プログラミング-言語・基礎・実践- 21－23
(1995. 3. 10)

ω言語上のリテラルシャッフル

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あらまし 本稿では、ω言語上でのリテラルシャッフル演算について研究を行なう。まず、duo(ε－フリー準同型写像写像およびε－フリー逆準同型写像の下で閉じている最小のω言語のクラス) がリテラルの下でシャッフルで閉じているための必要十分条件は、それが共通部分をとる演算の下で閉じていることであることを証明する。次に、ω正則言語のクラスのいくつかの部分クラスに対して、リテラルシャッフルの下での閉包性について論じる。最後に、通常のシャッフルとリテラルシャッフルの関係について考察を行なう。

キーワード ω正則言語，シャッフル演算，準同型写像，逆準同型写像

Literal shuffle on ω-languages

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Abstract

We study the literal shuffle operation on ω-languages. First we show that a duo (the smallest class closed under ε-free morphism and inverse ε-free morphism) is closed under literal shuffle operation if and only if it is closed under intersection. Next the closure properties of some classes of the ω-reglar languages under literal shuffle and shuffle operation are investigated. Furthermore we consider the relation between literal shuffle and shuffle operation.

key words ω－regular language, shuffle operation, morphism, inverse morphism

1. Introduction

The literal shuffle operation, introduced in [1] as a more constrained form of the shuffle operation, models the synchronomous behaviour while the shuffle corresponds asynchronomous one.

In this paper we study the literal shuffle operation, in relation with ω-languages. In section 2 basic definitions and notations are given. In section 3 we investigate the closure properties of some subfamilies of the ω-regular languages under the literal shuffle and the shuffle. We also give the necessary and saficient condition for a duo to be colsed under literal shuffle. In section 4, we consder the relation between literal shuffle and shuffle operation.

2. Preliminaries

Let Σ be an alphabet. Σ^* denotes the set of all finite words over Σ, and Σ^ω denotes the set of all ω-words over Σ, i.e., the set of all mappings $\alpha : \{0,1,2,...\} \to \Sigma$. Let $\Sigma^\infty = \Sigma \cup \Sigma^\omega$. An ω-word is written by $\alpha = a_0a_1...$ where $a_n = \alpha(n)$ $(n = 0,1,2,...)$. We call a subset of Σ^* (Σ^ω, resp.) a language (ω-language) over Σ.

A deterministic automaton (DA, for short) $\mathcal{A}$ over Σ is a 5-tuple $\mathcal{A} = \langle S, \Sigma, \delta, s_0, F\rangle$, where S is a finite set of states, Σ is an alphabet, $\delta : S \times \Sigma \to S$ is a next state function, $s_0 \in S$ is an initial state, and $F \subseteq S$ is a set of accepting states. Then the run Run($\mathcal{A}$, α) of a DA $\mathcal{A}$ on an ω-word α is an ω-word $q_0q_1... \in Q^\omega$ such that $q_0 = s_0$ and $q_{n+1} = \delta(q_n, \alpha(n))$ $(n=0,1,2,...)$.

For a run r of $\mathcal{A}$, let $Ex(r) = \{ q \in Q \mid q = q_n$ for some n }, and $Inf(r) = \{ q \in Q \mid q = q_n$ for infinitely many n }, and define the following six types of acceptances of the DA $\mathcal{A}$:

$E(\mathcal{A}) = \{ \alpha \in \Sigma^\omega \mid Ex(Run(\mathcal{A}, \alpha)) \cap F \neq \phi \}$,

$E'(\mathcal{A}) = \{ \alpha \in \Sigma^\omega \mid Ex(Run(\mathcal{A}, \alpha)) \subseteq F \}$,

$I(\mathcal{A}) = \{ \alpha \in \Sigma^\omega \mid Inf(Run(\mathcal{A}, \alpha)) \cap F \neq \phi \}$,

$I'(\mathcal{A}) = \{ \alpha \in \Sigma^\omega \mid Inf(Run(\mathcal{A}, \alpha)) \subseteq F \}$,

$L(\mathcal{A}) = \{ \alpha \in \Sigma^\omega \mid F \subseteq Inf(Run(\mathcal{A}, \alpha)) \}$,

$L'(\mathcal{A}) = \{ \alpha \in \Sigma^\omega \mid F \not\subseteq Inf(Run(\mathcal{A}, \alpha)) \}$.

