書換え系の Perpetual 性と一様停止性

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無限書換え列を持つ項を，無限書換え列を持つ項にうつす書換えを perpetual という．本稿では，様々な λ 計算の体系を含む直交な条件付き高階書換系のフレームワークとして Orthogonal Conditinal Expression Reduction Systems を用いて，Lévy 順序に関し極小となる perpetual な戦略の存在を示し，これに基づき perpetual なリデックスの特徴付けを与える二つの定理を証明する．

その結果，従来個別に提案や予想をされてきた様々な λ 計算のヴァリエーションにおける一様停止性（弱停止性と強停止性の等価性）を統一的かつ簡単に示す．

Perpetuality and Uniform Normalization (Extended abstract)

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We define a perpetual one-step reduction strategy which enables one to construct *minimal* (w.r.t. Lévy's ordering $\lhd$ on reductions) infinite reductions in Orthogonal Conditional Expression Reduction Systems. We use this strategy to derive two characterizations of *perpetual* redexes, i.e., redexes whose contractions retain the existence of infinite reductions. These characterizations generalize existing related criteria for perpetuality of redexes.

We give a number of applications of our results, demonstrating their usefulness. In particular, we prove equivalence of weak and strong normalization (the *uniform normalization* property) for various restricted λ-calculi, which cannot be derived from previously known perpetuality criteria.

1 Introduction

The objective of this paper is to study sufficient conditions for ***uniform normalization***, UN, of a term in an orthogonal (first or higher-order) rewrite system, and for the UN property of the rewrite system itself. Here a term is UN if either it does not have a normal form, or if any reduction eventually terminates in a normal form; the rewrite system is UN if every term is UN. Interest in criteria for UN arises, for example, in the proofs of strong normalization of typed λ-calculi, as it relates to the work on reducing strong normalization proofs to proving weak normalization [Ned73, Klo80, dGr93, Kha94c, KW95]. Further, the question: 'Which classes of terms have the uniform normalization property' is posed in [BI94] in connection with finding UN solutions to fixed point equations, and with representability of partial recursive functions by UN-terms only, in the λ-calculus.[1] The UN property is clearly useful as then all strategies are normalizing, and in particular, there is more room for optimality (cf. [GlKh96]).

It is easy to see that a rewriting system is UN iff all of its redexes are ***perpetual***. These are redexes that reduce terms having an infinite reduction, which we call ∞-terms, to ∞-terms. Therefore, studying the UN property reduces to studying perpetuality of redexes. The latter has already been studied quite extensively in the literature. The classical results in this direction are ***Church's Theorem*** [CR36], stating that the λ_I-calculus is uniformly normalizing, and the ***Conservation Theorem*** of Barendregt et al [BBKV76, Bar84], stating that β_I-redexes are perpetual in the λ-calculus. Bergstra and Klop [BK82] give a sufficient and necessary criterion for perpetuality of β_K-redexes in every context. Klop [Klo80] generalized Church's Theorem to all non-erasing orthogonal Combinatory Reduction Systems (CRSs) by showing that the latter are UN, and Khasidashvili [Kha94c] generalized the Conservation Theorem to all orthogonal Expression Reduction Systems (ERSs) [Kha92], by proving that all non-erasing redexes are perpetual in orthogonal ERSs.

For orthogonal Term Rewriting Systems (OTRSs), a very powerful perpetuality criterion was obtained by Klop [Klo92] in terms of ***critical*** redexes. These are redexes that are not perpetual, i.e., reduce ∞-terms to strongly normalizable terms (*SN-terms*). Klop showed that any critical redex u must erase an argument possessing an infinite reduction. The later is not valid for higher-order rewrite systems, because substitutions (from the outside) into the arguments of u may occur during rewrite steps, which may turn an SN argument of u into an ∞-term. However, we show that a critical redex u in a term t must necessarily erase a ***potentially infinite*** argument, i.e., an argument that would become an ∞-(sub)term after a number of steps in t. From this, we derive a criterion, called ***safety***, of perpetuality of redexes in every context, similar to the perpetuality criterion of β_K-redexes in [BK82]. These are the main results of this paper, and we will demonstrate their usefulness in applications.

