Technical Note

Optimum Estimation of Local Fractal Dimension Based on the Blanket Method

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We present an algorithm for estimating optimum local fractal dimension of textured images based on the blanket method. The proposed method determines a range of the optimum number of blankets to obtain the optimal local fractal dimension for a small local window. The robustness of the proposed method to stably estimate the local fractal dimension using up to a 3×3 local window is confirmed through experimental evaluations. The local fractal dimension maps created from natural scenes are presented that demonstrate the capability of the proposed method to extract the local image features from natural images.

1. Introduction

Fractal dimension (FD) of an image surface corresponds to human intuition of image roughness [1)]. A rough surface has a higher FD value than a smoother one. There are many methods available to estimate the FD of an image surface [2)~7)].

For an image which contains textures, local fractal dimension (LFD) is appropriate for good image segmentation. Although several attempts have been made to extract the LFD [2),5)~7)], these methods are inadequate to provide a precise estimate of LFD because of the inherent limitation on the available size of the local window in these methods. For example, Keller, et al. [5)] estimated the LFD by using box counting with local window size of 32×32 for segmenting texture image composites, while Chaudhuri, et al. [6),7)] proposed the differential box counting (DBC) method and used a window size of 17×17. Though the blanket method by Peleg, et al. [3)] has the possibility to use a small local window, Cheong, et al. [8)] used a 16×16 window in the blanket method. Our previous work has shown that a small 3×3 window in the LFD estimation is preferable for extracting the local image features and segmenting natural images [9)~11)]. The LFD by using a small local window is expected to provide the local image features that can be used to achieve detailed segmentation.

This paper proposes the optimum estimation of the LFD for up to a small 3×3 local window based on the blanket method. Optimization of the number of blankets is required in the blanket method since the LFD estimate varies according to the number of blankets, especially for a small local window. For a natural texture image, we find that the difference between the LFD and the global FD (GFD) is minimum at a certain range of the number of blankets, and we define it as the optimum number of blankets.

We have evaluated the proposed optimum LFD estimation method on two types of image surfaces: 2D fractional Brownian motion (fBm) image surfaces and the texture image surfaces from a Brodatz album [12)] that provide various kinds of natural texture images. We have compared the optimum LFD estimation with the other FD estimation methods based on the comparisons in Ref. 7). Since we have optimized the estimation of the LFD based on the natural texture images, the proposed LFD can be generalized to other natural images containing textures. We have demonstrated that LFD maps for natural scenes can be used as local image features for image segmentation.

2. Overview of the Blanket Method

Peleg, et al. [3)] introduced the covering algorithm of an image surface $g(i,j)$ by using a blanket with top u_ϵ and bottom b_ϵ surfaces, and $g(i,j) = u_0(i,j) = b_0(i,j)$. If ϵ is the number of blankets, the area of the blanket $A(\epsilon)$ is computed by:

$$A(\epsilon) = \frac{\sum_{i,j} \Big(u_\epsilon(i,j) - b_\epsilon(i,j)\Big)}{2\epsilon}. \qquad (1)$$

Mandelbrot defined the behaviour of a fractal surface as $A(\epsilon) = F\epsilon^{2-D}$, where F is a constant and D is the FD of the surface. The FD value is estimated from the linear fit of $\log\{A(\epsilon)\}$ against $\log\{\epsilon\}$ with the scale of the blanket

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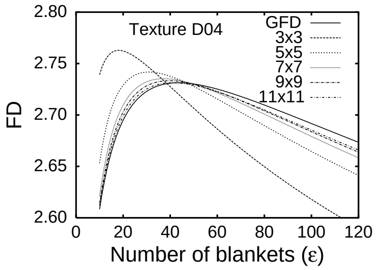

Fig. 1 Estimated LFDs for five sizes of local window and GFD versus ϵ for Brodatz texture D04.

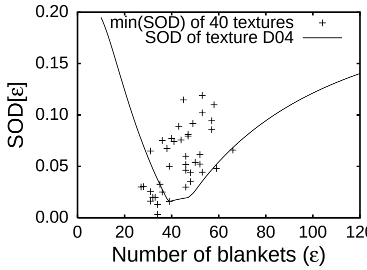

Fig. 2 Minimum SOD values for 40 Brodatz texture images and $SOD(\epsilon)$ for texture image D04.

ranging from $1 \sim \epsilon$, and the slope should be equal to $2 - D$.

3. Optimization of the Number of Blankets

The actual log-log plot of the blanket area $A(\epsilon)$ versus ϵ is a non-linear curve, especially for a small window. Thus, the estimated LFD varies according to ϵ. We have examined the behaviour of the estimated LFD for five sizes of local window (3×3, 5×5, 7×7, 9×9 and 11×11) when we changed ϵ from 10 to 120. We randomly chose 200 samples by using a certain size of local window from the Brodatz texture image and estimated each LFD with a certain ϵ, then calculated the average value of the LFDs.

