

Balanced (C_5, C_{12}) -Foil Designs and Related Designs

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1. Balanced (C_5, C_{12}) -Foil Designs

Let K_n denote the complete graph of n vertices. Let C_5 and C_{12} be the 5-cycle and the 12-cycle, respectively. The (C_5, C_{12}) - $2t$ -foil is a graph of t edge-disjoint C_5 's and t edge-disjoint C_{12} 's with a common vertex. When K_n is decomposed into edge-disjoint sum of (C_5, C_{12}) - $2t$ -foils and every vertex of K_n appears in the same number of (C_5, C_{12}) - $2t$ -foils, we say that K_n has a balanced (C_5, C_{12}) - $2t$ -foil decomposition. This decomposition is to be known as a balanced (C_5, C_{12}) - $2t$ -foil design.

Theorem 1. K_n has a balanced (C_5, C_{12}) - $2t$ -foil design if and only if $n \equiv 1 \pmod{34t}$.

Example 1.1. Balanced (C_5, C_{12}) -2-foil design of K_{35} . Starter :

$\{(35, 1, 16, 32, 14), (35, 5, 8, 18, 26, 13, 20, 11, 23, 21, 10, 4)\}$.

Example 1.2. Balanced (C_5, C_{12}) -4-foil design of K_{69} . Starter :

$\{(69, 2, 32, 63, 27), (69, 9, 14, 34, 49, 24, 37, 61, 43, 40, 18, 7), (69, 1, 30, 62, 28), (69, 10, 16, 35, 51, 25, 39, 22, 45, 41, 20, 8)\}$.

Example 1.3. Balanced (C_5, C_{12}) -6-foil design of K_{103} . Starter :

$\{(103, 3, 48, 94, 40), (103, 13, 20, 50, 72, 35, 54, 90, 63, 59, 26, 10), (103, 2, 46, 93, 41), (103, 14, 22, 51, 74, 36, 56, 91, 65, 60, 28, 11), (103, 1, 44, 92, 42), (103, 15, 24, 52, 76, 37, 58, 33, 67, 61, 30, 12)\}$.

Example 1.4. Balanced (C_5, C_{12}) -8-foil design of K_{137} . Starter :

$\{(137, 4, 64, 125, 53), (137, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13), (137, 3, 62, 124, 54), (137, 18, 28, 67, 97, 47, 73, 120, 85, 79, 36, 14), (137, 2, 60, 123, 55), (137, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15), (137, 1, 58, 122, 56), (137, 20, 32, 69, 101, 49, 77, 44, 89, 81, 40, 16)\}$.

2. Related Designs

Theorem 2. K_n has a balanced C_{17} - t -foil design if and only if $n \equiv 1 \pmod{34t}$.

Example 2.1. Balanced C_{17} design of K_{35} .

Starter : $\{(35, 1, 16, 32, 14, 19, 5, 8, 18, 26, 13, 20, 11, 23, 21, 10, 4)\}$.

Example 2.2. Balanced C_{17} -2-foil design of K_{69} . Starter :

$\{(69, 2, 32, 63, 27, 36, 9, 14, 34, 49, 24, 37, 61, 43, 40, 18, 7), (69, 1, 30, 62, 28, 38, 10, 16, 35, 51, 25, 39, 22, 45, 41, 20, 8)\}$.

Example 2.3. Balanced C_{17} -3-foil design of K_{103} . Starter :

$\{(103, 3, 48, 94, 40, 53, 13, 20, 50, 72, 35, 54, 90, 63, 59, 26, 10), (103, 2, 46, 93, 41, 55, 14, 22, 51, 74, 36, 56, 91, 65, 60, 28, 11), (103, 1, 44, 92, 42, 57, 15, 24, 52, 76, 37, 58, 33, 67, 61, 30, 12)\}$.

Example 2.4. Balanced C_{17} -4-foil design of K_{137} . Starter :

$\{(137, 4, 64, 125, 53, 70, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13), (137, 3, 62, 124, 54, 72, 18, 28, 67, 97, 47, 73, 120, 85, 79, 36, 14), (137, 2, 60, 123, 55, 74, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15), (137, 1, 58, 122, 56, 76, 20, 32, 69, 101, 49, 77, 44, 89, 81, 40, 16)\}$.

Theorem 3. K_n has a balanced (C_{10}, C_{24}) - $2t$ -foil design if and only if $n \equiv 1 \pmod{68t}$.

Example 3.1. Balanced (C_{10}, C_{24}) -2-foil design of K_{69} . Starter :

$\{(69, 2, 32, 63, 27, 55, 28, 62, 30, 1), (69, 9, 14, 34, 49, 24, 39, 61, 43, 40, 18, 7, 15, 8, 20, 41, 45, 22, 39, 25, 51, 35, 16, 10)\}$.

