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# ルートE重なりな深さ保存項書き換えシステムのチャーチ・ロッサー性について

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あらまし
項書き換えシステムが強深さ保存であるとは、すべての書き換え規則の両辺に現れる各変数において、その変数の左辺における出現位置の最小深さが右辺の最大深さ以上であるときである。本稿では強深さ保存項書き換えシステムのチャーチ・ロッサー性を保証する十分条件を与え、この条件の判定方法について述べる。各関数記号に対して正の整数 (重みと呼ぶ) を割り当てることにより、強深さ保存の概念を強重み保存の概念に自然に拡張することができ、強重み保存項書き換えシステムのチャーチ・ロッサー性を保証する同様な十分条件を導くことができる。

キーワード 項書き換えシステム、チャーチ・ロッサ性、非線形、深さ保存、重み保存、E重なり

# On the Church-Rosser Property of Root-E-overlapping and Depth-Preserving Term Rewriting Systems

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Abstract
A term rewriting system (TRS) is said to be strongly depth-preserving if for any rewrite rule and any variable appearing in the both sides, the minimal depth of the variable occurrences in the left-hand-side is greater than or equal to the maximal depth of the variable occurrences in the right-hand-side. This paper gives a sufficient condition for Church-Rosser of strongly depth-preserving TRS's and describes how to check this condition. By assigning a positive integer (called *weight*) to each function symbol, the notion of strongly depth-preserving is naturally extended to that of strongly weight-preserving and a similar sufficient condition for Church-Rosser of strongly weight-preserving TRS's is obtained.

key words Term Rewriting System, Church-Rosser property, non-linear, depth-preserving, weight-preserving, E-overlapping

# 1 Introduction

A term rewriting system (TRS) is a set of directed equations (called rewrite rules). A TRS is Church-Rosser (CR) if any two interconvertible terms reduce to some common term by applications of the rewrite rules. This CR property is important in various applications of TRS's and has received much attention so far [1)~3),5)~8)]. Although the CR property is undecidable for general TRS's, many sufficient conditions for ensuring this property have been obtained [1),2),5)~8)].

However, for nonlinear and nonterminating TRS's, a small number of results on the CR property have been obtained. Our previous papers [5),6)] may be pioneering ones which have first given nontrivial conditions for the CR property by using the notions of non-E-overlapping (stronger than nonoverlapping) and E-critical pairs extending that of critical pairs. In 5),6), some sufficient conditions for the CR property have been given which can be applied to subclasses of right-linear TRS's. In the case of non-right-linear TRS's, it has been shown that there exist non-E-overlapping and depth-preserving TRS's which do not satisfy the CR property, but all the non-E-overlapping and strongly depth-preserving* TRS's satisfy the CR property[9)~11)]. Here, a TRS is depth-preserving if for each rule $\alpha \to \beta$ and any variable $x$ appearing in both $\alpha$ and $\beta$, the maximal depth of the $x$ occurrences in $\alpha$ is greater than or equal to that of the $x$ occurrences in $\beta$ [3)]. A TRS is strongly depth-preserving* if it is depth-preserving and for each $\alpha \to \beta$ and for any variable $x$ appearing in $\alpha$, all the depths of the $x$ occurrences in $\alpha$ are the same[11)].

In this paper, we first slightly extends the definition of strongly depth-preserving* TRS's, i.e., a TRS is strongly depth-preserving if for each rule $\alpha \to \beta$ and any variable $x$ appearing in both $\alpha$ and $\beta$, the minimal depth of the $x$ occurrences in $\alpha$ is greater than or equal to the maximal depth of the $x$ occurrences in $\beta$. It is obvious that a TRS is strongly depth-preserving if it is strongly depth-preserving*. We extend the result in 11) by showing that in the class of strongly depth-preserving TRS's introduced here, non-E-overlapping also ensures the CR property.

Next we show that even if strongly depth-preserving TRS's are E-overlapping, a condition called root-E-closed (in Section 3) ensures their CR property (Theorem 1). And we give some decidable sufficient conditions which ensure this root-E-closed condition.

By assigning a positive integer (called *weight*) to each function symbol, the notion of depth is naturally extended to that of weight: the weight of the $x$ occurrence is the sum of weights of function symbols appearing in the path from the root to the $x$-occurrence. Using the notion of this weight, we can define strongly weight-preserving TRS's in a similar way to that of strongly depth-preserving TRS's and obtain the corresponding root-E-closed condition which is a sufficient condition for the CR property of the strongly weight-preserving TRS's (Theorem 2). For example, this result ensures that TRS $R = \{\ f(x) \to g(h(x), x),\ g(x, x) \to a,\ b \to h(b)\}$[12)] is CR. (It was described in 12) without proof that the TRS R is CR.)

This paper is organized as follows. Section 2 is devoted to definitions. In Section 3, we give the root-E-closed condition. Some assertions to prove Theorem 1 are given in Section 4. In Section 5, we give a sufficient condition for the CR property of strongly weight-preserving TRS's. In Section 6, some decidable sufficient conditions ensuring the root-E-closed condition are given.

# 2 Definitions

The following definitions and notations are similar to those in Refs. 2), 5) and 9) ~ 11). Let $X$ be a set of *variables*, F be a finite set of *function symbols* and $T$ be the set of *terms* constructed from $X$ and $F$.

For a term $M$, we use $O(M)$ to denote the set of *occurrences* (*positions*) of $M$, and $M/u$ to denote the subterm of $M$ at occurrence $u$, and $M[u \leftarrow N]$ to denote the term obtained from $M$ by replacing the subterm $M/u$ by term $N$. The set of occurrences $O(M)$ of $M$ is partially ordered by the prefix ordering: $u \leq v$ iff $\exists w.uw = v$. In this case, we denote $w$ by $v/u$. If $u \leq v$ and $u \neq v$, then $u < v$. If $u \not\leq v$ and $v \not\leq u$, then $u$ and $v$ are said to be *disjoint* and denoted $u|v$. Let $V(M)$ be the set of variables in $M$, $O_x(M)$ be the set of occurrences of variable $x \in V(M)$, and $O_X(M) = \cup_{x \in V(M)} O_x(M)$ i.e., the set of variable occurrences in $M$. Let $\bar{O}(M) = O(M) - O_X(M)$: the set of non-variable occurrences. We also use $N[u \leftarrow M/u \mid u \in U]$ to denote the term obtained from $N$ by replacing $N/u_i$ by $M/u_i$ where $U = \{u_1, \cdots, u_n\}$ and $u_1, \cdots, u_n$ are pairwise disjoint.