[1] The UN property is called in [BI94] ***strong normalization***, which we find inappropriate.

We obtain our results in the framework of ***Orthogonal Conditional Expression reduction Systems*** (OCERSs) [KO95]. CERS is a format for higher-order rewriting, or to be precise, second-order rewriting, which extends ERSs [Kha92] by allowing restrictions both on arguments of redexes and on the contexts in which the redexes can be contracted. Various interesting typed λ-calculi, including the simply typed λ-calculus and the system **F** [Bar92], can directly be encoded as OCERSs (see also [KOR93]); λ-calculi with specific reduction strategies (such as the call-by-value λ-calculus of Plotkin [Plo75]) can also be naturally encoded as OCERSs. ERSs are very close to the more familiar format of CRSs of Klop [Klo80], and we claim that all our results are valid for orthogonal CRSs as well (see [Raa96] for a detailed comparison of various forms of higher-order rewriting). However, using an example due to van Oostrom [Oos97], we will demonstrate that our results cannot be extended to higher-order rewriting systems where function variables can be bound [Wol93, Nip93, OR94], as they can exhibit pretty strange behaviour not characteristic of the λ-calculi.

In order to prove our perpetuality criteria, we first generalize the ***constricting*** or ***zoom-in*** perpetual strategy, independently discovered by Plaisted [Pla93], Sørensen [Sør95], Gramlich [Gra96], and Melliès [Mel96] (with small differences), from term rewriting and the λ-calculus to OCERSs. These strategies specify a construction of infinite reductions (whenever possible) such that all steps are performed in some smallest ∞-subterm. Our strategy is slightly more general than the above, and can be restricted so that the computation becomes constricting, and this allows for simple and concise proofs of our perpetuality criteria. We also show that constricting perpetual reductions are minimal w.r.t. Lévy's ordering on reductions in orthogonal rewriting systems [Lév80, HuLé91].

Despite the fact that our criteria are simple and intuitive, they appear to be strong tools in proving strong normalization from weak normalization in orthogonal (typed or type-free) rewrite systems. We will show that previously known related criteria [CR36, BBKV76, BK82, Klo80, Klo92, Kha94c] can be obtained as special cases. We will also derive the UN property for a number of variations of β-reductions [Plo75, dGr93, KW95], which cannot be derived from previously known perpetuality criteria, as immediate consequences of our criteria. Because of the space limitation, we omit all proofs, which can be found in [KO97].

2 Conditional Expression Reduction Systems

In this section, we recall the basic theory of orthogonal Conditional Expression Reduction Systems, OCERSs, as developed in [KO95], and some results concerning ***similarity*** of redexes in OERSs from [Kha94c]. CERSs extend ***Expression Reduction Systems*** [Kha92], a formalism of higher-order (rather, second-order) rewriting close to ***Combinatory Reduction Systems*** [Klo80]. We refer to [Raa96] for an extensive survey of the relationship among various formats of higher-order rewriting (such as [Klo80, Kha92, Wol93, Nip93, OR94, Oos94]) and refer to [Klo92] for a survey of results on conditional TRSs.

Terms in CERSs are built from the alphabet like in the first order case, except some symbols may have binding power. For example, a β-redex in the λ-calculus appears as $Ap(\lambda x\ t, s)$, where Ap is a function symbol of arity 2, and λ is an operator sign of arity $(1,1)$ and scope indicator (1). An integral $\int_s^t f(x)\,dx$ is represented as $\int x(s,t,f(x))$ using an operator sign $\int$ of arity $(1,3)$ and scope indicator (3). So $\int$ binds in the third argument.

