paper.

In this paper, we concentrate on one of the simplest types of cooperating distributed grammar systems introduced in [3], that is, grammar systems with regular components and extended axioms. We consider the language generation by one grammar system under the control of another grammar system; the latter grammar system allows the former grammar system to derive terminal strings if derivations in the former grammar system are acceptable by the latter grammar system. This can be understood as a model of cooperating distributed working of ordinary workers under the control of managers, which is typically observed in companies like ours. We show that the language generation by a grammar system under iterative controls is more powerful than the generation by one grammar system. This matches with the reality in practical situations; working with management is more powerful than without management. Finally, we show a case where a grammar system can be decomposed to two grammar systems with keeping the power of the language generation. Our results lead to an effective method of designing multi agent systems in a hierarchical way.

## 2 Preliminaries

For an alphabet $\Sigma$, we denote by $\Sigma^*$ the free monoid generated by $\Sigma$ under the operation of concatenation. The unity of $\Sigma^*$, called the *null string*, is denoted by $\lambda$. $|w|$ denotes the length of a string $w$. For strings $w_1, w_2, \ldots, w_n$, $\prod_{i=1}^{n} w_i$ denotes the string $w_1 w_2 \cdots w_n$. $[\![m:n]\!]$ denotes the set $\{i \mid m \leq i \leq n\}$ of integers. For a set $S$, $|S|$ denotes the cardinality of $S$.

**Definition 1** Let $m$ and $r$ be positive integers. An *extended cooperating distributed grammar system with m degree axioms and r regular components* (*ECDR*$(m,r)$ *grammar system*, for short) is an $(r+3)$-tuple $\Gamma = (N, \Sigma, P_1, P_2, \ldots, P_r, S)$, where $N$ is a finite set of nonterminals, $\Sigma$ is a finite set of terminals, each $P_i$ is a finite set of rules of the form $A \to uB$ or $A \to u$, where $A, B \in N$ and $u \in \Sigma^*$, and $S$ is a finite set of strings over $N$ such that for any $s \in S$, $1 \leq |s| \leq m$. Each $P_i$ is called a *component of* $\Gamma$ and each element of $S$ is called an *axiom of* $\Gamma$.

*ECDR*$(m,r)$ grammar systems are an extension of cooperating distributed grammar systems in [1] in the sense that axioms are sets of strings over nonterminals. In this paper, we follow the simplified definition of the grammar systems introduced in [3].

According to [1], we can understand an *ECDR*$(m,r)$ grammar system $\Gamma$ as follows. The components correspond to the agents solving a problem on the blackboard; any rule represents some pieces of knowledge which result in a possible change on the blackboard. Each axiom $A_1 \cdots A_k$ is the formal counterpart of a problem on the blackboard in the beginning; each $A_i$ is a *working unit* of the problem. The set of terminals $\Sigma$ contains the letters which correspond to such knowledge pieces accepted as solutions or part of solutions.

Let $\Gamma = (N, \Sigma, P_1, P_2, \ldots, P_r, S)$ be an *ECDR*$(m,r)$ grammar system. For any alphabet $\Psi = \{\psi_1, \psi_2, \ldots, \psi_r\}$, $\Gamma$ is said to be *labeled with* $\Psi$ if and only if $\Psi$ is associated in one-to-one manner to the components $P_1, P_2, \ldots, P_r$. Then each $\psi_i$ is called the label of $P_i$. Each $P_i$ is referable to its label $\psi_i$ and therefore, we may write $\psi_i$ instead of $P_i$

without any confusion.

Let $\Gamma = (N, \Sigma, P_1, P_2, \ldots, P_r, S)$ be an $ECDR(m,r)$ grammar system labeled with $\Psi = \{\psi_1, \psi_2, \ldots, \psi_r\}$. We denote $N \cup \Sigma$ by $V$.

**Definition 2** For any $x, y \in V^*$, we write $x \underset{\Gamma}{\Longrightarrow}^{\psi_i} y$ if and only if $x = uAv$, $y = uwv$, and $A \to w \in P_i$, where $u, v \in V^*$. We write $x \underset{\Gamma}{\overset{*}{\Longrightarrow}}^{\psi_i} y$ if and only if

$$x \underset{\Gamma}{\Longrightarrow}^{\psi_i} x_1, x_1 \underset{\Gamma}{\Longrightarrow}^{\psi_i} x_2, \ldots, x_n \underset{\Gamma}{\Longrightarrow}^{\psi_i} y.$$

Moreover, we write $x \underset{\Gamma}{\overset{t}{\Longrightarrow}}^{\psi_i} y$ if and only if $x \underset{\Gamma}{\overset{*}{\Longrightarrow}}^{\psi_i} y$ and there is no $z \neq y$ with $y \underset{\Gamma}{\overset{*}{\Longrightarrow}}^{\psi_i} z$.

