3B-7 Simple Method of Eliminating Blurs Based on Lane and Bates Algorithm

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1. Introduction

The Lane and Bates (LB) blind deconvolution [1] is based on analysis of the zero-sheets of the *z*-transform of a given image. In our papers [2] we presented determinant conditions (DCs) that make it possible to find zeros of blurs convolved in the image, without carrying out complicated analysis of the zero-sheets of a given image.

In this paper we present an algorithm for finding blurs, which is a new tool for the LB method but different from the DCs. This algorithm can be extended to more realistic 8bits images.

2. Search Algorithm

We assume an ideal situation where the noise is minute enough to be neglected and the observed image $g(x,y)$ is given as the convolution $f(x,y)$ and $h(x,y)$, where $f(x,y)$ and $h(x,y)$ represent a true image and a blur image, respectively. The *z*-transform $G(u,v)$ of the given image function $g(x,y)$ of size $M\times N$ is written as

$$G(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}g(x,y)u^x v^y=F(u,v)H(u,v), \qquad (1)$$

where u and v are complex variables and $F(u,v)$ and $H(u,v)$ are respectively *z*-transforms of *f* and *h* such as

$$F(u,v)=\frac{1}{m'n'}\sum_{x=0}^{m'-1}\sum_{y=0}^{n'-1}f(x,y)u^x v^y, \qquad (2)$$

$$H(u,v)=\frac{1}{mn}\sum_{x=0}^{m-1}\sum_{y=0}^{n-1}h(x,y)u^x v^y. \qquad (3)$$

For a given *u*, the solutions *v* of the equation $G(u,v)=0$ are denoted by $\beta^u{}_i\ (i=1,2,\cdots,N';\ \ N'\le N-1)$. We denote the *i*-th solution of $H(u_j,v)=0$ with $\gamma^j{}_i$. Then, from (1) it follows that $\{\gamma^j\}\subset\{\beta^j\}$. From (3) we obtain n equations

$$\sum_{x=0}^{m-1}h(x,0)u_j{}^x=p_jc_0=(-1)^{n-1}p_j\prod_{i=1}^{n-1}\gamma^j{}_i$$

$$\sum_{x=0}^{m-1}h(x,1)u_j{}^x=p_jc_1=(-1)^{n-2}p_j\left(\prod_{i=2}^{n-1}\gamma^j{}_i+\prod_{i=1(i\ne 2)}^{n-1}\gamma^j{}_i+\cdots+\prod_{i=1}^{n-2}\gamma^j{}_i\right) \qquad (4)$$

$$\vdots$$

$$\sum_{x=0}^{m-1}h(x,n-1)u_j{}^x=p_jc_{n-1}=p_j,$$

which can be considered as a set of simultaneous equations for unknowns *h*(*x*,*y*) and p_j.

Our aim is to determine the blur functions $h(x,y)$ from (4). In order to obtain $h(x,y)$ uniquely, one has to evaluate (4) at *q* different values of u_j (i.e., $j=1,\cdots,q$), where *q* can be taken as the smallest integer that satisfies $q\ge mn/(n-1)$. Then, one can solve (4) with $j=1,\cdots,q$ for $h(x,y)$ and p_j. If eqs. (4) have nontrivial solutions for $h(x,y)$ and p_j's the zeros $\gamma^j{}_i\ \ (i=1,2,\cdots,\ n-1)$ are those of an $m\times n$ blur and the solutions $h(x,y)$ form the blur matrix. For all combinations of $n-1$ β^j's, one repeats this procedure. The number of the possible combinations is ${}_{N-1}C_{n-1}$ for a given $M\times N$ image. The search algorithm for finding $m\times n$ blur matrix $h(x,y)$ is summarized as follows:

1. Determine the smallest integer q that satisfies $q\le mn/(n-1)$.
2. Pick up combinations of $n-1$ zero-values β from $\beta_i\ (i=1,2,\cdots,N';\ \ N'\ge N-1)$ of the variable v of the *z*-transform of the given image.
3. Evaluate the $n-1$ zero-values β at q different points. Then, solve (4) for the blur $h(x,y)$ and unknown constants $p_j\ (j=1,\cdots,q)$.
4. If the set of equations give a non-trivial solution, then those β's are identified as the γ's that are solutions of $H(u_j,v)=0$.
5. Find nontrivial solutions by repeating the procedures 1. ~ 3. at most ${}_{N-1}C_{n-1}$ times until the condition 4. is met.
6. Restore the true image $f(x,y)$ by removing the blur using $F(u,v)=G(u,v)/H(u,v)$.