The class of ω-languages of the form $E(\mathcal{A})$, $E'(\mathcal{A})$, $I(\mathcal{A})$, $I'(\mathcal{A})$, $L(\mathcal{A})$, $L'(\mathcal{A})$ resp.) for some automaton $\mathcal{A}$ over Σ is denoted by $\mathbb{E}_\Sigma$ ($\mathbb{E}'_\Sigma$, $\mathbb{I}_\Sigma$, $\mathbb{I}'_\Sigma$, $\mathbb{L}_\Sigma$, $\mathbb{L}'_\Sigma$, resp.). All these classes are included in the class $\mathbb{R}^\omega{}_\Sigma$ of ω-regular languages. (For the definition of an ω-regular language and inclusion relations among these classes, see [6],[8], and [9]). If Σ is clear in the context, we omit Σ and simply write $\mathbb{E}$, $\mathbb{E}'$, and so on.

Let $h: \Sigma \to \Delta$ be a morphism. We say that a class $\mathbb{C}$ of ω-languages is closed under morphism h if $h(X) \in \mathbb{C}_\Delta$ for any $X \in \mathbb{C}_\Sigma$, and that $\mathbb{C}$ is closed under an inverse morphism h^{-1} if $h^{-1}(Y) \in \mathbb{C}_\Sigma$ for any $Y \in \mathbb{C}_\Delta$.

A class of ω-languages closed under any ε-free morphism and any inverse ε-free morphism is called a duo. Only three classes $\mathbb{R}^\omega$, $\mathbb{I}'$ and $\mathbb{E}'$ are duos among those defined above.([5])

For X and $Y \subseteq \Sigma^\omega$, the shuffle Sh(X, Y) and the literal shuffle LSh(X, Y) are defined by $Sh(X, Y) = \{u_0v_0u_1v_1\cdots \mid u_0u_1\cdots \in X,\ v_0v_1\cdots \in Y,\ u_i, v_i \in \Sigma^+\}$, and $LSh(X, Y) = \{x_0y_0x_1y_1\cdots \mid x_0x_1\cdots \in X,\ y_0y_1\cdots \in Y,\ x_i, y_i \in \Sigma\}$. We say that a class $\mathbb{C}$ is closed under Sh (or LSh) if for any X, $Y \subseteq \Sigma^\omega$ in $\mathbb{C}$, Sh(X,Y) (or LSh(X, Y), resp.) is in $\mathbb{C}$.

Facts. For X and Y $\subseteq \Sigma^{\omega}$, the followings are easily obtained.

2.1) $LSh(\Sigma^{\omega}-X, \Sigma^{\omega}-Y) = \Sigma^{\omega} - (LSh(X, \Sigma^{\omega}) \cup LSh(\Sigma^{\omega}, Y))$.

2.2) $LSh(X, Y) = LSh(X, \Sigma^{\omega}) \cap LSh(\Sigma^{\omega}, Y)$.

2.3) $X \cap Y = h^{-1}(LSh(X, Y))$, where $h: \Sigma \to \Sigma$ is a morphism defined by $h(a) = aa$ for $a \in \Sigma$.

3. Closure properties under literal shuffle and shuffle

Lemma 3.1 Let $\mathbb{C}$ be a duo and X be an ω-language over Σ in $\mathbb{C}$. Then both $LSh(X, \Sigma^{\omega})$ and $LSh(\Sigma^{\omega}, X)$ are in $\mathbb{C}$.

(Proof) Let $X \subseteq \Sigma^{\omega}$ be in $\mathbb{C}$. We define ε-free morphisms $g: \Sigma \to (\Sigma' \cup \{\#\})$ and $h_0: \Sigma \to (\Sigma' \cup \{\#\})$, $h_1: \Sigma \to (\Sigma' \cup \{\#\})$, where $\Sigma' = \{\sigma' \mid \sigma \in \Sigma\}$ and $\# \notin \Sigma$, as follows: (1) $h_0(a) = a'\#$ and $h_1(a) = \#a'$ for $a \in \Sigma$. (2) $g(a) = \#$, $g(a') = a$ for $a \in \Sigma$.