Metaterms will be used to write rewrite rules. They are constructed from metavariables and meta-expressions for substitutions, called *metasubstitutions*. Instantiation of metavariables in metaterms yields terms. Metavariables play the role of variables in TRS rules, and function variables in HRS and HORS rules [Nip93, OR94]; unlike HRSs and HORSs, metavariables *cannot* be bound.

Definition 2.1 Let Σ be an *alphabet* comprising *variables*, denoted by x, y, z, and *symbols* (*signs*). A symbol σ can be either a *function symbol* (*simple operator*) having an *arity* $n \in N$, or an *operator sign* (*quantifier sign*) having *arity* $(m,n) \in N \times N$. In the latter case σ needs to be supplied with m *binding* variables $x_1,\ldots,x_m$ to form the *quantifier* (*compound operator*) $\sigma x_1 \ldots x_m$. If σ is an operator sign it has a *scope indicator* specifying in which of the n arguments it has binding power. *Terms* t, s, e, o are constructed from variables, function symbols and quantifiers in the usual first-order way. A predicate AT on terms specifies which terms are *admissible*.

Metaterms are constructed like terms, but also allowing as basic constructions *metavariables* A, B, ... and *metasubstitutions* $(t_1/x_1,\ldots,t_n/x_n)t_0$, where each t_i is an arbitrary metaterm and the x_i have a binding effect in t_0. An *assignment* θ maps each metavariable to some term. The application of θ to a metaterm t is written $t\theta$, and is obtained from t by replacing metavariables with their values under θ, and by replacing metasubstitutions $(t_1/x_1,\ldots,t_n/x_n)t_0$, in right to left order, with the result of substitution of terms $t_1,\ldots,t_n$ for free occurrences of $x_1,\ldots,x_n$ in t_0.

Definition 2.2 A rewrite rule is a pair of metaterms $r : t \to s$, such that t and s do not contain free variables. For an assignment θ satisfying the following condition (needed to avoid the capturing of free variables), the term $t\theta$ is called a *redex* and $s\theta$ its *contractum*:

- for a metavariable A, each free variable occurring in the term $A\theta$ is either bound in the θ-instance of each occurrence of A in the rule or in none of them.

The ERS-rewrite relation consists of pairs $C[t\theta] \to C[s\theta]$, where $C[]$ is a context (with one hole). If restrictions are put via an *admissibility* predicate on rules, assignments, and contexts, the rewrite relation is called *conditional*. [2]

[2] Conditional ERSs are called *context-sensitive* in [KO95]. Unlike our definition, *conditional* rewriting systems, usually, allow restrictions only on assignments, but not on contexts.

A *Conditional ERS* (CERS) R is a pair consisting of an alphabet and a set of rewrite rules, described above, both possibly restricted. Redexes that are instances of the same rule will be called *weakly similar*.

Notation 2.3 *We use a,b,c,d for constants, t,s,e,o for terms and metaterms, u,v,w for redexes, and P,Q for reductions. We write $s \subseteq t$ if s is a subterm of t. A one-step reduction in which a redex $u \subseteq t$ is contracted is written as $t \xrightarrow{u} s$. We write $P : t \twoheadrightarrow s$ if P denotes a reduction sequence from t to s, and write $P : t \twoheadrightarrow \infty$ if P is infinite. $P+Q$ denotes the concatenation of P and Q. $FV(t)$ denotes the set of free variables of t.*

2.1 Orthogonal CERSs

The idea of orthogonality is that contraction of a redex does not destroy other redexes (in whatever way), but rather leaves a number of their *residuals*. A prerequisite for the definition of residual is the notion of *descendant*, also called *trace*. It is a standard technique in higher-order rewriting [Klo80] to *decompose* or *refine* each rewrite step into two parts: a *TRS*-part replacing the left-hand side by the right-hand side, and a *substitution*-part evaluating the substitutions. To express substitution, we use the S-reduction rules

$$S^{n+1}x_1 \ldots x_n A_1 \ldots A_n A_0 \to (A_1/x_1,\ldots,A_n/x_n)A_0.$$

The descendant relation of a rewrite step can then be obtained by composing the descendant relation of the TRS-step and the descendant relations of the evaluation steps.