The *depth* of occurrence $u \in O(M)$ is defined by $|u|$, i.e., the length of $u$, Let $H(M) = Max\{|u| \mid u \in O(M)\}$: the *height* of $M$. For example, $H(f(g(x))) = 2, H(a) = 0$.

A rewrite rule is a directed equation $\alpha \rightarrow \beta$ such that $\alpha \in T - X, \beta \in T$ and $V(\alpha) \supseteq V(\beta)$. A term-rewriting system (TRS) is a set of rewrite rules.

A term $M$ reduces to a term $N$ if $M/u = \sigma(\alpha)$ and $N = M[u \leftarrow \sigma(\beta)]$ for some $\alpha \rightarrow \beta \in R$ and $\sigma : X \rightarrow T$. We denote this reduction by $M \overset{u}{\rightarrow} N$. In this notation $u$ may be omitted (i.e., $M \rightarrow N$) and $\rightarrow^*$ is the reflexive-transitive closure of $\rightarrow$. Let $M \overset{u}{\leftrightarrow} N$ be $M \overset{u}{\rightarrow} N$ or $N \overset{u}{\rightarrow} M$.

A parallel reduction $M \leftarrow\!\!\!+\!\!\!\rightarrow N$ is defined as follows: $M \leftarrow\!\!\!+\!\!\!\rightarrow N$ iff $\exists U \subseteq O(M)$ such that $\forall u, v \in U$ $u \neq v \Rightarrow u|v$, $\forall u \in U$ $M/u \overset{\varepsilon}{\leftrightarrow} N/u$ and $N = M[u \leftarrow N/u \mid u \in U]$. In this case, let $R(M \leftarrow\!\!\!+\!\!\!\rightarrow N) = U$. (Note. $U = \phi$ is allowed.) Let $\leftarrow\!\!\!+\!\!\!\rightarrow^*$ be the reflexive-transitive closure of $\leftarrow\!\!\!+\!\!\!\rightarrow$. If $u < v$ for all $v \in R(M \leftarrow\!\!\!+\!\!\!\rightarrow N)$, then we denote this reduction by $M \overset{>u}{\leftarrow\!\!\!+\!\!\!\rightarrow} N$. If $M \leftarrow\!\!\!+\!\!\!\rightarrow N$ is reduced by the only not $\varepsilon$, then we denote this reduction by $M \overset{\varepsilon-inv}{\leftarrow\!\!\!+\!\!\!\rightarrow} N$. Let $\overset{>u}{\leftarrow\!\!\!+\!\!\!\rightarrow}{}^*$ be the reflexive-transitive closure of $\overset{>u}{\leftarrow\!\!\!+\!\!\!\rightarrow}$ and let $\overset{\varepsilon-inv}{\leftarrow\!\!\!+\!\!\!\rightarrow}{}^*$ be the reflexive-transitive closure of $\overset{\varepsilon-inv}{\leftarrow\!\!\!+\!\!\!\rightarrow}$.

We assume that $\gamma : M_0 \leftarrow\!\!\!+\!\!\!\rightarrow M_1 \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_n$ in the following definitions.

Let $R(\gamma) = \bigcup_{0 \leq i < n} R(M_i \leftarrow\!\!\!+\!\!\!\rightarrow M_{i+1})$ and $MR(\gamma)$ be the set of *minimal* occurrences in $R(\gamma)$ under the prefix ordering. For $u \in O(M_0)$, if there exists no $v \in R(\gamma)$ such that $v \leq u$, then $\gamma$ is said to be *u-invariant*. Let $M_0 = \sigma(\alpha)$ or $M_n = \sigma(\alpha)$ for some $\alpha \rightarrow \beta \in R$ and $\sigma : X \rightarrow T$. Then, $\gamma$ is said to be *$\alpha$-keeping* if $\gamma$ is $u$-invariant for all $u \in \bar{O}(\alpha)$. That is, $\gamma$ is $\alpha$-keeping iff all reductions of $\gamma$ occur in the variable parts of $\alpha$. Reduction sequence $\gamma$ is said to be a *peak* if $\gamma : M_0 \overset{\varepsilon}{\longleftarrow} M_1 \overset{\varepsilon-inv}{\leftarrow\!\!\!+\!\!\!\rightarrow}{}^* M_{n-1} \overset{\varepsilon}{\longrightarrow} M_n$. We denote by $\gamma[i, j]$ the subsequence $M_i \leftarrow\!\!\!+\!\!\!\rightarrow M_{i+1} \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_j$ of $\gamma$ where $i \geq 0$ and $j \leq n$. Let $u \in MR(\gamma)$. Then, the *cut* sequence of $\gamma$ at $u$ is $\gamma/u = (M_0/u \leftarrow\!\!\!+\!\!\!\rightarrow M_1/u \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_n/u)$. We denote by $\gamma[\xi'/\xi]$ the sequence obtained from reduction sequence $\gamma$ by replacing subsequence or cut sequence (or cut subsequence) $\xi$ of $\gamma$ by sequence $\xi'$. The composition of $\gamma$ and $\delta : N_0 \leftarrow\!\!\!+\!\!\!\rightarrow N_1 \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow N_k$ where $N_0 = M_n$ is denoted by $(\gamma; \delta)$.

Let $\gamma^R$ be the reverse sequence of $\gamma$, i.e., $\gamma^R : M_n \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_1 \leftarrow\!\!\!+\!\!\!\rightarrow M_0$. The number of parallel reduction steps of $\gamma$ is $|\gamma|_p = n$. (Note. If $\delta : M \leftarrow\!\!\!+\!\!\!\rightarrow M$, then $|\delta|_p = 1$.) Let $net(\gamma)$ be the sequence obtained from $\gamma$ by removing all $M_i \leftarrow\!\!\!+\!\!\!\rightarrow M_{i+1}$ satisfying that $M_i = M_{i+1}$, $0 \leq i < n$. We use $|\gamma|_{np}$ to denote $|net(\gamma)|_p$. Let $H(\gamma) = Max\{H(M_i) \mid 0 \leq i \leq n\}$.