It is obvious that $LSh(X, \Sigma^{\omega}) = g^{-1}(h_0(X))$ and $LSh(\Sigma^{\omega}, X) = g^{-1}(h_1(X))$. Thus the result holds. ::

Lemma 3.2 Let $\mathbb{C} \in \{\mathbb{I}, \mathbb{E}, \mathbb{L}, \mathbb{L}'\}$, and $X \subseteq \Sigma^{\omega}$ be in $\mathbb{C}$. Then both $LSh(X, \Sigma^{\omega})$ and $LSh(\Sigma^{\omega}, X)$ are in $\mathbb{C}$.

(Proof) Suppose that $X \in \mathbb{C}$, and that X is accepted by a DA $\mathcal{A} = \langle Q, \Sigma, \delta, q_0, F\rangle$. We difine the DA $\mathcal{A}' = \langle Q \cup Q', \Sigma, \delta', q_0, F\rangle$, where $Q' = \{q' \mid q \in Q\}$, as follows: $\delta'(q, \sigma) = (\delta(q, \sigma))'$, and $\delta'(q', \sigma) = q$ for $q \in Q$ and $\sigma \in \Sigma$. It is obvious that $I(\mathcal{A})' = LSh(I(\mathcal{A}'), \Sigma^{\omega})$, $E(\mathcal{A}') = LSh(E(\mathcal{A}), \Sigma^{\omega})$ and so on. For the case of $LSh(\Sigma^{\omega}, X)$, the proof is similar to that for the case of $LSh(X, \Sigma^{\omega})$ and is omitted. ::

Theorem 3.3 For a class $\mathbb{C}$ of ω-languages, we define $\mathbb{C}^c = \{X^c \mid X \in \mathbb{C}\}$, where $X^c = \Sigma^{\omega}-X$. If $\mathbb{C}$ or $\mathbb{C}^c$ is a duo, then the followings are equivalent.

(1)$\mathbb{C}$ is closed under intersection. (2)$\mathbb{C}$ is closed under LSh.

(Proof) Let $\mathbb{C}$ be a duo. Then (1)==>(2) is a consequence of Fact 2.2 and Lemma 3.1. (2)==>(1) is directly obtained by Fact 2.3. Let $\mathbb{C}^c$ be a duo. (1)==>(2): From Fact 2.1, we have that LSh(X, Y) = $LSh(X^c, \Sigma^\omega)^c \cap (LSh(\Sigma^\omega, Y^c))^c$. The result holds from Lemma 3.1. (2)==>(1): For $\alpha \in \Sigma^\omega$, and the morphism h in Fact 2.3, we have that $\alpha \in X^c \cup Y^c$ iff $h(\alpha) \in LSh(X^c, \Sigma^\omega) \cup LSh(\Sigma^\omega, Y^c)$. Hence $X^c \cup Y^c = h^{-1}(LSh(X^c, \Sigma^\omega) \cup LSh(\Sigma^\omega, Y^c)) = h^{-1}(LSh(X,Y)^c)$ from Fact 2.1. Thus $X \cap Y = (h^{-1}(LSh(X, Y)^c))^c$. ::

Colorally 3.4 The classes $\mathbb{R}^\omega$, $\mathbb{I}$, $\mathbb{I}'$, $\mathbb{E}$, and $\mathbb{E}'$ are closed under LSh. ::

Theorem 3.5 The class $\mathbb{L}$ is not closed under LSh.

(Proof) Let X=L($\mathcal{A}$), where $\mathcal{A}$ is the DA defined in Fig.1. Suppose that LSh(X, X) = L($\mathcal{A}'$) for a DA $\mathcal{A}'$ with an initial state q_0 and a single accepting state q_f. Consider the ω-words $\alpha = b^\omega$ and $\beta = LSh(b^3(a^5b)^\omega, (a^5b)^\omega)$. Since $\alpha = LSh(\alpha, \alpha)$ with $\alpha \in L(\mathcal{A})$, there exists an integer i such that $\delta(q_0, b^i) = q_f$. Since β is in L($\mathcal{A}'$), either of the followings holds for some $k \geq 0$, $0 \leq j \leq 2$, $-1 \leq i \leq 0$, and $0 \leq m \leq 1$.

