A Method For Rendering Water Flow Based On Measured Data

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1 Intrduction

This paper gives a description of how to rendering water flow in a river by means of flow nets based on measured data. The algorithmes of calculating the flow lines and same potential lines of the flow nets were showed.

Experiments were done using the measured data of Edo river in Tokyo. In order to control the density and distribution of the flow nets to represent the different velocity and direction of water flow distribution function $\mu = g(r)$ and density function $n = f(V)$ were introduced.

2 The water flow in rivers

Though water flow problem can be descriped in mathmatics equation, it is a huge time consumption task in digital computer systems to get the answer. Water flow can be rendered by the flow nets which are composed of flow lines and same potential lines [1]. We will try to get these lines from the measured data [2]

$P^k, O^k, W^k k = 1, 2, ..., N$ (Fig. 1)

instead of to resolve the tideous mathmatics equations.

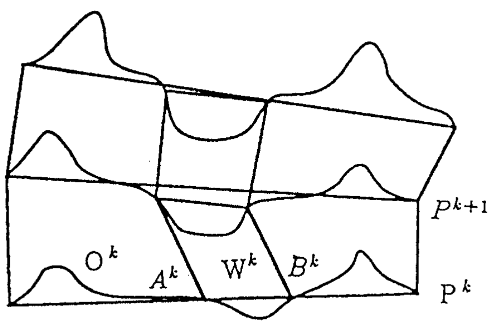

Fig. 1 The measured data of a rive

2.1 The primary interpolation method

the basic algorithm to calculate the flow lines from the the four points P^k, P^{k+1} is given below (Fig. 2).

Point $P_i = (X_i, Y_i)$ in the flow line can be computed by

$X_i = x_o + (d^k + i\Delta d)\cos(\theta^k + i\Delta\theta)$

$Y_i = y_o + (d^k + i\Delta d)\sin(\theta^k + i\Delta\theta)$ (i = 1, 2, ..., n)

$\Delta d = (d^{k+1} - d^k)/n, \qquad \Delta\theta = (\theta^{k+1} - \theta^k)/n.$

The n is the order of the interpolation, which is proportional to the velocity of the water flow. $O = (x_o, y_o)$ is the cross point decided by position data P^k and P^{k+1}, θ^k is the angle of P^k to X axis, $\theta^{k+1}$is that of P^{k+1}, $d^k = \overline{OP_l^k}$, $d^{k+1} = \overline{OP_l^{k+1}}$.

2.2 The density and distribution functions

The order of interpolate n is depended on the velocity of the water flow. It is used to manage the density of the flow nets. The most simple one is const, ie. $f(V) = const$. The more complicated one can be decided by the average velocity of the water flow in P^k and P^{k+1}. Surppose that the density function is proportional to the velocity V, then $n = f(V) = \kappa V^k$ κ is a const coeffincient.

The procedure used to compute the speed of water flow is given below:

1). Find the main cross points A^k, B^k

The main cross points of O^k and W^k are cross points found from the deepest point to the two side because there may be more than two cross points defined by data O^k and W^k.

$Y_A^k = Y_B^k = W^k$

Assume that Y_A^k is in between Y_r^k and Y_{r+1}^k, Y_B^k is in between Y_s^k and Y_{s+1}^k, ie., $Y_r^k <= Y_A^k <= Y_{r+1}^k$, $Y_s^k <= Y_B^k <= Y_{s+1}^k$, the X_A^k and X_B^k can be calculated by the follow expressions:

$$X_A^k = \frac{(X_{r+1}^k - X_r^k)(Y_A^k - Y_r^k)}{Y_{r+1}^k - Y_r^k} + X_r^k$$

$$X_B^k = \frac{(X_{s+1}^k - X_s^k)(Y_B^k - Y_s^k)}{Y_{s+1}^k - Y_s^k} + X_s^k$$

2). Compute the dynamic radius of the water R^k.

$$R^k = \sum_{i=r}^{s-1} \sqrt{(X_{i+1}^k - X_i^k)^2 + (Y_{i+1}^k - Y_i^k)^2}$$

$+\sqrt{(X_A^k - X_r^k)^2 + (Y_A^k - Y_r^k)^2} + \sqrt{(X_B^k - X_s^k)^2 + (Y_B^k - Y_s^k)^2}$.

3). Calculate the average bed slope I^k.

$I^k = (\min Y_i^{k+1} - \min Y_i^k)/\delta P$,

δP is the distance between P^k, P^{k+1}, it may be 500, or 1000, or calculated by the distance of their central line

4). Get the average velocity of the water

The average velocity of water in a river can be calculated by equation

$A^k = \frac{R^{k\frac{1}{6}}}{\lambda}\sqrt{R^k I^k}$,

λ is called Manning coefficient.