Let $t \xrightarrow{u} s$ in a CERS R, let $v \subseteq t$ be an admissible redex, and let $w \in s$ be a u-descendant of v. We call w a u-*residual* of v if: the patterns of u and v do not overlap; w is a redex weakly similar to v; and w is admissible.

Definition 2.4 A CERS is *orthogonal* (OCERS) if:

1. the left-hand side of a rule is not a metavariable;
2. the left-hand side of a rule does not contain metasubstitutions, and its metavariables contain those of the right-hand side;
3. in no term can admissible redex-patterns overlap;
4. all the descendants of an admissible redex u in a term t under the contraction of any other admissible redex $v \subseteq t$ are residuals of u.

As in the case of the λ-calculus [Bar84], for any co-initial (i.e., with the same initial term) reductions P and Q, one can define in OCERSs the notion of *residual of P under Q*, written P/Q, due to Lévy [Lév80]. We write $P \trianglelefteq Q$ if $P/Q = \emptyset$; P and Q are called *Lévy-equivalent* (written $P \approx_L Q$) if $P \trianglelefteq Q$ and $Q \trianglelefteq P$.

Theorem 2.5 ([KO95]) **(Strong Church-Rosser)** For any finite co-initial reductions P and Q in an OCERS, $P + Q/P \approx_L Q + P/Q$.

2.2 Similarity of redexes

We can write a CERS redex as $u = C[\overline{x_1}t_1, \ldots, \overline{x_n}t_n]$, where C is the pattern, $t_1, \ldots, t_n$ are the arguments, and $\overline{x_i} = \{x_{i_1}, \ldots, x_{i_{n_i}}\}$ is the subset of binding variables of C such that t_i is in the scope of an occurrence of each x_{i_j}, $i = 1, \ldots, n$. Let us call the maximal subsequence $j_1, \ldots, j_k$ of $1, \ldots, n$, such that $t_{j_1}, \ldots, t_{j_k}$ have u-descendants, the *main sequence* of u or the *u-main sequence*, call $t_{j_1}, \ldots, t_{j_k}$ (u-)*main arguments*, and call the remaining arguments (u)-*erased*. Now the similarity of redexes can be defined as follows [Kha94]: Let $u = C[\overline{x_1}t_1, \ldots, \overline{x_n}t_n]$ and $v = C[\overline{x_1}s_1, \ldots, \overline{x_n}s_n]$ be weakly similar. We call u and v *similar*, written $u \sim v$, if the main sequences of u and v coincide, and for any main argument t_i of u, $\overline{x_i} \cap FV(t_i) = \overline{x_i} \cap FV(s_i)$.

Lemma 2.6 ([Kha94c]) Let $u = C[\overline{x_1}t_1, \ldots, \overline{x_n}t_n]$ and $v = C[\overline{x_1}s_1, \ldots, \overline{x_n}s_n]$ be weakly similar redexes, and let for any main argument t_i of u, $\overline{x_i} \cap FV(t_i) = \overline{x_i} \cap FV(s_i)$. Then the main sequences of u and v coincide, and consequently, $u \sim v$. In particular, if $u = v\theta$, then $u \sim v$.

3 Two Characterizations of Critical Redexes

In this section, we generalize the constricting perpetual strategy [Pla93, Sør95, Gra96, Mel96] to OCERSs, and use it to derive two characterizations of critical redexes, generalizing the results of Bergstra and Klop [BK82, Klo92].

Let us first fix the terminology. Recall that a term t is called *weakly normalizable*, a *WN-term*, written $WN(t)$, if it is reducible to a *normal form*, i.e., a term without a redex. t is called *strongly normalizable*, an *SN-term*, written $SN(t)$, if it does not possess an infinite reduction. We call t an *∞-term*, $\infty(t)$, if $\neg SN(t)$. Clearly, for any term t, $SN(t) \Rightarrow WN(t)$. If the converse is also true, then we call t *uniformly normalizable*, or a *UN-term*, $UN(t)$. Correspondingly, a rewrite system R is called respectively WN, SN, or UN if so is any term in R.