(1) $\delta(q_0, LSh(b^3(a^5b)^k a^j, (a^5b)^k a^{j+3+i})) = q_f$,

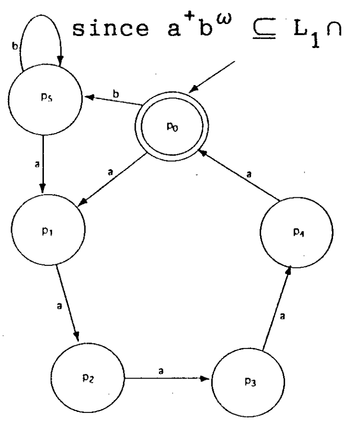
(2) $\delta(q_0, LSh(b^3(a^5b)^k a^3, (a^5b)^k a^5 b^m) = q_f$,

(3) $\delta(q_0, LSh(b^3(a^5b)^k a^{m+4}, (a^5b)^{k+1} a^{m+i+1})) = q_f$,

Suppose that (1) holds. For ω-word $\alpha_1 = LSh(b^3(a^5b)^k a^j, (a^5b)^k a^{j+3+i}) b^\omega$, q_f is in Inf(Run($\mathcal{A}'$, α_1)). But α_1 is not in LSh(X, X). This is a contradiction. For other cases, the proof is similar to that of case (1) and is omitted. ::

Theorem 3.6 The class $\mathbb{L}'$ is not closed under LSh.

(Proof)Let $X_1 = L'(\mathcal{A}_1)$ and $X_2 = L'(\mathcal{A}_2)$ where $\mathcal{A}_1$ and $\mathcal{A}_2$ are defined in Fig.2. Suppose that $X = LSh(X_1, X_2)$ is in $\mathbb{L}'$. Let $X = L'(\mathcal{A}_3)$ for a DA $\mathcal{A}_3 = \langle Q, \{a,b\}, \delta, q_0, \{q_f\} \rangle$. It is clear that both a^ω and b^ω are not in X, i.e., they are in X^c. Since $X^c = L(\mathcal{A}_3)$, $\delta(q_0, a^n) = \delta(q_0, b^m) = q_f$ for sufficiently large m and n. Thus $a^n b^\omega \in L(\mathcal{A}_3) = X^c$. However $a^n b^\omega \in X$ for $n \geqq 2$, since $a^+ b^\omega \subseteq L_1 \cap L_2$. This is a contradiction. ::



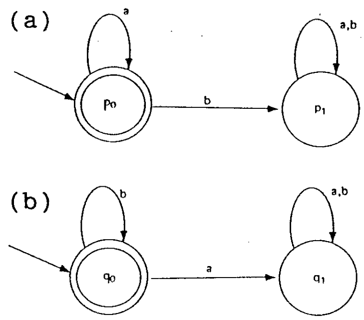


Fig.1. DA $\mathcal{A}$ in Th.3.5 Fig.2 (a)DA $\mathcal{A}_1$ (b)DA $\mathcal{A}_2$ in Th. 3.6

<u>Theorem 3.7</u> The classes $\mathbb{L}$ and Π are not closed under Sh.

(Proof) Let $\Sigma = \{a, b\}$. Take two ω-languages $\{a^\omega\}$ and $\{b^* a^\omega\}$ $\in \mathbb{L} \subseteq \Pi$. Then $Sh(\{a^\omega\}, \{b^* a^\omega\}) = \Sigma^* a^\omega \notin \Pi$ ([8],[9]).

<u>Lemma 3.8</u> For $\Sigma = \{ a, b \}$, the ω-language $X_0 = \{ w \mid Inf(w) = \Sigma \}$ over Σ is in $\Pi - \Pi'$.

(Proof) It is obvious that $X_0 = (a^* b)^\omega \cap (b^* a)^\omega$.