**Example.** Let $\delta : f(c, c) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(c), g(c)) \leftarrow\!\!\!+\!\!\!\rightarrow a \leftarrow\!\!\!+\!\!\!\rightarrow a$, then $|\delta|_p = 3$, $net(\delta) : f(c, c) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(c), g(c)) \leftarrow\!\!\!+\!\!\!\rightarrow a$, $|\delta|_{np} = 2$ and $H(\delta) = H(f(g(c), g(c))) = 2$.

We use the definitions of $left(\gamma, h)$, $right(\gamma, h)$, $ldis(\gamma, h)$ and $width(\gamma, h)$ in 11): $left(\gamma, h)$ is the first position whose term height is $h$ from the left end of $\gamma$ and each term height in the left side from this position is less than $h$ where if there exist no such position, then $left(\gamma, h)$ is undefined; $right(\gamma, h)$ is defined in a similar way $left(\gamma, h)$ by replacing the term left with right, i.e., $|\gamma|_p - left(\gamma^R, h)$; $ldis(\gamma, h)$ is $|\gamma|_p - left(\gamma, h)$; $width(\gamma, h)$ is defined if either $left(\gamma, h)$ or $right(\gamma, h)$ is defined and $right(\gamma, h')$ $- left(\gamma, h'')$ where $h'$ (resp. $h''$) is the first position whose the term height is equal or greater than $h$ from the right (resp. left) end of $\gamma$. And we use the definitions of $K_{ldis}(\gamma)$, $K_{right}(\gamma)$ and $K_{width}(\gamma)$ in 11) such that $K_Y(\gamma)$ is the set of pairs $(h, Y(\gamma, h))$ where $Y \in \{ldis, width, right\} \wedge Y(\gamma, h)$ is defined. These formal definitions are given in 11).

**Example.** Let $\delta : f(c) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(g(c))) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(c)) \leftarrow\!\!\!+\!\!\!\rightarrow f(f(g(g(c)))) \leftarrow\!\!\!+\!\!\!\rightarrow f(f(c)) \leftarrow\!\!\!+\!\!\!\rightarrow g(c)$. Then, we have $left(\delta, 1) = 0$, $left(\delta, 3) = 1$, $left(\delta, 4) = 3$, $right(\delta, 1) = 5$, $right(\delta, 2) = 4$, $right(\delta, 4) = 3$, $ldis(\delta, 1) = 5$, $ldis(\delta, 3) = 4$, $ldis(\delta, 4) = 2$, $width(\delta, 1) = 5$, $width(\delta, 2) = 3$, $width(\delta, 3) = 2$, $width(\delta, 4) = 0$. And we have $K_{ldis}(\delta) = \{(1, 5), (3, 4), (4, 2)\}$, $K_{width}(\delta) = \{(1, 5), (2, 3), (3, 2), (4, 0)\}$ and $K_{right}(\delta) = \{(1, 5), (2, 4), (4, 3)\}$.

We define an ordering $<_s \subseteq N \times N$ (where $N = \{0, 1, 2, \cdots\}$) as follows: $(a, b) <_s (a', b') \Leftrightarrow (a < a' \bigwedge b \leq b') \bigvee (a = a' \bigwedge b < b')$. Let $\leq_s$ be $<_s \bigcup =$. We use $\ll_s$ to denote the multiset ordering of this ordering $<_s$. Let $\lessapprox_s$ be $\ll_s \bigcup =$. We use $\{\cdots\}_m$ to denote a multiset, e.g., $\{1, 1, 2\}_m$. We use $\ll_w$ to denote the multiset ordering of a lexicographic ordering $<$ (i.e., $(a, b) < (a', b') \Leftrightarrow a < a' \bigvee (a = a' \bigwedge b < b')$). Let $\lessapprox_w$ be $\ll_w \bigcup =$. Note that if $(a, b) <_s (a', b')$, then $(a, b) < (a', b')$, but the converse does not necessarily hold. And if $A \ll_s B$, then $A \ll_w B$. The orderings of $>_s$ and $>$ are well-founded, so that $\gg_s$ and $\gg_w$ are well-founded[1]).

**Definition of $\langle\ \delta \preceq \gamma\ \rangle$**

We define a relation $\preceq$ over parallel reduction sequences as follows. Let $\gamma : M \leftarrow\!\!\!+\!\!\!\rightarrow^* N$ and $\delta : M \leftarrow\!\!\!+\!\!\!\rightarrow^* N$. Then, $\delta \preceq \gamma$ if $|\delta|_p = |\gamma|_p$, $|\delta|_{np} \leq$

$|\gamma|_{np}$, $K_{ldis}(\delta)\leqq_s K_{ldis}(\gamma)$ and $K_{right}(\delta)\leqq_s K_{right}(\gamma)$. (Note that if $\delta \preceq \gamma$, then $H(\delta) \leq H(\gamma)$ holds by $K_{ldis}(\delta)\lesseqqgtr_s K_{ldis}(\gamma)$, and $\preceq$ is reflexive and transitive.)

A pair of rewrite rules $\alpha_1 \to \beta_1$ and $\alpha_2 \to \beta_2$ is *overlapping* iff there exist $u \in \bar{O}(\alpha_1)$ and mappings $\sigma, \sigma'$ such that $\sigma(\alpha_1/u) = \sigma'(\alpha_2)$, where $u = \varepsilon$ implies that $(\alpha_1 \to \beta_1) \neq (\alpha_2 \to \beta_2)$. In this case, the pair is overlapping at $u$, and *root-overlapping* if $u = \varepsilon$.

**Definition of ⟨ E-overlapping TRS, root-E-overlapping TRS ⟩**

A reduction sequence is *E-overlapping* if the reduction sequence is $\sigma(\alpha_1)[u \leftarrow \sigma'(\beta_2)] \leftarrow \sigma(\alpha_1)[u \leftarrow \sigma'(\alpha_2)] \overset{>u}{\longleftrightarrow^*} \sigma(\alpha_1) \to \sigma(\beta_1)$ for some $\alpha_1 \to \beta_1, \alpha_2 \to \beta_2 \in R$, $u \in \bar{O}(\alpha_1)$ and mappings $\sigma, \sigma' : X \to T$ where $u = \varepsilon$ implies that $(\alpha_1 \to \beta_1) \neq (\alpha_2 \to \beta_2)$. TRS $R$ is E-overlapping iff there exists an E-overlapping reduction sequence. If $u = \varepsilon$, then the reduction sequence $\sigma(\beta_1) \leftarrow \sigma(\alpha_1) \overset{\varepsilon-inv}{\longleftrightarrow^*} \sigma'(\alpha_2) \to \sigma'(\beta_2)$ is *root-E-overlapping*. In this case, the pair $(\alpha_1 \to \beta_1, \alpha_2 \to \beta_2)$ has a root-E-overlapping sequence. TRS $R$ is root-E-overlapping if all the E-overlapping sequences are root-E-overlapping.