A string $x$ over $V$ is called a *sentential form* of $\Gamma$ if $x \in S$ or $s \underset{\Gamma}{\overset{*}{\Longrightarrow}}^{\alpha} x$ for some $s \in S$.

**Definition 3** For any $x, y \in V^*$, we write $x \underset{\Gamma}{\overset{t}{\Longrightarrow}}^{\alpha} y$, which is called the *derivation from x to y with the control word* $\alpha$, if and only if

$$x \underset{\Gamma}{\overset{t}{\Longrightarrow}}^{\psi_{i_1}} x_1, x_1 \underset{\Gamma}{\overset{t}{\Longrightarrow}}^{\psi_{i_2}} x_2, \ldots, x_{n-1} \underset{\Gamma}{\overset{t}{\Longrightarrow}}^{\psi_{i_n}} y$$

and $\alpha = \psi_{i_1}\psi_{i_2}\cdots\psi_{i_n}$.

In this paper, we consider only so called $t$-mode in [1].

**Definition 4** The *language generated by* $\Gamma$, denoted by $L(\Gamma)$, is the set

$$L(\Gamma) = \{w \mid s \underset{\Gamma}{\overset{t}{\Longrightarrow}}^{\alpha} w, s \in S, w \in \Sigma^*\}$$

and the *Szilard language of* $\Gamma$ is the set

$$Sz(\Gamma) = \{\alpha \mid s \underset{\Gamma}{\overset{t}{\Longrightarrow}}^{\alpha} w, s \in S, w \in \Sigma^*\}.$$

**Definition 5** Let $m$ be a positive integer. Then $\mathcal{ECDRL}[m]$ denotes the family of languages

$\mathcal{ECDRL}[m] = \{L(\Gamma) \mid r$ is a positive integer and $\Gamma$ is an $ECDR(m,r)$ grammar system$\}$.

## 3 Basic Properties

In this section, we shall show some basic properties of $ECDR(m,r)$ grammar systems necessary for the following sections.

**Proposition 6** *For any $ECDR(m,r)$ grammar system $\Gamma$, $Sz(\Gamma)$ is regular.*

Let $\Gamma = (N, \Sigma, P_1, P_2, \ldots, P_r, S)$ be an $ECDR(m,r)$ grammar system. Then, for any integer $i \in [\![1:m]\!]$, $N(i)$ denotes the set of all nonterminals $N(i) = \{B \mid s \in S, A$ is $i$th occurred nonterminal in $s$, $A \underset{\Gamma}{\overset{*}{\Longrightarrow}}^{\alpha} uB\}$. For each $P_i$, $N(P_i)$ denotes the subset of $N$ such that for any $A \in N(P_i)$ there exists a rule $A \to w$ in $P_i$. Then $T(P_i)$ denotes the set $N - N(P_i)$. For any $P_i$ and any $A \in N(P_i)$, $G[P_i, A]$ denotes the regular grammar $G[P_i, A] = (N(P_i), \Sigma \cup T(P_i), P_i, A)$.

**Definition 7** An $ECDR(m,r)$ grammar system $\Gamma = (N, \Sigma, P_1, P_2, \ldots, P_r, S)$ is said to be *in the normal form* if and only if

1. for each axiom $s \in S$, $|s| = m$,
2. each $P_i$ has rules only of the form $A \to aB$ or $A \to a$, where $A, B \in N$ and $a \in \Sigma \cup \{\lambda\}$, and
3. for any $i, j \in [\![1:m]\!]$, $i \neq j$ implies $N(i) \cap N(j) = \varnothing$.
4. there is no $A \in N$ such that $A \underset{\Gamma}{\overset{*}{\Longrightarrow}}^{\alpha} A$ and
5. for any $P_i$ and any $A \in N(P_i)$, the language generated by $G[P_i, A]$ is finite.