Figure 1 shows the estimated LFD and GFD versus ϵ for the Brodatz texture image D04, in which the GFD uses a 256×256 window that means the entire image. It should be noted that the estimated LFD values for the five sizes of local window and the GFD value are similar at a certain range of the number of blankets. If a larger or smaller number of blankets is used, there is a large difference of FD values between the estimated LFD and the GFD.

We have evaluated the sum of the difference (SOD) between the LFD for five sizes of local window and the GFD:

$$SOD(\epsilon) = \sum_{k=1}^{5} \left| LFD_k(\epsilon) - GFD(\epsilon) \right|. \quad (2)$$

In this experiment, we have used 40 kinds of natural texture images from the Brodatz album and the minimum values of the SODs for those texture images are plotted in **Fig. 2**. The solid line in this figure represents the $SOD(\epsilon)$ of the texture image D04 which has the minimum value at $\epsilon = 39$. The minimum and maximum numbers of blankets from the 40 textures in Fig. 2 are 27 and 66, respectively. The average and the standard deviation of the number of blankets are 43.75 and 9.58, respectively. Regarding the average $\pm$ standard deviation as the optimal range, we have determined the range of the optimum number of blankets $34 \leq \epsilon \leq 53$, and have used the number of blankets 44 to calculate the optimal LFD in our algorithm.

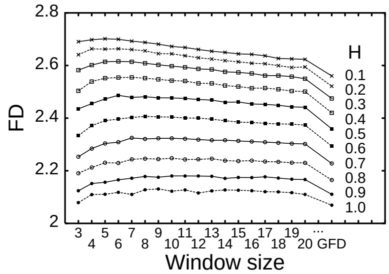

Fig. 3 Estimated LFDs of 2D-fBm images ($\epsilon = 44$).

4. Experimental Evaluations

4.1 Evaluations on 2D-fBm Images

In order to evaluate FD estimation by using the optimum number of blankets quantitatively, we have employed the artificial 2D-fBm surfaces generated by using the successive random addition algorithm introduced by Voss [4)]. The 2D-fBm images are generated for ten kinds of Hurst parameter values, i.e., $H = 0.1, 0.2, 0.3, \ldots, 1.0$ that correspond to FD $= 2.9, 2.8, 2.7, \ldots, 2.0$, respectively. For each Hurst parameter value, we generated 10 images of size 513×513 by using different seeds for the random generator, estimated the LFDs of 20 samples that were randomly chosen from each of the generated image by using a certain window size, and took the average of the estimated 200 LFDs. The experimental results in **Fig. 3** show that the optimum number of blankets produces LFD estimates which are relatively stable over the wide range of window sizes. Moreover, the results demonstrate the FD values within the theoretical range ($2.0 \sim 3.0$), though the estimated

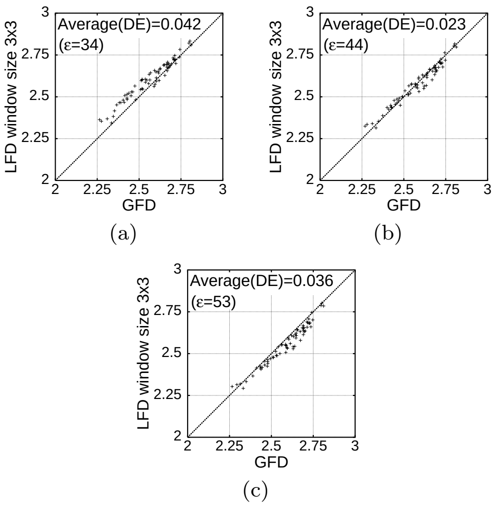

Fig. 4 Scatterplot of LFD versus GFD for 70 Brodatz texture images using (a) $\epsilon = 34$, (b) $\epsilon = 44$ and (c) $\epsilon = 53$.

LFD values show a slight shrinkage toward the FD value of 2.4 from the ideal one.

4.2 Evaluations on Brodatz Textures

Figure 4 shows the scatterplots of the LFD by using a 3×3 local window versus the GFD for 70 Brodatz texture images. Each point in the plot represents the relationship between the average LFD values from 200 local windows randomly selected from one texture image and the GFD of the same image. The FDs in Figs. 4 (a)–(c) are estimated by using the number of blankets $\epsilon = 34$, 44 and 53, respectively.

Let DE denote the difference error between the GFD and LFD. Then the average DE values for Figs. 4 (a)–(c) are 0.042, 0.023 and 0.036, respectively, the standard deviations of the DEs are 0.029, 0.018 and 0.024, respectively, and the average error rates of DE to the GFD are 1.68%, 0.90% and 1.37%, respectively. The number of blankets 44 provides the smallest average and standard deviation of DE values, and demonstrates the best LFD estimation.