We have constructed the algorithm in terms of the zero-values of the variable *v*. Of course we may use the zero-values $\alpha_i\ \ (i=1,2,\cdots,\ m-1)$ of u instead of v in constructing the same algorithm. This version can simply be obtained by the replacements $N\to M,\ \ n\to m,\ \ \beta_i\to\alpha_i$, and $u_j\to\ v_j$

3. Test of the search algorithm

Fig. 1(a) shows a test image of the size 54×55 [3]. The test image was obtained from the true image of Fig. 1(d) by convolving blur images shown in Figs. (e), (f), and (g), of which sizes are respectively 2×2, 2×3, and 3×3. The image is regarded as 35bit image.

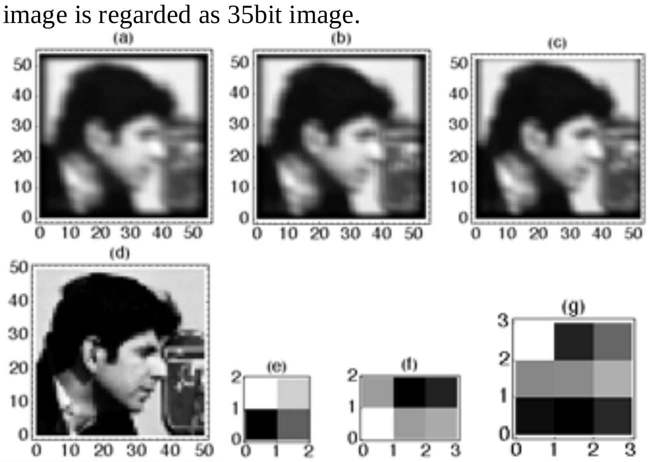

Fig. 1. (a): Test image obtained from a true image of (d) by convolving the three blurs of (e), (f), and (g). The image size is 44×45. (b): Image restored by removing the blur of (e). (c): Image restored by removing blurs of (e) and (f). (d): Image restored by removing blurs of (e), (f), and (g).

We test how the search algorithm works in finding each single blur convolved in the true image. To choose u_j's we

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imposed $|u_j|=1$, and changed only its phase. We used *Mathematica* to obtain the zeros β_i of $G(u,v)$ numerically. The parameter q is 4, 3, and 5 for each blur of Fig. 1(e), 1(f), and 1(g). Fig. 1(b) shows the image restored by removing the 2×2 blur that has been detected by searching the test image of Fig. 1(a) for the 2×2 blur. Fig. 1(c) shows the image restored by removing the 2×3 blur that has been detected by searching the image of Fig. 1(b) for the 2×3 blur. Finally, Fig. 1(d) shows the image restored by removing the 3×3 blur that has been detected by searching the image of Fig. 1(c) for 3×3 blur. In each search, the algorithm worked very well for blurs of different sizes. All calculations in this test have been performed using *Mathematica*. In this way we verified that the search algorithm works well in finding a single blur convolved in a given image.

By modifying the algorithm slightly, one can make it robust against a small error in the given image. In the case of the foregoing test image, the convolved image had 13-bit gray levels. Suppose we compress the gray levels to say, 256, as in the most common case, the eqs. (4) no longer hold exactly. In that case, instead of eqs. (4), we subtract the right-hand side from the left-hand side of the equations, and make the sum of the remaining difference squared. By varying the elements of blur h and the coefficient p_j so that we obtain the minimum of this sum, we can reproduce the true image approximately. Note that in doing so, one can eventually increase q, the number of set of equations to vary more widely u to obtain the optimum result. The true image (b) in Fig. 2 was obtained from the test image (a) in the same figure, whose gray levels are compressed to 256 from the test image used in the previous part. We chose 6 different values of u to obtain 12 equalities to obtain (b).

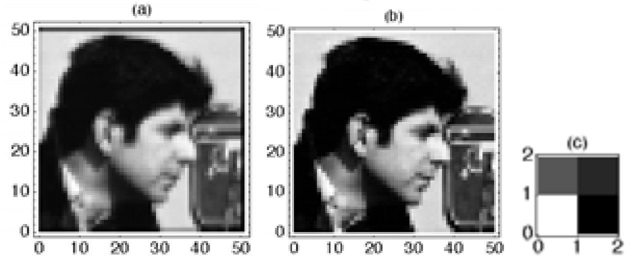

Fig. 2. (a): An 8-bit test image reduced from a 13-bit image of Fig. 1(c). (b): Image restored by removing 2×2blur.

4. Summary

A simple search algorithm for blur images convolved in a given image has been given in the form of solving simultaneous equations for a blur matrix elements. The algorithm is for finding a single blur of a specified size. Once a blur of a given size is detected, and the matrix elements are obtained, one can easily reconstruct the unblurred image. The advantage of this method is that it can be extended to more realistic images where the number of gray levels is reduced from the exact convolution.

This method will be very useful in making use of the LB blind deconvolution.

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