An E-overlapping reduction sequence $\gamma : \sigma(\alpha)[u \leftarrow \sigma'(\beta')] \leftarrow \sigma(\alpha)[u \leftarrow \sigma'(\alpha')] \overset{>u}{\longleftrightarrow^*} \sigma(\alpha) \to \sigma(\beta)$ is *standard* iff for the subsequence $\gamma' : \sigma'(\alpha') \longleftrightarrow^* \sigma(\alpha/u)$, $R(\gamma') \cap (\bar{O}(\alpha') \cap \bar{O}(\alpha/u)) = \phi$, i.e., any reduction in $\gamma'$ occurs in the variable parts in $\sigma'(\alpha')$ or $\sigma(\alpha/u)$.

**Definition of ⟨ strongly depth-preserving TRS ⟩**

TRS $R$ is *strongly depth-preserving* if $\forall \alpha \to \beta \in R\ \forall x \in V(\alpha) \bigcap V(\beta)\quad Max\{|v| \mid v \in O_x(\beta)\} \leq Min\{|u| \mid u \in O_x(\alpha)\}$.

Note. This definition slightly extend the previous one[9)~11)] (i.e., TRS $R$ was said to be strongly depth-preserving if $Max\{|v| \mid v \in O_x(\beta)\} \leq Max\{|u| \mid u \in O_x(\alpha)\}$ and $\forall \alpha \to \beta \in R\ \forall x \in V(\alpha)\ \forall u, v \in O_x(\alpha)\ |u| = |v|$.)

**Example.** Let $R_1 = \{f(x,x) \to a,\ c \to g(c),\ g(x) \to f(x,x)\}$ and $R_2 = \{f(g(x), g(g(x))) \to h(g(x), g(c)),\ c \to g(c)\}$ where $x$ is a variable. Both $R_1$ and $R_2$ are strongly depth-preserving. (But $R_2$ was not strongly depth-preserving in the previous definition[11)].)

# 3 Root-E-Closed Condition

In this paper, we give a sufficient condition for the CR property of root-E-overlapping and strongly depth-preserving TRS's. We first give the definition of *root-E-closed* TRS's.

**Definition of ⟨ root-E-closed TRS ⟩**

A TRS $R$ is *root-E-closed* if $R$ is root-E-overlapping and satisfies the following condition (∗):

(∗) For all standard root-E-overlapping reduction sequences $\gamma : \sigma(\beta) \leftarrow \sigma(\alpha) \overset{\varepsilon-inv}{\longleftrightarrow^*} \sigma'(\alpha') \to \sigma'(\beta')$ for some rule $\alpha \to \beta$, $\alpha' \to \beta' \in R$ and mappings $\sigma, \sigma'$, there exists $\delta : \sigma(\beta) \longleftrightarrow^* \sigma'(\beta')$ such that the following conditions (i)–(ii) hold:

(i) $\delta \preceq \gamma$

(ii) At least one of the following conditions (1)–(3) holds.

(1) $|\delta|_{np} < |\gamma|_{np}$

(2) $K_{ldis}(\delta) \ll_s K_{ldis}(\gamma)$

(3) If $\delta[0,1] : \sigma(\beta) \overset{\varepsilon}{\longleftarrow} M$ ∗ for some $M$,then $\delta[1, |\delta|_p] : M \longleftrightarrow^* \sigma'(\beta')$ is $\varepsilon$-invariant.

Note that if $\delta \preceq \gamma$ and $H(\delta) < H(\gamma)$, then (2) holds since $K_{ldis}(\delta)\lesseqqgtr_s K_{ldis}(\gamma)$.

**Example 1.** TRS $R_3 = \{f(g(x), x) \to h(x,x),\ f(x,x) \to a,\ c \to g(c),\ h(x,x) \to a\}$ is strongly depth-preserving. There exists a root-E-overlapping sequence $\gamma : h(g^n(c), g^n(c)) \overset{\varepsilon}{\longleftarrow} f(g^{n+1}(c), g^n(c)) \longleftrightarrow f(g^{n+1}(c), g^{n+1}(c)) \overset{\varepsilon}{\longrightarrow} a$.

Then, there exists a sequence $\delta : h(g^n(c), g^n(c)) \to a \longleftrightarrow a \longleftrightarrow a$. By $H(\delta) < H(\gamma)$ and $|\delta|_p = |\gamma|_p$, $\delta \preceq \gamma$ holds. Since $|\delta|_{np} = 1 < |\gamma|_{np} = 3$ holds, (1) of the root-E-closed condition (ii) holds ((2) and (3) hold, too). Hence, $R_3$ satisfies (i) and (ii) of the root-E-closed condition.

**Example 2.** TRS $R_4 = \{f(x,x) \to h(x,x), f(g(x), x) \to a,\ c \to g(c),\ h(g(x), x) \to a\}$. There exists a root-E-overlapping sequence $\gamma : h(g^n(c), g^n(c)) \overset{\varepsilon}{\longleftarrow} f(g^n(c), g^n(c)) \longleftrightarrow f(g^{n+1}(c), g^n(c)) \overset{\varepsilon}{\longrightarrow} a$.

Then, there exists a sequence $\delta : h(g^n(c), g^n(c)) \longleftrightarrow h(g^n(c), g^n(c)) \to h(g^{n+1}(c), g^n(c)) \to a$. Then, $R_4$ satisfies (i) and (3) of the root-E-closed condition (ii).

# 4 Assertions

We use the following five assertions $S(n)$, $P(k)$, $P'(k)$, $Q(k)$ and $Q'(k)$ (where $n \geq 0$, $k \geq 0$) to prove that root-E-closed and strongly depth-preserving TRS $R$ is CR. Assertion $Q(k)$ ensures that TRS $R$ is CR.