The distribution function $g(r)$ is used to control the distribution of the flow lines at the vertical direction, it is a function of curvature radius, ie., $g(r) = \frac{\mu}{r}$. μ is a coeffincient. The average curvature radius of the river in between P^k and P^{k+1} can be expressed by the ratio of the difference of their angles to the arc $\widehat{AB}$.

$r = (\Delta\theta^k)/\sum_{i=1}^{n-1}\sqrt{(x_{i+1} - xi)^2 + (y_{i+1} - y_i)^2}$

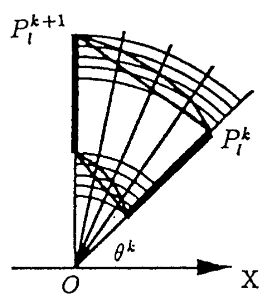

Fig. 2 The primary interpolation method

2.3 The flow nets between P^k and P^{k+1}

Algorithm to decide the flow lines can be expressed by the following steps:

1). Decide the density function and distribution function $f(V)$ and $g(r)$ by using the method described in the previous subsection.

2). The position and length of the flow lines

For $X^k = A^k + i * g(r)\frac{\overline{A^k B^k}}{f(V)}, X^{k+1} = A^{k+1} + i * g(r)\frac{\overline{A^{k+1}B^{k+1}}}{f(V)}, i = 1, 2, ..., n$ use the interpolate method in subsection 2.1 to calculate the interpolated points between A^k and A^{k+1}, B^k and B^{k+1}. The larger the r is, the shorter the distance between the adjacent lines.

3). The position and length of the same potential lines

The simplest logic to get the same potential line is calculating it at every flow line. We suppose that a same potential line crossed with a flow line in the middle of the flow line, the length of the same potential line is decided by the distance of two adjacent flow lines. The end point of the same potential line is just in the middle of them. The directions of the same potential lines are vertical to the flow lines according to their attributes.

3 The experiments

Fig. 3 shows an example of our test case. The measured data are from Edo river in Tokyo. k = 0.0, 0.5, ..., 59.5,

From the simulation results, we can find the bottle neck of a river, analysis the dangerous places, or even understand the flow quantity of the river.

For further research, the liquid fluid Stock's equation may be applied to get more precise result, and the discrete points may be used to draw a surface instead of lines.

4 References

[1] 日野 幹雄 水理学　丸善株式会社　平成3年9月

[2] 吉　鴻賓等 ”ＣＧ用の河川３次元モデルと描画アルゴリズムの研究” １９９３年三月電子情報通信学会

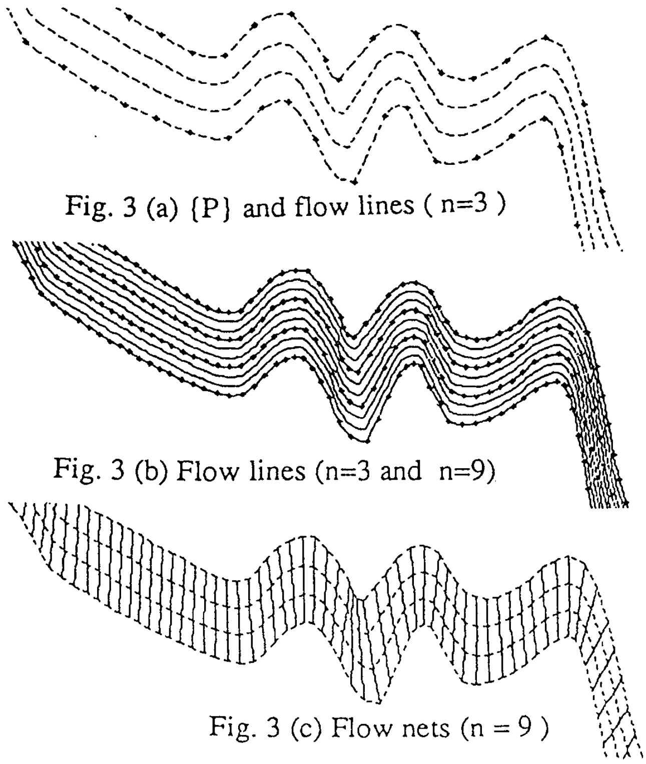

Fig. 3 (a) {P} and flow lines (n=3)

Fig. 3 (b) Flow lines (n=3 and n=9)

Fig. 3 (c) Flow nets (n = 9)