**Lemma 8** *For any $ECDR(m,r)$ grammar system $\Gamma$, there effectively exists an $ECDR(m,r')$ grammar system $\Gamma'$ in the normal form such that $L(\Gamma) = L(\Gamma')$ holds.*

**Lemma 9** *Let $\Gamma = (N, \Sigma, P_1, P_2, \ldots, P_r, S)$ be an $ECDR(m,r)$ grammar system in the normal form. Then there exists an integer $p$ such that if $w \in L(\Gamma)$ and $|w| > p$ then there exists a factorization*

$$w = u_1v_1w_1u_2v_2w_2\cdots u_nv_nw_nu_{n+1} \ (1 \leq n \leq m)$$

*that satisfies*

1. *there exist nonterminals* $B_1, B_2, \ldots, B_n$ *such that*

$$A_1 A_2 \cdots A_m \overset{t}{\underset{\Gamma}{\Longrightarrow}}{}^{\alpha} x_1 B_1 x_2 B_2 \cdots x_n B_n x_{n+1}$$

$$x_1 B_1 x_2 B_2 \cdots x_n B_n x_{n+1} \overset{t}{\underset{\Gamma}{\Longrightarrow}}{}^{\beta} x_1 v_1 B_1 x_2 v_2 B_2 \cdots x_n v_n B_n x_{n+1}$$

$$x_1 B_1 x_2 B_2 \cdots x_n B_n x_{n+1} \overset{t}{\underset{\Gamma}{\Longrightarrow}}{}^{\gamma} u_1 w_1 u_2 w_2 \cdots u_n w_n u_{n+1},$$

2. *for each* $k \geq 0$, $u_1(v_1)^k w_1 u_2(v_2)^k w_2 \cdots u_n(v_n)^k w_n u_{n+1} \in L(\Gamma)$,
3. *for each* $i$, $|v_i| > 0$,

*where* $A_1 A_2 \cdots A_m \in S$ *and each* $x_i \in V^*$.

## 4 Generation under Iterative Controls

In this section, we consider the language generated by an $ECDR(m,r)$ grammar system under the control of another $ECDR(n,q)$ grammar system. The latter grammar system allows the former grammar system to derive terminal strings if control words are acceptable by the latter grammar. This is formally defined in the following way.

**Definition 10** Let $\Gamma_1 = (N_1, \Sigma, P_1, \ldots, P_r, S_1)$ be an $ECDR(m,r)$ grammar system and $\Gamma_2 = (N_2, \Psi, Q_1, \ldots, Q_q, S_2)$ be an $ECDR(n,q)$ grammar system. The *language generated by* $\Gamma_1$ *under the control of* $\Gamma_2$, denoted by $L(\Gamma_1 \triangleleft L(\Gamma_2))$, is the set

$$\{w \in \Sigma^* \mid s \overset{t}{\underset{\Gamma_1}{\Longrightarrow}}{}^{\alpha} w, s \in S_1, \alpha \in L(\Gamma_2)\}.$$

This can be understood as follows. We have two agent systems. One consists of actual worker agents and the other consists of management agents. Each agent system has its own blackboard and agents in the system can cooperate through the blackboard. The management agent system can monitor the cooperation of the worker agent system and can refuse some solutions of the worker agent system according to its own solutions. This kind of situation is quite common in companies; ordinary workers are controlled by managers.

We shall show the following theorem, which plays the main role in this section.

**Theorem 11** *Let* $\Gamma_1$ *and* $\Gamma_2$ *be* $ECDR(m,r)$ *and* $ECDR(n,q)$ *grammar systems in the normal form, respectively. Then there effectively exists an* $ECDR(m \times n, q)$ *grammar system* $\Gamma$ *such that* $L(\Gamma) = L(\Gamma_1 \triangleleft L(\Gamma_2))$ *holds.*

We show the method of composing $\Gamma$ from $\Gamma_1$ and $\Gamma_2$.