**Assertion** $S(n)$

Let $\gamma : M \leftrightarrow\!\!\!\rightarrow^* N$ where $|\gamma|_p = n$.

Then $\exists\delta : M \leftrightarrow\!\!\!\rightarrow^* N$ such that the following conditions (i)–(ii) hold:

(i) There are no peaks in $\delta$

(ii) $\delta \preceq \gamma$

**Assertion** $P(k)$

Let $\gamma : M \overset{\varepsilon-inv}{\leftrightarrow\!\!\!\rightarrow^*} \sigma(\alpha) \rightarrow \sigma(\beta)$ for some rule $\alpha \rightarrow \beta \in R$ and mapping $\sigma$ where $H(\gamma) \leq k$.

Then, $\exists\delta : N \leftrightarrow\!\!\!\rightarrow^* \sigma(\beta)$ for some $N$ such that the following conditions (i) and (ii) hold:

(i) $M \rightarrow^* N$

(ii) Either $H(\delta) < H(\gamma)$ or $\delta$ is $\varepsilon$-invariant and $H(\delta) = H(\gamma)$.

**Assertion** $P'(k)$

Let $\gamma : M_0 \leftrightarrow\!\!\!\rightarrow M_1 \leftrightarrow\!\!\!\rightarrow M_2 \cdots \leftrightarrow\!\!\!\rightarrow M_n$ where $H(\gamma) \leq k$, the number of $\varepsilon$-reductions in $\gamma$ is $l (> 0)$ and each $\varepsilon$-reduction is $M_i \xrightarrow{\varepsilon} M_{i+1}$ for some $i$ $(0 \leq i < n)$. Let $M_{i_1} \xrightarrow{\varepsilon} M_{i_1+1}, \cdots, M_{i_l} \xrightarrow{\varepsilon} M_{i_l+1}$ be the $\varepsilon$-reductions of $\gamma$, $0 \leq i_1 < i_2 \cdots < i_l < n$. Then, there exist $i_j$ $(1 \leq j \leq l)$ and $\delta : N \leftrightarrow\!\!\!\rightarrow^* M_{i_j+1}$ for some $N$ such that the following conditions (i) and (ii) hold:

(i) $M_0 \rightarrow^* N$

(ii) Either $H(\delta) < H(\gamma[0, i_j+1])$ holds or $i_j = i_l$, $H(\delta) = H(\gamma[0, i_j+1])$ and $\delta$ is $\varepsilon$-invariant.

**Assertion** $Q(k)$[11]

Let $\gamma : M \leftrightarrow\!\!\!\rightarrow^* N$ where $H(\gamma) \leq k$. Then, $\exists\delta : M \leftrightarrow\!\!\!\rightarrow^* L \leftrightarrow\!\!\!\rightarrow^* N$ for some $L$ such that $H(\delta) \leq k$, $M \rightarrow^* L$ and $N \rightarrow^* L$.

**Assertion** $Q'(k)$[11]

Let $\gamma_i : M \leftrightarrow\!\!\!\rightarrow^* M_i$, where $H(\gamma_i) \leq k$, $1 \leq i \leq n$ and $n \geq 2$. Then, $\exists\ \delta : M \leftrightarrow\!\!\!\rightarrow^* N$ for some $N$ such that $H(\delta) \leq k$ and $\forall i$ $(1 \leq i \leq n)$ $M_i \rightarrow^* N$.

$S(n)$ says that for given sequence $\gamma$, there exists $\delta$ such that $\delta$ has no peaks and $\delta \preceq \gamma$. On the other hand, the previous definitions of $S(n)$ and $S'(n)$[11] said that it is possible to remove the outermost peak, so that by repeating this process, we can obtain sequence $\delta$ having no peaks. The current $S(n)$ describes and extends this result. $P(k)$ and $P'(k)$ are slightly simpler than those of 11) in the sense that the condition that $M \rightarrow^* N$ had to satisfy in the previous definitions is removed.

To prove these assertions, we use the following properties of *ldis*, *right*, *width*.

**Property 3**[11]

Let $\gamma : M_0 \leftrightarrow\!\!\!\rightarrow M_1 \cdots \leftrightarrow\!\!\!\rightarrow M_n$. Let $u \in MR(\gamma)$ and $\bar{\gamma} = \gamma[i,j]/u$ where $0 \leq i < j \leq n$. Let $\delta : L_i \leftrightarrow\!\!\!\rightarrow L_{i+1} \cdots \leftrightarrow\!\!\!\rightarrow L_j$ where $L_i = M_i/u, L_j = M_j/u$, $|\delta|_p = |\bar{\gamma}|_p$ and $H(\delta) \leq H(\bar{\gamma})$. Let $\gamma' = \gamma[\delta/\bar{\gamma}]$.

1. If $K_{ldis}(\delta) \lesseqqgtr_s K_{ldis}(\bar{\gamma})$, then $K_{ldis}(\gamma') \lesseqqgtr_s K_{ldis}(\gamma)$.
2. If $K_{right}(\delta) \lesseqqgtr_s K_{right}(\bar{\gamma})$, then $K_{right}(\gamma') \lesseqqgtr_s K_{right}(\gamma)$.
3. If $K_{ldis}(\delta) \lesseqqgtr_s K_{ldis}(\bar{\gamma})$ and $K_{right}(\delta) \lesseqqgtr_s K_{right}(\bar{\gamma})$, then $K_{width}(\gamma') \lesseqqgtr_s K_{width}(\gamma)$.

**Property 4**[11]

Let $\gamma$ be a parallel reduction sequence. Then, $K_{ldis}(net(\gamma)) \lesseqqgtr_s K_{ldis}(\gamma)$ and $K_{right}(net(\gamma)) \lesseqqgtr_s K_{right}(\gamma)$.

**Property 5**[11]

Let $\gamma : M_0 \leftrightarrow\!\!\!\rightarrow M_1 \cdots \leftrightarrow\!\!\!\rightarrow M_n$ and $\bar{\gamma} = \gamma[0,i]$ where $0 \leq i \leq n$. Let $\delta : L_0 \leftrightarrow\!\!\!\rightarrow L_1 \cdots \leftrightarrow\!\!\!\rightarrow L_j$ where $0 \leq j$, $L_j = M_i$ and $H(\delta) < H(\bar{\gamma})$. Let $\gamma' = \gamma[\delta/\bar{\gamma}]$. Then, $K_{ldis}(\gamma') \ll_w K_{ldis}(\gamma)$ and $K_{width}(\gamma') \ll_w K_{width}(\gamma)$.