Following [BK82, Klo92], we call a redex occurrence $u \subseteq t$ *perpetual* if $\infty(t) \Rightarrow \infty(s)$, where $t \xrightarrow{u} s$, and call u *critical* otherwise. A *perpetual strategy* is a function on terms which selects a perpetual redex in any ∞-term, and selects any redex (if any) otherwise [Bar84]. A redex (not an occurrence) is called *perpetual* iff its occurrence in any (admissible) context is perpetual.

Finally, let us recall the concept of *external* redexes due to Huet and Lévy [HuLé91]. These are redexes whose residuals or descendants can never occur in an argument of another redex. Any external redex is outermost, but not vice versa (e.g., the first a in $f(a, a)$, in $R = \{f(x, g(y)) \to y, a \to g(b)\}$, is outermost but not external).

Definition 3.1 (1) Let $\infty(t)$, in an OCERS, and let $s \subseteq t$ be a smallest subterm of t such that $\infty(s)$ (here s is smallest means that $SN(e)$ for every proper subterm $e \subseteq s$). We call s a *minimal perpetual subterm* of t, and call every external redex of s a *minimal perpetual redex* of t.

(2) Let F_m^∞ be a one-step strategy which in each ∞-term t contracts a minimal perpetual redex if $\infty(t)$, and contracts any redex otherwise. Then we call F_m^∞ a *minimal perpetual strategy*. We call F_m^∞ the *leftmost* minimal perpetual strategy if in each term it contracts the leftmost minimal perpetual redex. We call F_m^∞ *constricting* if for any F_m^∞-reduction $P : t_0 \xrightarrow{u_0} t_1 \xrightarrow{u_1} \ldots$ (i.e., constructed using F_m^∞), s_{i+1} is in the descendant of s_i, where s_i is the minimal perpetual subterm containing u_i; we then call P a *constricting* minimal perpetual reduction.

Theorem 3.2 Any minimal perpetual strategy F_m^∞ is a perpetual strategy, in OCERSs.

It is easy to show that the leftmost minimal perpetual strategy is constricting. This fact, and the next lemma, which follows from Lemma 2.6 and expresses a key property of similarity of redexes, are used essentially in our proof of Theorem 3.5.

Lemma 3.3 Let $t \xrightarrow{u} s$, in an OCERS. Let $o \subseteq t$ be either in an argument of u or not overlapping with u, and let $o' \subseteq s$ be a u-descendant of o. Then $o' = o\theta$ for a substitution θ. If moreover o is a redex, then so is o' and $o \sim o'$.

Definition 3.4 (1) Let $P : t_0 \xrightarrow{u_0} t_1 \xrightarrow{u_1} \ldots \xrightarrow{u_{k-1}} t_k$, and let $s_0, s_1, \ldots, s_k$ be a chain of descendants of s_0 in t_k (i.e, s_{i+1} is a u_i-descendant of s_i). Then, following [BK82], we call P *passive* w.r.t. $s_0, s_1, \ldots, s_k$ if the pattern of u_i does not overlap with s_i (s_i may be in an argument of u_i or be disjoint from u_i) for $0 \leq i < k$. In the latter case, we call s_k a *passive* descendant of s_0.

(2) Let t be a term in an OCERS, and let $s \subseteq t$. We call s a *potentially infinite* subterm of t if s has a passive descendant s' (along some reduction starting from t) s.t. $\infty(s')$ and $s' = s\theta$ for some substitution θ.

Theorem 3.5 Let $\infty(t)$ and let $t \xrightarrow{v} s$ be a critical step, in an OCERS. Then v erases a potentially infinite argument o (thus $\infty(o\theta)$ for some substitution θ).