Since the class Π is closed under intersection([8,9]), and both $(a*b)^\omega$ and $(b*a)^\omega$ are in Π, X is in Π. Suppose that $X \in \Pi'$. Let $h: \Sigma^\infty \to \Sigma^\infty$ be a morphism defined by $h(a) = a$ and $h(b) = ba$. It follows that $h^{-1}(X_0) = (a^* b)^\omega$. We know that Π' is closed under inverse morphism. However, $(a^* b)^\omega$ is in $\Pi - \Pi'$([8, 9]). This is a contradiction. Thus X_0 is in $\Pi - \Pi'$.

Theorem 3.9 Let $\Sigma = \{a, b\}$. The classes $\mathbb{E}'$, $\mathbb{L}'$ and $\mathbb{I}'$ are not closed under Sh.

(Proof) Consider the two ω-languages $\{a^{\omega}\}$ and $\{b^{\omega}\} \in \mathbb{E}' \subseteq \mathbb{L}' \subseteq \mathbb{I}'$. By the previous Lemma, $Sh(\{a^{\omega}\}, \{b^{\omega}\}) = X_0 \notin \mathbb{I}'$. Thus $\mathbb{E}'$, $\mathbb{L}'$ and $\mathbb{I}'$ are not closed under Sh. ::

Theorem 3.10 The class $\mathbb{E}$ is closed under Sh. ::

Theorem 3.11 ([7]) The class $\mathbb{R}^{\omega}$ is closed under Sh. ::

4. Relation between shuffle and literal shuffle

We define a morphism $h_{\$}:(\Sigma \cup \{\$\})^{\infty} \rightarrow \Sigma^{\infty}$, where $\$ \notin \Sigma$, by $h_{\$}(\sigma) = \sigma$ for $\sigma \in \Sigma$ and $h_{\$}(\$) = \varepsilon$.

Facts. 4.1) For $X \subseteq \Sigma^{\omega}$ and $Y \subseteq \Delta^{\omega}$ such that $\Sigma \cap \Delta = \phi$, $LSh(X, Y) = Sh(X, Y) \cap (\Sigma\Delta)^{\omega}$. 4.2) For X and $Y \subseteq \Sigma^{\omega}$, $Sh(X, Y) = h_{\$}(LSh(h_{\$}^{-1}(X), h_{\$}^{-1}(Y)))$.

Theorem 4.3 Let X and Y be ω-languages. Then $Sh(X, Y) = f(g^{-1}(LSh(h_{\$}^{-1}(X), h_{\$}^{-1}(Y)))) \cap W$ for ε-free morphisms f and g, the morphism $h_{\$}$ defined above, and $W \in \mathbb{I}$.

(Proof) For X and $Y \subseteq \Sigma^{\omega}$, define morphisms f, g, and ω-regular language W as follows.

(1) $f:(\Sigma\times\Sigma\cup\Sigma\times\{\$\}\cup\{\$\}\times\Sigma)^{\infty} \rightarrow \Sigma^{\infty}$

by $f(\langle\sigma, \eta\rangle) = \sigma\eta$ for σ and $\eta \in \Sigma$, $f(\langle\sigma, \$\rangle) = \sigma$ for $\sigma \in \Sigma$, and $f(\langle\$,\eta\rangle) = \eta$ for $\eta \in \Sigma$.

(2) $g: (\Sigma\times\Sigma\cup\Sigma\times\{\$\}\cup\{\$\}\times\Sigma)^{\omega} \rightarrow (\Sigma \cup \{\$\})^{\omega}$

by $g(\langle\sigma, \eta\rangle) = \sigma\eta$ for $\sigma \in \Sigma$ and $\eta \in \Delta$, $g(\langle\sigma, \$\rangle) = \sigma\$$ for $\sigma \in \Sigma$, and $g(\langle\$, \eta\rangle) = \$\eta$ for $\eta \in \Sigma$.

(3) $W = (\Sigma\Sigma \cup \Sigma\{\$\} \cup \{\$\}\Sigma)^{\omega} - U$,

where $U = (\Sigma\Sigma \cup \Sigma\{\$\} \cup \{\$\}\Sigma)^{*}((\{\$\}\Sigma)^{\omega} \cup (\Sigma\{\$\}))))^{\omega}$

It is obvious that $Sh(X,Y) = f(g^{-1}(LShuf(h_{\$}^{-1}(X), h_{\$}^{-1}(Y) \cap W)$, and W $\in$ II.   ::

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