**Property 6**

Let $\gamma : M_0 \leftrightarrow\!\!\!\rightarrow M_1 \cdots \leftrightarrow\!\!\!\rightarrow M_n$. Let $u \in MR(\gamma)$ and $\bar{\gamma} = \gamma[i,j]/u$ where $0 \leq i < j \leq n$. Let $\delta : L_i \leftrightarrow\!\!\!\rightarrow L_{i+1} \cdots \leftrightarrow\!\!\!\rightarrow L_j$ where $L_i = M_i/u, L_j = M_j/u$. Let $\gamma' = \gamma[\delta/\bar{\gamma}]$.

If $\delta \preceq \bar{\gamma}$, then $\gamma' \preceq \gamma$.

The proofs of assertions and properties are omitted.

By $Q(k)$ holds for all $k \geq 0$, we have the following Theorem 1.

**Theorem 1.** All the root-E-closed and strongly depth-preserving TRS's are CR.

Theorem 1 obviously implies that non-E-overlapping and strongly depth-preserving TRS's are CR. The proof of the non-E-overlapping case derived from this proof of Theorem 1 is more improved than the old one in 11).

## 5 Weight-Preserving TRS

By assigning a positive integer (we call *weight*) to each function symbol, we naturally extend the notion of depth to that of weight, and we show that the similar result to Theorem 1 is obtained for strongly weight-preserving TRS's.

**Definition of ⟨ strongly weight-preserving TRS $R$ ⟩**

For a weight-assigning function $w$, let $W_w(u, M)$ be the total of weights of function symbols occurring from the root to occurrence $u$ of term $M$. Formally, $W_w(\varepsilon, x) = 0$, $W_w(\varepsilon, fM_1 \cdots M_n) = w(f)$, $W_w(iu, fM_1 \cdots M_n) = w(f) + W_w(u, M_i)$ where $x \in X$, $f \in F$ and $M_i \in T$ $(1 \leq i \leq n)$. A TRS $R$ is *strongly $w$-weight-preserving* if $\forall \alpha \rightarrow \beta \in R$ $\forall x \in V(\alpha) \bigcap V(\beta)$ $Max\{W_w(v, \beta) \mid v \in O_x(\beta)\} \leq Min\{W_w(u, \alpha) \mid u \in O_x(\alpha)\}$.

A TRS $R$ is *strongly weight-preserving* if $\exists w : F \rightarrow \{1, 2, 3, \cdots\}$ $R$ is strongly $w$-weight-preserving.

**Example 3.** $R_5 = \{f(x, x) \rightarrow a,\ c \rightarrow h(c, g_1(g_2(c))),\ h(g_3(x), g_1(g_2(x))) \rightarrow f(x, h(x, g(x)))\}$. $R_5$ is strongly $w$-weight-preserving for weight-assigning function $w$ such that $w(g_3) = 2$ and the weights of the other symbols are 1. But $R_5$ is not strongly depth-preserving. For the weight-assigning function $w$, $W_w(1, f(g_3(x), x)) = 1+2 = 3$ and $W_w(2, f(g_3(x), x)) = 1$.

The problem of deciding for a given TRS whether it is strongly weight-preserving or not can reduce to that of solving integer programming.

**Example 4.** For a TRS $R_5$, we have an integer programming problem such that

$$\begin{aligned} k &\geq 1 \\ &\quad \text{for all } k \in K = \{f, g_1, g_2, g_3, h\} \\ h + g_3 &\geq f + h + g \\ h + g_1 + g_2 &\geq f + h + g \end{aligned}$$

These inequalities hold for weight-assigning function $w$ such that $w(g_3) = 2$ and $w(k) = 1$ for all $k \in K - \{g_3\}$. Thus, $R_5$ is strongly $w$-weight-preserving.

If TRS $R$ is strongly depth-preserving, then $R$ is strongly weight-preserving, since $R$ is strongly $w_1$-weight-preserving for the weight-assigning function $w_1$ such that $w_1(f) = 1$ for all $f \in F$.

For any strongly weight-preserving TRS $R$, we construct a strongly depth-preserving TRS $\bar{R}$ which can simulate reductions of $R$. For this purpose, we define a set of new function symbols $\bar{F}$ and a translation $\psi : F \rightarrow \bar{F}^*$ as follows:

$$\bar{F} = \{f_1, f_2, \cdots, f_k \mid f \in F, w(f) = k\}$$

where $arity(f_i) = 1,\ 1 \leq i < k$ and $arity(f_k) = arity(f)$

$$\psi(f) = f_1 \cdot f_2 \cdot \cdots \cdot f_k \text{ for } f \in F \text{ of } w(f) = k$$

Here, $(f_1 \cdot f_2 \cdot \cdots \cdot f_k)M_1 \cdots M_n = f_1(f_2 \cdots (f_k M_1 \cdots M_n) \cdots)$ for $M_1, \cdots, M_n \in T$.

Translation $\psi$ is extended to the translation: $T \rightarrow \bar{T}^*$ as follows:

$$\begin{aligned} \psi(x) &= x \text{ for } x \in X \\ \psi(fM_1 \cdots M_n) &= \psi(f)\psi(M_1) \cdots \psi(M_n) \end{aligned}$$

for $f \in F$, $M_1, \cdots, M_n \in T$. Here, $\bar{T}$ is the set of terms constructed from $X$ and $\bar{F}$.

Using this translation $\psi$, we define a new TRS $\bar{R}$ by

$$\bar{R} = \{\psi(\alpha) \rightarrow \psi(\beta) \mid \alpha \rightarrow \beta \in R\}$$

It is straightforward that $R$ is root-E-closed and strongly $w$-weight-preserving iff $\bar{R}$ is root-E-closed and strongly depth-preserving.

And $\bar{R}$ is CR iff $R$ is CR. Hence, we have the following result by Theorem 1.

**Theorem 2.** All the root-E-closed and strongly weight-preserving TRS's are CR.

Since non-E-overlapping TRS's are obviously root-E-closed TRS's, we have the following corollary.

