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Parametric Analysis of Optimal Static Load Balancing

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1. Introduction One of the attractive features of distributed computer systems is the capability to share processing of jobs in the event of overloads. This study focuses on the issue of balancing loads between nodes of a distributed system in response to imbalances in loads. We study optimal static load balancing problems in distributed computer systems that consist of a set of heterogeneous host computers connected by a single channel communications network such as LANs.

Tantawi and Towsley [1] considered an overall optimal policy which optimizes the overall mean job response time. They derived the conditions that the optimal solution should satisfy. We call the solution the *optimum*. In this paper, we study an individually optimal policy whereby job scheduling is determined so that every job may feel that its own expected response time is minimum, in the same model as the Tantawi and Towsley single c[illegible]s model. We show the conditions that the solution of the individually optimal policy satisfies. Then we study the characteristics of the overall and individually optimal policies and the effects of varying the system parameters on the performance variables of these policies.

2. Model Description We consider a distributed computer system model that consists of n nodes (host computers) connected by a single channel communications network as shown in Figure 1. Let us have the following notation.

- n Number of nodes
- ϕ_i External arrival rate to node i
- Φ Total external job arrival rate, i.e., $\Phi = \sum_{i=1}^{n} \phi_i$
- β_i Job processing rate (load) at node i, $\beta_i > 0$
- $\boldsymbol{\beta}$ $[\beta_1, \beta_2, \ldots, \beta_n]$
- λ Total traffic through the network
- $F_i(\beta_i)$ Expected node delay of jobs processed at node i (We assume that it is differentiable, increasing, and convex with respect to β_i)
- $G(\lambda)$ Expected communication delay of jobs (We assume that it is differentiable, nondecreasing, and convex with respect to λ)
- $T(\boldsymbol{\beta})$ Overall mean job response time (It is a convex function with respect to $\boldsymbol{\beta}$).

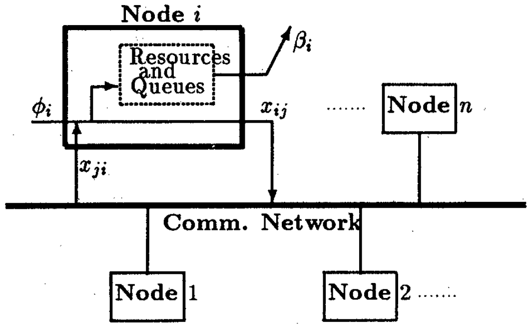

Figure 1. A model of a distributed computer system.

Jobs arrive at each node according to a time-invariant Poisson process. A job that arrives at node i (origin node) may either be processed at node i or be transferred to another node j (processing node). We classify nodes into either set of idle source (R_d), active source (R_a), or neutral (N).

3. Optimal Solution According to the results of Tantawi and Towsley [1], we have the following theorem for the overall optimal policy.

Theorem 1 *The optimal solution, $\boldsymbol{\beta}$ satisfies the relations*

$$\begin{aligned} f_i(\beta_i) &\geq \alpha + g(\lambda), \quad \beta_i = 0 \qquad (i \in R_d), \\ f_i(\beta_i) &= \alpha + g(\lambda), \quad 0 < \beta_i < \phi_i \qquad (i \in R_a), \\ \alpha &\leq f_i(\beta_i) \leq \alpha + g(\lambda), \quad \beta_i = \phi_i \ (i \in N), \\ \alpha &= f_i(\beta_i) \qquad (i \in S), \end{aligned}$$

where α is the Lagrange multiplier and

$$f_i(\beta_i) = d(\beta_i F_i(\beta_i))/d\beta_i, \quad g(\lambda) = d(\lambda G(\lambda))/d\lambda.$$

For the individually optimal policy, we have the following definition.

Definition: $\boldsymbol{\beta}$ is said to satisfy the equilibrium conditions for the individually optimal policy, if the following relations hold:

$$\begin{aligned} F_i(\beta_i) &\geq R + G(\lambda), \quad \beta_i = 0 \qquad (i \in R_d), \\ F_i(\beta_i) &= R + G(\lambda), \quad 0 < \beta_i < \phi_i \qquad (i \in R_a), \\ R &\leq F_i(\beta_i) \leq R + G(\lambda), \quad \beta_i = \phi_i \ (i \in N), \\ R &= F_i(\beta_i), \qquad (i \in S), \end{aligned}$$

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We call $\boldsymbol{\beta}$ the solution of the individually optimal policy if it satisfies the above equilibrium conditions. We also call such a $\boldsymbol{\beta}$ the *equilibrium*. For the individually optimal policy, we have the following theorem.

Theorem 2 *The individually optimal policy has a solution. That is, there exists one and only one $\boldsymbol{\beta}$ that satisfies the above equilibrium conditions.*

4. Parametric Analysis In this section, we study the effects of the system parameters on the behavior of the system in the optimum and in the equilibrium while node partition remains the same, respectively. We consider the communication time t, the node i processing time u_i, and the node i job arrival rate ϕ_i as system parameters. We use a vector $\mathbf{p}$ to denote $[t, u_1, u_2, \ldots, u_n, \phi_1, \phi_2, \ldots, \phi_n]$.