Let $\Gamma_1 = (N_1, \Sigma, R_1, \ldots, R_r, S_1)$ be an $ECDR(m,r)$ grammar system labeled with $\Psi = \{\psi_1, \ldots, \psi_r\}$ and $\Gamma_2 = (N_2, \Psi, Q_1, \ldots, Q_q, S_2)$ be an $ECDR(n,q)$ grammar system labeled with $\Phi = \{\phi_1, \ldots, \phi_q\}$. Lemma 8 ensures that we have only to consider the case where $\Gamma_1$ and $\Gamma_2$ are in the normal form. We define the $ECDR(m \times n, q)$ grammar system $\Gamma = (N_1 \times N_2, \Sigma, P_1, \ldots, P_q, S)$ with $P_1, \ldots, P_q$ labeled with $\Delta = \{\delta_1, \ldots, \delta_q\}$ in the following way. Let $h$ be a homomorphism such that $\delta_i = h(\phi_i)$ for any $i \in [\![1:q]\!]$. For any $\prod_{i=1}^{m}(\prod_{j=1}^{l_i}(A_{i,j}, X_j))$ such that each $A_{i,j} \in N_1$, each $X_j \in N_2$ and each $l_i \leq n$, if $A_{1,1} \cdots A_{m,1} \in S_1$, $X_1 \cdots X_n \in S_2$ and there exists a derivation $X_1 \cdots X_n \overset{t}{\underset{\Gamma_2}{\Longrightarrow}}{}^{\beta} \alpha_1 \cdots \alpha_n$ such that for each $j \in [\![1:n]\!]$, $\alpha_j \in \Psi^*$ and $X_j \overset{t}{\underset{\Gamma_2}{\Longrightarrow}}{}^{\beta_j} \alpha_j$ and for each $i \in [\![1:m]\!]$

$$A_{i,1} \overset{t}{\underset{\Gamma_1}{\Longrightarrow}}{}^{\alpha_1} u_1 A_{i,2} \overset{t}{\underset{\Gamma_1}{\Longrightarrow}}{}^{\alpha_2} \cdots \overset{t}{\underset{\Gamma_1}{\Longrightarrow}}{}^{\alpha_{l_i-1}} u_1 \cdots u_{l_i-1} A_{i,l_i} \overset{t}{\underset{\Gamma_1}{\Longrightarrow}}{}^{\alpha_{l_i}} u_1 \cdots u_{l_i},$$

where each $u_j$ is in $\Sigma^*$, then $\prod_{i=1}^{m}(\prod_{j=1}^{l_i}(A_{i,j}, X_j))$ is in $S$. $P_1, \ldots, P_q$ are defined in the following way. For any $i \in [\![1:q]\!]$,

1. for each $X \rightarrow \psi_j Y$ in $Q_i$,
   (a) for each $A \rightarrow aB$ in $R_j$, if $B$ is in $N(R_j)$ then $(A,X) \rightarrow a(B,X)$ is in $P_i$, otherwise, $(A,X) \rightarrow a(B,Y)$ is in $P_i$,

(b) for each $A \to a$ in $R_j$, $(A, X) \to a$ is in $P_i$,

2. for each $X \to \psi_j$ in $Q_i$,

(a) for each $A \to aB$ in $R_j$, if $B$ is in $N(R_j)$ then $(A, X) \to a(B, X)$ is in $P_i$, otherwise, $(A, X) \to a$ is in $P_i$,

(b) for each $A \to a$ in $R_j$, $(A, X) \to a$ is in $P_i$,

3. for each $X \to Y$ in $Q_i$, $(C, X) \to (C, Y)$ is in $P_i$ for any $C \in N_1$, and

4. for each $X \to \lambda$ in $Q_i$, $(C, X) \to \lambda$ is in $P_i$ for any $C \in N_1$,

where $A, B \in N_1$, $X, Y \in N_2$, $a \in \Sigma \cup \{\lambda\}$ and $\psi_j$ is the label of $R_j$.

Since $N_1$ and $N_2$ are finite and the number of elements of $S$ is bounded by $m$ and $n$, $S$ is effectively constructed in finite steps. Since $\Gamma_1$ and $\Gamma_2$ have finite numbers of rules, each $P_i$ is also effectively constructed in finite steps. Hence, the construction of $\Gamma$ is effective and $\Gamma$ is finitely defined.

**Lemma 12** *For any* $\phi \in \Phi$, $A \underset{\Gamma_1}{\overset{t}{\Longrightarrow}}{}^{\alpha} uB$ *and* $X \underset{\Gamma_2}{\overset{t}{\Longrightarrow}}{}^{\phi} \alpha Y$ *if and only if* $(A, X) \underset{\Gamma}{\overset{t}{\Longrightarrow}}{}^{h(\phi)} u(B, Y)$, *where* $u \in \Sigma^*$, $A \in N_1$, $B \in N_1 \cup \{\lambda\}$, $X \in N_2$, $Y \in N_2 \cup \{\lambda\}$ *and* $B = \lambda$ *or* $Y = \lambda$ *if and only if* $(B, Y) = \lambda$.