**Corollary 1.** All the non-E-overlapping and strongly weight-preserving TRS's are CR.

**Example 5.** Since $R_5$ is non-E-overlapping and strongly weight-preserving, Corollary 1 ensures that $R_5$ is CR. TRS $R = \{\ f(x) \rightarrow g(h(x), x),\ g(x, x) \rightarrow a,\ b \rightarrow h(b)\}$[12] in Section 1 is non-E-overlapping and strongly weight-preserving for a weight-assigning function $w$ such that $w(f) = 2$ and the weight of the other symbols are 1, so that this $R$ is CR.

## 6 Sufficient Conditions

In this section, we first show some sufficient conditions for satisfying root-E-closed condition and we next show the following Lemma 2 which says a relation between root-E-overlapping and strongly root-overlapping conditions [1].

Henceforth we assume that $\gamma$ is a standard root-E-overlapping sequence such that $\gamma : \sigma(\beta) \leftarrow \sigma(\alpha) \overset{\varepsilon-inv}{\leftrightarrow\!\!\!\to^*} \sigma'(\alpha') \rightarrow \sigma'(\beta')$ for some $\alpha \rightarrow \beta$, $\alpha' \rightarrow \beta' \in R$, mappings $\sigma, \sigma' : X \rightarrow T$ and $\gamma = net(\gamma)$. Let $\gamma : M_0 \leftrightarrow\!\!\!\to M_1 \cdots \leftrightarrow\!\!\!\to M_{n_1}$.

**Definition of $\langle$ $\delta$ is covered in $\gamma$ by $k$-shift $\rangle$**

A reduction sequence $\delta : N_0(= M_0) \leftrightarrow\!\!\!\to N_1 \cdots \leftrightarrow\!\!\!\to N_{n_2}(= M_{n_1})$ is said to *be covered in $\gamma$ by $k(\geq 0)$-shift* if $\forall i (0 \leq i \leq n_2)$ $\exists j_1$ $H(N_i) \leq H(M_{j_1})$ $(0 \leq j_1 \leq i+k)$ and $\exists j_2$ $H(N_i) \leq H(M_{j_2})$ $(i+k \leq j_2 \leq n_1)$.

Note that $|\delta|_p \leq |\gamma|_p$ by this definition. Note that for the pairs $(\gamma, \delta)$ given in Examples 1 and 2, $\delta$ is covered in $\gamma$ by 0-shift.

**Lemma 1.** Let $\delta$ be covered in $\gamma$ by $k$-shift for some $k \geq 0$. Then, there exists $\delta'$ such that $net(\delta') = \delta$ and $\delta' \preceq \gamma$.

The proof of Lemma 1 is omitted.

**Condition I.** Standard root-E-overlapping sequence $\gamma$ satisfies Condition I if there exists $\delta : \sigma(\beta) \leftrightarrow\!\!\!\to P' \leftrightarrow\!\!\!\to^* Q' \leftrightarrow\!\!\!\to \sigma'(\beta')$ for some $P', Q'$ such that $H(P' \leftrightarrow\!\!\!\to^* Q') \leq min(max(H(\sigma(\alpha)), H(\sigma(\beta))), max(H(\sigma'(\alpha')), H(\sigma'(\beta'))))$ and $|\delta|_p < |\gamma|_p$.

**Theorem 3.** If every standard root-E-overlapping reduction sequence $\gamma$ satisfies Condition I, then TRS $R$ is root-E-closed.

**Lemma 2.** If TRS $R$ is strongly root-overlapping and satisfies the $(*)$ condition in the definition of root-E-closed TRS, then $R$ is root-E-overlapping or non-E-overlapping.

The proofs of theorems and lemma are omitted.

Using Condition I and Lemma 2, we can give a sufficient condition for CR, whose minor alteration can be used as a procedure to decide CR.

**Condition I'.** For any pair $(\alpha \rightarrow \beta, \alpha' \rightarrow \beta')$ which is strongly overlapping at $\varepsilon$, there exists $k > 0$ satisfying the following two conditions (1) and (2).

(1) for any standard root-E-overlapping sequence $\gamma : \sigma(\beta) \leftarrow \sigma(\alpha) \overset{\varepsilon-inv}{\leftrightarrow\!\!\!\to^*} \sigma'(\alpha') \rightarrow \sigma'(\beta')$, $|\gamma|_p \geq k$ holds.

(2) there exists $\delta : \beta \leftrightarrow\!\!\!\to P' \leftrightarrow\!\!\!\to^* Q' \leftrightarrow\!\!\!\to \beta'$ * for some $P', Q'$ * such that $H(P' \leftrightarrow\!\!\!\to^* Q') \leq min(max(H(\alpha), H(\beta)), max(H(\alpha'), H(\beta')))$ and $|\delta|_p < k$.

**Theorem 4.** If TRS $R$ satisfies Condition I', then TRS $R$ is root-E-closed.

A procedure to check Condition I' is given as follows. *

```
Procedure I'
For each pair p = (α → β, α' → β') of
    rules which is strongly overlapping
do begin
   if p is overlapping
   then begin
      if (2) of Condition I' does not hold for k = 2,
      then return 'False'
      end
   else begin
        let k = 3;
        while (1) of Condition I' holds
          do begin if (2) of Condition I' holds
                   then break,
                        i.e, exit from this loop
                   else let k = k + 1
             end;
        if (1) of Condition I' does not hold,
        then return 'False'
        end
   end;
return 'True'
```

In Procedure I', for each pair $p$ of rules which is strongly overlapping, we first check whether the pair $p$ is overlapping at $u$. If it is true, then (1) of Condition I' holds only for $k = 2$, so we check whether (2) of Condition I' holds for $k = 2$. If (2) holds, then we continue to check for the next strongly overlapping pair. Otherwise, Condition I' doesn't hold, so that 'False' is returned.

If the pair $p$ is not overlapping, then (1) of Condition I' holds for $k = 3$. So, we check whether (2) of Condition I' holds for $k = 3$. If it is true, then we continue to check for the next strongly overlapping pair. Otherwise, we let $k = k + 1$ and check whether root-E-overlapping holds for $k = 4$, i.e., whether there exists no standard root-E-overlapping sequence $\gamma$ of (1) with $|\gamma|_p = 3$. If (1) does not hold, then Condition I' does not hold, so that 'False' is returned. Otherwise, we check whether (2) holds for $k = 4$. We continue the same process as the above until (1) does not hold, or both (1) and (2) hold for some $k \geq 4$.