Example 3.2. Balanced (C_{10}, C_{24}) -4-foil design of K_{137} . Starter :

$\{(137, 4, 64, 125, 53, 107, 54, 124, 62, 3), (137, 2, 60, 123, 55, 111, 56, 122, 58, 1), (137, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13, 27, 14, 36, 79, 85, 120, 73, 47, 97, 67, 28, 18), (137, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15, 31, 16, 40, 81, 89, 44, 77, 49, 101, 69, 32, 20)\}$.

Example 3.3. Balanced (C_{10}, C_{24}) -6-foil design of K_{205} . Starter :

$\{(205, 6, 96, 187, 79, 159, 80, 186, 94, 5), (205, 4, 92, 185, 81, 163, 82, 184, 90, 3), (205, 2, 88, 183, 83, 167, 84, 182, 86, 1), (205, 25, 38, 98, 141, 68, 105, 177, 123, 116, 50, 19, 39, 20, 52, 117, 125, 178, 107, 69, 143, 99, 40, 26), (205, 27, 42, 100, 145, 70, 109, 179, 127, 118, 54, 21, 43, 22, 56, 119, 129, 180, 111, 71, 147, 101, 44, 28), (205, 29, 46, 102, 149, 72, 113, 181, 131, 120, 58, 23, 47, 24, 60, 121, 133, 66, 115, 73, 151, 103, 48, 30)\}$.

Example 3.4. Balanced (C_{10}, C_{24}) -8-foil design of K_{273} . Starter :

$\{(273, 8, 128, 249, 105, 211, 106, 248, 126, 7), (273, 6, 124, 247, 107, 215, 108, 246, 122, 5), (273, 4, 120, 245, 109, 219, 110, 244, 118, 3), (273, 2, 116, 243, 111, 223, 112, 242, 114, 1), (273, 33, 50, 130, 187, 90, 139, 235, 163, 154, 66, 25, 51, 26, 68, 155, 165, 236, 141, 91, 189, 131, 52, 34), (273, 35, 54, 132, 191, 92, 143, 237, 167, 156, 70, 27, 55, 28, 72, 157, 169, 238, 145, 93, 193, 133, 56, 36), (273, 37, 58, 134, 195, 94, 147, 239, 171, 158, 74, 29, 59, 30, 76, 159, 173, 240, 149, 95, 197, 135, 60, 38), (273, 39, 62, 136, 199, 96, 151, 241, 175, 160, 78, 31, 63, 32, 80,$

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161, 177, 88, 153, 97, 201, 137, 64, 40)}.

Theorem 4. K_n has a balanced C_{34} - t -foil design if and only if $n \equiv 1 \pmod{68t}$.

Example 4.1. Balanced C_{34} design of K_{69} .

Starter : $\{(69, 2, 32, 63, 27, 36, 9, 14, 34, 49, 24, 37, 61, 43, 40, 18, 7, 15, 8, 20, 41, 45, 22, 39, 25, 51, 35, 16, 10, 38, 28, 62, 30, 1)\}$.

Example 4.2. Balanced C_{34} -2-foil design of K_{137} . Starter :

$\{(137, 4, 64, 125, 53, 70, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13, 27, 14, 36, 79, 85, 120, 73, 47, 97, 67, 28, 18, 72, 54, 124, 62, 3), (137, 2, 60, 123, 55, 74, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15, 31, 16, 40, 81, 89, 44, 77, 49, 101, 69, 32, 20, 76, 56, 122, 58, 1)\}$.

Example 4.3. Balanced C_{34} -3-foil design of K_{205} . Starter :

$\{(205, 6, 96, 187, 79, 104, 25, 38, 98, 141, 68, 105, 177, 123, 116, 50, 19, 39, 20, 52, 117, 125, 178, 107, 69, 143, 99, 40, 26, 106, 80, 186, 94, 5), (205, 4, 92, 185, 81, 108, 27, 42, 100, 145, 70, 109, 179, 127, 118, 54, 21, 43, 22, 56, 119, 129, 180, 111, 71, 147, 101, 44, 28, 110, 82, 184, 90, 3), (205, 2, 88, 183, 83, 112, 29, 46, 102, 149, 72, 113, 181, 131, 120, 58, 23, 47, 24, 60, 121, 133, 66, 115, 73, 151, 103, 48, 30, 114, 84, 182, 86, 1)\}$.