Note that if the OCERS is an OTRSs, the potentially infinite argument o of $v \subseteq t$ is actually an ∞-term, implying Klop's perpetuality lemma [Klo92]. O'Donnell's [O'Do77] lemma, stating that any term from which an innermost reduction is normalizing is strongly normalizable, is an immediate consequence of Klop's Lemma.

Using Theorem 3.5, we can derive the following characterization of perpetuality of erasing redexes, similar to the perpetuality criterion of β_K-redexes in [BK82].

Definition 3.6 A substitution θ is SN iff $SN(x\theta)$ for every variable x. We call a redex u *safe* (respectively, *SN-safe*) if (either it is non-erasing, or else it is erasing and) for any (resp. SN-) substitution θ, if $u\theta$ erases an ∞-argument, then the contractum of $u\theta$ is an ∞-term.

Theorem 3.7 Safe redexes are perpetual in OCERSs.

Although we do not use it in the following, it is interesting to note that the constricting perpetual reductions are minimal w.r.t. Lévy's embedding relation $\trianglelefteq$, hence the name *minimal*.

The relations $\trianglelefteq, \approx_L$ and $/$ are extended to co-initial possibly infinite reductions N, N' as follows. $N \trianglelefteq N'$, or equivalently, $N/N' = \emptyset$ if, for any redex v contracted in N, say $N = N_1 + v + N_2$, $v/(N'/N_1) = \emptyset$; and $N \approx_L N'$ iff $N \trianglelefteq N'$ and $N' \trianglelefteq N$. Here, for any infinite P, $u/P = \emptyset$ if $u/P' = \emptyset$ for some finite initial part P' of P, and P/Q is only defined for finite Q.

Theorem 3.8 Let $P : t \twoheadrightarrow \infty$ be a constricting minimal perpetual reduction and let $Q : t \twoheadrightarrow \infty$ be any infinite reduction such that $Q \trianglelefteq P$, in an OCERS. Then $Q \approx_L P$.

4 Applications

4.1 The restricted orthogonal λ-calculi

Let us call *orthogonal restricted λ-calculi* (ORLC) the calculi that are obtained from the λ-calculus by restricting the β-rule (by some conditions on arguments and contexts) and that are orthogonal CERSs. Examples include the λ_I-calculus, the call-by-value λ-calculus [Plo75], as well as a large class of typed λ-calculi.

If R is an ORLC, more information can be extracted from our proofs of Theorem 3.5 and Theorem 3.7. Then, we have the following two corollaries, of which the second is a mere extension of Bergstra-Klop criterion [BK82].

Corollary 4.1 Let $\infty(t)$ and let $t \xrightarrow{v} s$ be a critical step, in an ORLC. Then v erases a potentially infinite non-variable argument o such that $\infty(o\theta)$ for some SN-substitution θ.

Corollary 4.2 Any SN-safe redex v, in an ORLC, is perpetual (in every admissible context).

Note that these corollaries are not valid for OCERSs in general: Let $R = \{\sigma x AB \rightarrow Sx\omega(A/x)B,\ E(x) \rightarrow a\}$ where $\omega = \lambda x.Ap(x,x)$. Then the step $\sigma x Ap(x,x)E(x) \rightarrow \sigma x Ap(x,x)a$ is SN-safe but critical since $\sigma x Ap(x,x)E(x) \rightarrow Sx\omega E(Ap(x,x)) \rightarrow E(Ap(\omega,\omega)) \rightarrow E(Ap(\omega,\omega)) \rightarrow \ldots$

4.2 Plotkin's call-by-value λ-calculus

Plotkin [Plo75] introduced the *call-by-value* λ-calculus, λ_V, which restricts the usual λ-calculus by allowing the contraction of redexes whose arguments are *values*, i.e., either abstractions $\lambda x.t$ or variables. Let the *lazy* call-by-value λ-calculus λ_{LV} be obtained from λ_V by allowing only call-by-value redexes that are not in the scope of a λ-occurrence (λ_{LV} is enough for computing values in λ_V, see Corollary 1 in [Plo75]). Then it follows from Corollaries 4.1, 4.2 that any λ_{LV}-redex is perpetual, hence λ_{LV} is UN. Indeed, let $v = (\lambda x.s)o$ be a λ_{LV}-redex. Then, if o is a variable, then it is immediate that v cannot be critical, and if o is an abstraction, any of its instances is an abstraction too, hence is a λ_{LV}-normal form. This is not surprising, however, as λ_{LV}-redexes are disjoint, and there is no duplication or erasure of (admissible) redexes.