For the overall optimal policy, we have the following relations.

Theorem 3 *The following relations hold for the incremental node delay $\alpha(\mathbf{p})$ at sinks.*

$$
\begin{aligned}
\frac{\partial \alpha(\mathbf{p})}{\partial t} &< 0. \\
\frac{\partial \alpha(\mathbf{p})}{\partial u_i} &> 0, \quad i \in S \cup R_a, \\
&= 0, \quad i \in N \cup R_d. \\
\frac{\partial \alpha(\mathbf{p})}{\partial \phi_i} &> 0, \quad i \in S \cup R_a, \\
&= 0, \quad i \in N \cup R_d.
\end{aligned}
$$

Denote the overall mean job response time in the optimum under the overall optimal policy by $T(\mathbf{p})$. Then we have the following theorem.

Theorem 4 *The following relations hold for the overall mean job response time in the optimum, $T(\mathbf{p})$.*

$$
\begin{aligned}
\frac{\partial T(\mathbf{p})}{\partial t} &> 0, \\
\frac{\partial T(\mathbf{p})}{\partial u_i} &> 0, \quad i \in S \cup R_a \cup N, \\
&= 0, \quad i \in R_d.
\end{aligned}
$$

For the individually optimal policy, we have the following theorems.

Theorem 5 *The following relations hold for the expected node delay $R(\mathbf{p})$ at sinks.*

$$
\begin{aligned}
\frac{\partial R(\mathbf{p})}{\partial t} &< 0. \\
\frac{\partial R(\mathbf{p})}{\partial u_i} &> 0, \quad i \in S \cup R_a, \\
&= 0, \quad i \in N \cup R_d. \\
\frac{\partial R(\mathbf{p})}{\partial \phi_i} &> 0, \quad i \in S \cup R_a, \\
&= 0, \quad i \in N \cup R_d.
\end{aligned}
$$

Theorem 6 *The following relations hold for the overall mean job response time in the equilibrium, $T(\mathbf{p})$.*

$$
\begin{aligned}
\frac{\partial T(\mathbf{p})}{\partial u_i} &> 0, \quad i \in S \cup R_a \cup N, \\
&= 0, \quad i \in R_d. \\
\frac{\partial T(\mathbf{p})}{\partial \phi_i} &> 0, \quad i \in R_a \cup R_d \cup N.
\end{aligned}
$$

5. Numerical Examination We have examined numerically the effects of the system parameters in several examples of a distributed computer system that consists of four nodes connected via a single channel. Each node is modeled as a central-server model. We consider processor sharing M/G/1 model for the single channel communications network.

We have observed (results not presented here), in most cases, that the results of the numerical examination agree with our intuition and that the overall mean job response time of the equilibrium is close to that of the optimum. We also observed that, in most cases, the individually optimal policy is more sensitive to the system parameters than the overall optimal policy.

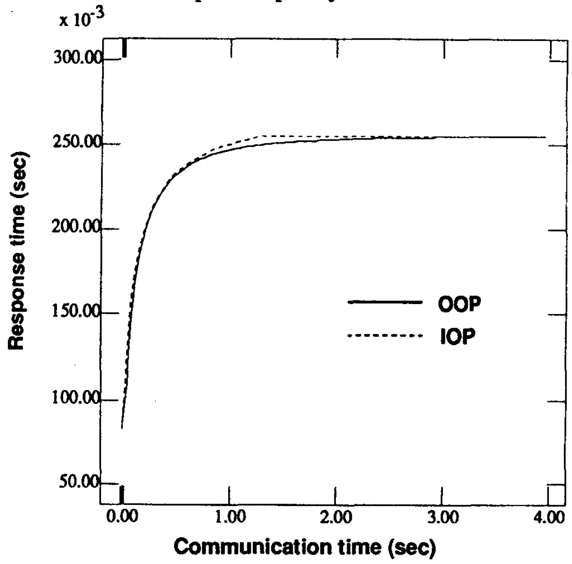

Figure 2. The overall mean job response times ($T(\boldsymbol{\beta})$) under the overall and individually optimal policies in the case where $\phi_1 = 80$, $\phi_2 = 7$, $\phi_3 = 7$, and $\phi_4 = 7.5$.

6. Conclusion We found that the two policies have very similar characteristics even though they are of the nature entirely different from each other. We observed that two policies can be implemented in the similar way. We observed, however, such anomalous phenomena that there are cases where in the equilibrium, the overall mean job response time decreases even though the communication time increases and that there are cases where in the optimum and in the equilibrium, the overall mean job response time decreases even though the job arrival rates increase.

Reference [1] Tantawi, A. N. and Towsley, D.: Optimal static load balancing in distributed computer systems, *J. ACM 32*, 2 (April 1985), 445-465.