4.3 Comparisons of FD Estimations

In order to demonstrate the reliability of the proposed method, we have compared the estimated FD obtained using the proposed algorithm with other algorithms based on the comparisons in Ref. 7) as shown in **Table 1**. The LFD estimations for 12 Brodatz texture images by the proposed method using a 3×3 window and $\epsilon = 44$ were compared with the estimated GFD using the DBC algorithm of Sarkar[7], with the estimated GFD of Peleg[3], and with the GFD of Keller[5]. The average differences in the FD estimated by the proposed algorithm and the one estimated by each of the above three algorithms are 4.2%, 2.4% and 1.9%, respectively, and the maximum differences between them are 11.0%, 5.0% and 5.8%, respectively. These results represent good agreement between the proposed algorithm and each of the above three algorithms. Thus, the proposed algorithm can produce a relatively correct LFD for various types of natural images.

Table 1 Comparisons of the estimated FDs for 12 Brodatz texture images.

Texture	FD estimated in Ref. 7)			Proposed
	DBC	Peleg	Keller	LFD $\pm \frac{1}{2}$SD
D03	2.60	2.69	2.63	2.65 ± 0.08
D04	2.66	2.72	2.68	2.68 ± 0.04
D05	2.45	2.52	2.57	2.58 ± 0.06
D09	2.59	2.65	2.65	2.73 ± 0.03
D24	2.45	2.59	2.57	2.72 ± 0.05
D28	2.55	2.61	2.62	2.54 ± 0.07
D33	2.23	2.34	2.36	2.43 ± 0.11
D54	2.39	2.53	2.51	2.55 ± 0.08
D55	2.48	2.60	2.59	2.56 ± 0.06
D68	2.52	2.63	2.60	2.57 ± 0.08
D84	2.60	2.68	2.65	2.60 ± 0.06
D92	2.50	2.59	2.59	2.63 ± 0.04

5. Creation of LFD Maps

We have created LFD maps for natural scenes by using the proposed optimum LFD estimation. Each position (i, j) in the LFD map shows the LFD value estimated from a 3×3 local window centered at (i, j) in the original image. **Figures 5** (a) and (b) are the original images of size 400×400 pixels with 256 gray levels. These images contain various kinds of textures with different kinds of roughness which are used to demonstrate the ability of the proposed algorithm to extract the local image features. The LFD maps of images in Figs. 5 (a) and (b) are shown in Figs. 5 (c) and (d), respectively, and the frequency distributions of the LFD maps with FD interval of 0.01 are shown in Figs. 5 (e) and (f), respectively. The bright level in the LFD map represents a higher value of LFD than the dark level.

The LFD maps in Figs. 5 (c) and (d) show homogeneous LFDs for smooth regions (the sky) and fine-texture regions (the trees and walls) which are suitable for a similarity measure in image segmentation. The other merit of using a small local window is that it preserves details of local image features especially in the vicinity of strong-edge regions (the structure of the

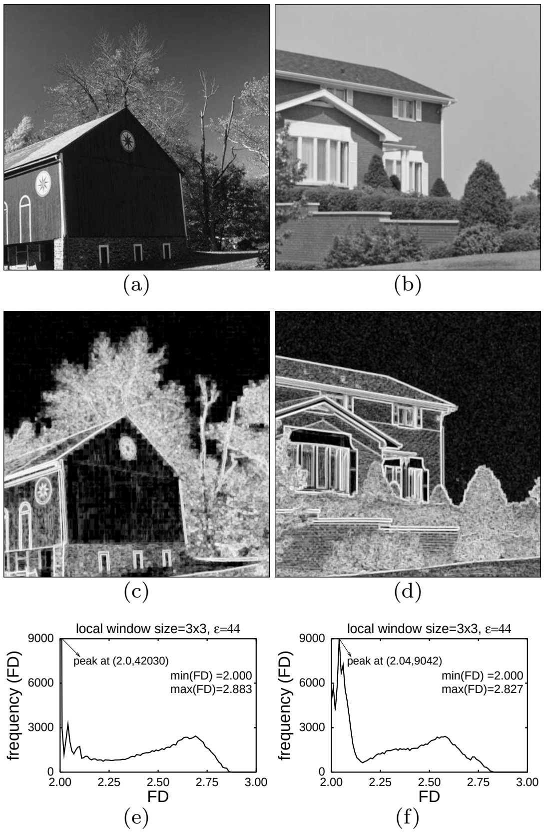

Fig. 5 (a) and (b) are original images, (c) and (d) are the LFD maps of (a) and (b), respectively, (e) and (f) are the frequency distributions of (c) and (d), respectively.

house) that can be used as a discontinuity measure in image segmentation. It is obvious from Figs. 5 (e) and (f) that the optimized estimation method provides the LFD maps with frequency distributions within the proper range of the FD values (2.0 $\sim$ 3.0).

6. Conclusions

In this paper we have described the optimization of the LFD estimation based on the blanket method. We have proposed the range of the optimum number of blankets to estimate the precise LFD for a small local window that has been determined based on the difference between the LFD and the GFD. The experimental evaluations have demonstrated that the proposed estimation method stably provides a relatively correct LFD for various types of images even with a 3×3 small window, and it also provides LFD maps for natural images.

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