**Lemma 13** *For any* $A \in N_1$, *any* $X \in N_2$ *and any* $w \in \Sigma^*$, *either* $A \underset{\Gamma_1}{\overset{t}{\Longrightarrow}}{}^{\alpha} w$ *and* $X \underset{\Gamma_2}{\overset{t}{\Longrightarrow}}{}^{\mu} \alpha Y$ *or* $A \underset{\Gamma_1}{\overset{t}{\Longrightarrow}}{}^{\alpha} wB$ *and* $X \underset{\Gamma_2}{\overset{t}{\Longrightarrow}}{}^{\mu} \alpha$ *if and only if* $(A, X) \underset{\Gamma}{\overset{t}{\Longrightarrow}}{}^{h(\mu)} w$, *where* $B \in N_1 \cup \{\lambda\}$ *and* $Y \in N_2$.

**Lemma 14** *Let* $k \in [\![1 : m]\!]$ *and* $l \in [\![1 : n]\!]$. *For any* $A_{1,1}, \ldots, A_{1,l}, \ldots, A_{k,1}, \ldots, A_{k,l} \in N_1 \cup \{\lambda\}$ *and any* $X_1, \ldots, X_n \in N_2$,

$$A_{1,1} \cdots A_{k,1} \underset{\Gamma_1}{\overset{t}{\Longrightarrow}}{}^{\alpha_1} u_{1,1}B_{1,1} \cdots u_{k,1}B_{k,1},$$

$$\vdots$$

$$A_{1,l-1} \cdots A_{k,l-1} \underset{\Gamma_1}{\overset{t}{\Longrightarrow}}{}^{\alpha_{l-1}} u_{1,l-1}B_{1,l-1} \cdots u_{k,l-1}B_{k,l-1},$$

*and*

$$A_{1,l} \cdots A_{k,l} \underset{\Gamma_1}{\overset{t}{\Longrightarrow}}{}^{\alpha_l} u_{1,l} \cdots u_{k,l},$$

$X_1 \cdots X_l \underset{\Gamma_2}{\overset{t}{\Longrightarrow}}{}^{\mu} \alpha_1 Y_1 \cdots \alpha_l Y_l$, *and for each* $j$ ($j \in [\![1 : l]\!]$) *either* $Y_j \in N_2$ *and* $B_{i,j} = \lambda$ *for any* $i$ *or* $Y_j = \lambda$ *and* $B_{i,j} \in N_1 \cup \{\lambda\}$ *if and only if*

$$(A_{1,1}, X_1) \cdots (A_{1,l}, X_l) \cdots (A_{k,1}, X_1) \cdots (A_{k,l}, X_l) \underset{\Gamma}{\overset{t}{\Longrightarrow}}{}^{h(\mu)} u_{1,1} \cdots u_{1,l} \cdots u_{k,1} \cdots u_{k,l},$$

*where each* $u_{i,j} \in \Sigma^*$ *and if* $A_{i,j} = \lambda$ *then* $(A_{i,j}, X_j) = \lambda$ *and vice versa and then* $B_{i,j} = u_{i,j} = \lambda$.

**Lemma 15** *For any* $w \in \Sigma^*$, $w \in L(\Gamma_1 \lhd L(\Gamma_2))$ *if and only if* $w \in L(\Gamma)$.

**Definition 16** Let $n$ and $m_1, m_2, \ldots, m_n$ be positive integers. Then $\mathcal{ECDRL}[m_1 \lhd m_2 \lhd \cdots \lhd m_n]$ denotes the family of languages

$$\{L(\Gamma_1 \lhd L(\Gamma_2 \lhd \ldots \lhd L(\Gamma_n) \cdots)) \mid \text{each } \Gamma_i \text{ is an } ECDR(m_i, r_i) \text{ grammar system in the normal form}\}.$$

**Theorem 17** *For any integers* $n \geq 2$ *and* $m_1, m_2, \ldots, m_n \geq 1$,

$$\mathcal{ECDRL}[m_1 \lhd m_2 \lhd \cdots \lhd m_n] \subseteq \mathcal{ECDRL}[m_1 \times m_2 \times \cdots \times m_n].$$