4.3 De Groote's β_{IS}-reduction and Kfoury and Wells' $\star$-reduction

De Groote [dGr93] introduced β_S-reduction on λ-terms by the following rule: $\beta_S : (((\lambda x.M)N)O) \rightarrow ((\lambda x.(MO)N)$, where $x \notin FV(M)$. He proved that the β_{IS}-calculus is UN. Clearly, this is immediate from Theorem 3.5 as the β_S- and β_I-rules are non-erasing (note that these rules do not conflict because of the conditions on bound variables). Using this result, the author proves strong normalization of a number of typed λ-calculi.

Kfoury and Wells [KW95], for the same purpose, study a similar, what they call *structural*, rule $\gamma : (\lambda x.(\lambda y.A))B \rightarrow \lambda y.((\lambda x.A)B)$, where $y \notin FV(B)$, and study its combination with the β_I-rule, called the $\star$-reduction. Note that γ conflicts with β_I. However, in $\star$-reductions, β_I-steps are only allowed in terms in γ-normal form, therefore $\star$-reduction on λ-terms forms an orthogonal CERS, and the UN property of $\star$-reductions is a corollary of Theorem 3.5. Similarly to [dGr93], UN of of $\star$-reductions can be shown to imply β-SN of terms that are $\star$-WN.

4.4 Böhm and Intrigila's $\lambda - \delta_k$-calculus

Böhm and Intrigila [BI94] introduced the $\lambda - \delta_k$-calculus to study UN solutions to fixed point equations in the $\lambda\eta$-calculus. Since the K-redexes are the source of failure of the UN property in the $\lambda(\eta)$-calculus, they define a 'restricted K combinator' δ_K by the following rule: $\delta_K AB \rightarrow A$, where B can be instantiated to closed $\lambda - \delta_k$-normal forms. Theorem 3.5 implies the UN property of the $\lambda - \delta_k$-calculus only when the η-rule is dropped.

However, Sørensen shows that η-redexes are perpetual [Sør96], and we hope that our results can be generalized to weakly-orthogonal CERSs (and thus cover the η-rule since η-redexes are non-erasing) using van Oostrom and van Raamsdonk's technique for simulating $\beta\eta$ reductions with β-reductions [OR94].

5 Concluding Remarks

We have obtained two criteria for perpetuality of redexes in orthogonal CERSs, and demonstrated their usefulness in applications. We claim that our results are also valid for Klop's orthogonal *substructure* CRSs [KOR93].

However, they cannot be generalized (at least, directly) to orthogonal *Pattern Rewrite Systems* (OPRSs) [Nip93], as witnessed by the following example due to van Oostrom [Oos97]. It shows that already the Conservation Theorem fails for OPRSs:

Let $R = \{g(M.N.X(x.M(x),N)) \rightarrow_g X(x.I,\Omega),\ @(\lambda(x.M(x)),N) \rightarrow_\beta M(N)\}$ where $\Omega = @(\lambda(x.xx),\lambda(x.xx))$. Then $g(M.N.@(\lambda(x.M(x)),N)) \rightarrow_\beta g(M.N.M(N))$ is non-erasing but critical, since $g(M.N.M(N))$ is SN, while $g(M.N.@(\lambda(x.M(x)),N)) \rightarrow_g @(\lambda(x.I),\Omega) \rightarrow @(\lambda(x.I),\Omega) \rightarrow \ldots$.

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