**Example 6.** To show CR of $R_3$ in Example 1, we can apply Procedure I' to $R_3$. Since strongly over-

[1] For a term $\alpha$, let $\bar{\alpha}$ be a linearization of $\alpha$, i.e., a linear term obtained from $\alpha$ by renaming any variables in $V(\alpha)$, e.g., if $\alpha = f(x, x)$, then $\bar{\alpha} = f(x, y)$ where $f \in F$ and $x, y \in X$. A pair of rewrite rules $\alpha_1 \rightarrow \beta_1$ and $\alpha_2 \rightarrow \beta_2$ is *strongly overlapping (at $u$)* if the pair of $\bar{\alpha}_1 \rightarrow \beta_1$ and $\bar{\alpha}_2 \rightarrow \beta_2$ is overlapping (at $u$). If $u = \varepsilon$, then the pair is *strongly root-overlapping*. TRS $R$ is strongly root-overlapping if all strongly overlapping pairs of rewrite rules are strongly root-overlapping.

lapping pair $(f(g(x),x) \to h(x,x),\ f(x,x) \to a)$ is not overlapping, let $k = 3$. (1) of Condition I' obviously holds for $k = 3$. So, consider (2) of Condition I'. There exists $h(x,x) \to a \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow a \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow a$, so that (2) of Condition I' holds for $k = 3$. Thus, Procedure I' returns "True", i.e., $R_3$ is CR.

However, note that Procedure I' is not a decision procedure, since it may not terminate when (1) holds, but (2) does not hold for any $k > 0$. And note that for a given $k$ we do not give how to check whether (1) of Condition I' holds in this paper. However, we strongly suggest that this problem is decidable.

**Condition II.** Standard root-E-overlapping sequence $\gamma$ satisfies Condition II if there exists a reduction sequence $\beta \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'} M \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow \beta'$ or $\beta \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow M \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'} \beta'$ where $\leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow$ is a one-step parallel reduction of the original TRS $R$ and $\leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'}$ is a parallel reduction of the TRS $R'$ where TRS $R' = \{\alpha/u \to \alpha'/u \mid u \in Min\{O_X(\alpha) \cup O_X(\alpha')\}\}$. Here, any variable appearing in rewrite rules in TRS $R'$ is regarded as a constant and no substitution for the variable is allowed in rewriting $\leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'}$.

**Theorem 5.** If every standard root-E-overlapping reduction sequence $\gamma$ satisfies Condition II, then TRS $R$ is root-E-closed.

The proof of Theorem 5 is omitted.
A decision procedure to check Condition II is given as follows. *

**Procedure II**
**For** each pair $p = (\alpha \to \beta,\ \alpha' \to \beta')$ of rules
    which is strongly overlapping
**do begin**
    **if** there exist no $M$ such that $\beta \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'} M \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow \beta'$
        or $\beta \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow M \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'} \beta'$
    **then return** 'False'
    **end**
**return** 'True'

In Procedure II, for each pair $p$ of rules which is strongly overlapping, we check whether there exist no $M$ such that $\beta \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'} M \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow \beta'$ or $\beta \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow M \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'} \beta'$. If it is true, then Condition II does not hold, so that 'False' is returned. Otherwise, continue to check for the next strongly overlapping pair.

**Example 7.** To show CR of $R_4$ in Example 2, we can apply Procedure II to $R_4$. For strongly overlapping pair $(f(g(x),x) \to h(x,x),\ f(x,x) \to a)$, there exists a reduction sequence $h(x,x) \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow_{R'} h(g(x),x) \leftarrow\!\!\!+\!\!\!\!+\!\!\!\rightarrow a$. Thus, Procedure II returns "True", i.e., $R_4$ is CR.

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## 参考文献


[1] Dershowitz,N. and Jouannaud,J.-P.: Rewrite Systems, Handbook of Theoretical Computer Science, Vol.B, ed. J.van Leeuwen, pp.243-320, North-Holland, Amsterdam(1990).

[2] Huet,G.: Confluent reductions:abstract properties and applications to term rewrite systems, J.ACM, 27, pp.797-821(1980).

[3] Matsuura,K., Oyamaguchi,M. and Ohta,Y.: On the E-overlapping property of nonlinear term rewriting systems, LA symposium(Winter)(1994).

[4] Ogawa,M. and Ono,S.: On the uniquely converging property of nonlinear term rewriting systems, Technical Report of IEICE, COMP 89-7, pp.61-70(1989).

[5] Oyamaguchi,M. and Ohta,Y.: On the Confluent Property of Right-Ground Term Rewriting Systems, Trans,IEICE Japan, J76-D-I, pp.39-45(1993).

[6] Toyama,Y. and Oyamaguchi,M.: Church- Rosser Property and Unique Normal Form Property of Non-Duplicating Term Rewriting Systems, LA Symposium(Winter) (1993) and 4th International Workshop on CTRS(1994).

[7] Ohta,Y. Oyamaguchi,M. and Toyama,Y.: On the Church-Rosser Property of Simple-Right-Linear TRS's, Trans. IEICE Japan, J78-D-I, pp.263-268(1995).

[8] Oyamaguchi,M. and Toyama,Y.: On the Church-Rosser property of E-overlapping and simple-right-linear TRS's, Technical Report of IEICE, COMP 94-29, pp.47-56(1994).

[9] Oyamaguchi,M. and Gomi,H.: Some Results on the CR property of Non-E-overlapping and Depth-Preserving TRS's, RIMS Kokyuroku 918, pp.150-159(1995).

[10] Gomi,H., Oyamaguchi,M. and Ohta,Y.: On the Church-Rosser Property of Non-E-overlapping and Weight-Preserving TRS's, RIMS Kokyuroku 950, pp.167-173(1996).

[11] Gomi,H., Oyamaguchi,M. and Ohta,Y.: On the Church-Rosser Property of Non-E-overlapping and Strongly Depth-preserving Term Rewriting Systems, to appear in Trans.IPS.Japan.

[12] Middeldorp,A., Okui,S. and Ida,T.: Lazy Narrowing: Strong Completeness and Eager Variable Elimination, Theoretical Computer Science 167(1,2),pp.95-130(1996).