**Lemma 18** *Let* $n$ *and* $m_1, m_2, \ldots, m_n$ *be positive integers. Then*

$$\{(a_1\sharp)^k (a_2\sharp)^k \cdots (a_{m_1 \times m_2 \times \cdots \times m_n}\sharp)^k \flat \mid k \geq 0\} \in \mathcal{ECDRL}[m_1, m_2, \ldots, m_n].$$

**Theorem 19** *For any integers* $m_1, m_2, \ldots, m_n \geq 1$ *and* $k \geq 1$,

$$\mathcal{ECDRL}[m_1 \lhd m_2 \lhd \cdots \lhd m_n] = \mathcal{ECDRL}[m_1 \lhd m_2 \lhd \cdots \lhd m_n \lhd 1],$$

$$\mathcal{ECDRL}[m_1 \lhd m_2 \lhd \cdots m_n \lhd k] \subsetneq \mathcal{ECDRL}[m_1 \lhd m_2 \lhd \cdots m_n \lhd k + 1].$$

Hence, for extended grammar systems with regular components, the language-generation by a grammar system under iterative controls is more powerful than the generation by one grammar system.

## 5 Decomposing Grammar Systems

In this section, we show a case of decomposing one grammar system to two grammar systems with keeping the power of the language generation. The basic idea is to decompose components of one grammar system and to control the decomposed components by another grammar system.

**Proposition 20** *Let $\Gamma$ be ECDR$(m,r)$ grammar system in the normal form. Then there exist a regular grammar $G$ and an ECDR$(m,r)$ grammar system $\Gamma'$ such that $L = L(G \triangleleft L(\Gamma'))$.*

To decompose a general *ECDR* grammar system to two *ECDR* grammar systems, a complicated procedure are required. Here, we only show an example of decomposition.

Let us consider the following *ECDR*(4, 3) grammar systems $\Gamma = (N, \Sigma, P_1, P_2, P_3, S)$:

$$
\begin{aligned}
N &= \{A, A', B, B', C, C', D, D'\},\\
\Sigma &= \{a, b, c, d\},\\
P_1 &= \{A \to aA', B \to bB', C \to cC',\\
&\qquad D \to dD'\},\\
P_2 &= \{A \to \lambda, B \to \lambda, C \to \lambda, D \to \lambda\},\\
P_3 &= \{A' \to A, B' \to B, C' \to C, D' \to D\},\\
S &= \{ABCD\}.
\end{aligned}
$$

We can easily show that $L(\Gamma) = \{a^n b^n c^n d^n \mid n \geq 0\}$. This grammar system can be decomposed to $\Gamma_1 = (N_1, \Sigma_1, P_{1,1}, P_{1,2}, P_{1,3}, P_{1,4}, S_1)$ and $\Gamma_2 = (N_2, \Sigma_2, P_{2,1}, P_{2,2}, P_{2,3}, P_{2,4}, S_2)$ that satisfy that $L(\Gamma) = L(\Gamma_1 \triangleleft \Gamma_2)$:

$$
\begin{aligned}
N_1 &= \{A, A', C, C'\},\\
\Sigma_1 &= \{a, b, c, d\},\\
P_{1,1} &= \psi_1 : \{A \to aA', C \to cC'\}\\
P_{1,2} &= \psi_2 : \{A \to bA', C \to dC'\}\\
P_{1,3} &= \psi_3 : \{A \to \lambda, C \to \lambda\}\\
P_{1,4} &= \psi_4 : \{A' \to A, C' \to C\},\\
S_1 &= \{AC\},
\end{aligned}
$$

and

$$
\begin{aligned}
N_2 &= \{E, E', E'', F, F', F''\},\\
\Sigma_2 &= \{\psi_1, \psi_2, \psi_3, \psi_4\},\\
P_{2,1} &= \{E \to \psi_1 E', F \to \psi_2 F'\},\\
P_{2,2} &= \{E' \to \psi_4 E'', F' \to \psi_4 F''\},\\
P_{2,3} &= \{E'' \to E, F'' \to F\},\\
P_{2,4} &= \{E \to \lambda, F \to \psi_3\},\\
S_2 &= \{EF\}.
\end{aligned}
$$

We note that $L(\Gamma_2) = \{(\psi_1\psi_4)^n(\psi_2\psi_4)^n\psi_3 \mid n \geq 0\}$. Thus, it is easy to show that $L(\Gamma) = L(\Gamma_1 \triangleleft \Gamma_2)$. We can easily see that both $\Gamma_1$ and $\Gamma_2$ are *ECDR*(2, 4) grammar systems.

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