Example 4.4. Balanced C_{34} -4-foil design of K_{273} . Starter :

$\{(273, 8, 128, 249, 105, 138, 33, 50, 130, 187, 90, 139, 235, 163, 154, 66, 25, 51, 26, 68, 155, 165, 236, 141, 91, 189, 131, 52, 34, 140, 106, 248, 126, 7), (273, 6, 124, 247, 107, 142, 35, 54, 132, 191, 92, 143, 237, 167, 156, 70, 27, 55, 28, 72, 157, 169, 238, 145, 93, 193, 133, 56, 36, 144, 108, 246, 122, 5), (273, 4, 120, 245, 109, 146, 37, 58, 134, 195, 94, 147, 239, 171, 158, 74, 29, 59, 30, 76, 159, 173, 240, 149, 95, 197, 135, 60, 38, 148, 110, 244, 118, 3), (273, 2, 116, 243, 111, 150, 39, 62, 136, 199, 96, 151, 241, 175, 160, 78, 31, 63, 32, 80, 161, 177, 88, 153, 97, 201, 137, 64, 40, 152, 112, 242, 114, 1)\}$.

Theorem 5. K_n has a balanced C_{51} - t -foil design if and only if $n \equiv 1 \pmod{102t}$.

Example 5.1. Balanced C_{51} design of K_{103} .

Starter : $\{(103, 3, 48, 94, 40, 53, 13, 20, 50, 72, 35, 54, 90, 63, 59, 26, 10, 21, 11, 28, 60, 65, 91, 56, 36, 74, 51, 22, 14, 55, 41, 93, 46, 2, 45, 43, 44, 92, 42, 57, 15, 24, 52, 76, 37, 58, 33, 67, 61, 30, 12)\}$.

Example 5.2. Balanced C_{51} -2-foil design of K_{205} . Starter :

$\{(205, 6, 96, 187, 79, 104, 25, 38, 98, 141, 68, 105, 177, 123, 116, 50, 19, 39, 20, 52, 117, 125, 178, 107, 69, 143, 99, 40, 26, 106, 80, 186, 94, 89, 93, 4, 92, 185, 81, 108, 27, 42, 100, 145, 70, 109, 179, 127, 118, 54, 21), (205, 3, 90, 184, 82, 110, 28, 44, 101, 147, 71, 111, 180, 129, 119, 56, 22, 45, 23, 58, 120, 131, 181, 113, 72, 149, 102, 46, 29, 112, 83, 183, 88, 2, 87, 85, 86, 182, 84, 114, 30, 48, 103, 151, 73, 115, 66, 133, 121, 60, 24)\}$.

Theorem 6. K_n has a balanced C_{68} - t -foil design if

and only if $n \equiv 1 \pmod{136t}$.

Example 6.1. Balanced C_{68} design of K_{137} .

Starter : $\{(137, 4, 64, 125, 53, 70, 17, 26, 66, 95, 46, 71, 119, 83, 78, 34, 13, 27, 14, 36, 79, 85, 120, 73, 47, 97, 67, 28, 18, 72, 54, 124, 62, 59, 61, 2, 60, 123, 55, 74, 19, 30, 68, 99, 48, 75, 121, 87, 80, 38, 15, 31, 16, 40, 81, 89, 44, 77, 49, 101, 69, 32, 20, 76, 56, 122, 58, 1)\}$.

Example 6.2. Balanced C_{68} -2-foil design of K_{273} . Starter :

$\{(273, 8, 128, 249, 105, 138, 33, 50, 130, 187, 90, 139, 235, 163, 154, 66, 25, 51, 26, 68, 155, 165, 236, 141, 91, 189, 131, 52, 34, 140, 106, 248, 126, 119, 125, 6, 124, 247, 107, 142, 35, 54, 132, 191, 92, 143, 237, 167, 156, 70, 27, 55, 28, 72, 157, 169, 238, 145, 93, 193, 133, 56, 36, 144, 108, 246, 122, 5), (273, 4, 120, 245, 109, 146, 37, 58, 134, 195, 94, 147, 239, 171, 158, 74, 29, 59, 30, 76, 159, 173, 240, 149, 95, 197, 135, 60, 38, 148, 110, 244, 118, 115, 117, 2, 116, 243, 111, 150, 39, 62, 136, 199, 96, 151, 241, 175, 160, 78, 31, 63, 32, 80, 161, 177, 88, 153, 97, 201, 137, 64, 40, 152, 112, 242, 114, 1)\}$.

Theorem 7. K_n has a balanced C_{85} - t -foil design if and only if $n \equiv 1 \pmod{170t}$.

Example 7.1. Balanced C_{85} design of K_{171} .

Starter : $\{(171, 5, 80, 156, 66, 87, 21, 32, 82, 118, 57, 88, 148, 103, 97, 42, 16, 33, 17, 44, 98, 105, 149, 90, 58, 120, 83, 34, 22, 89, 67, 155, 78, 4, 77, 73, 76, 154, 68, 91, 23, 36, 84, 122, 59, 92, 150, 107, 99, 46, 18, 37, 19, 48, 100, 109, 151, 94, 60, 124, 85, 38, 24, 93, 69, 153, 74, 2, 3, 1, 72, 152, 70, 95, 25, 40, 86, 126, 61, 96, 55, 111, 101, 50, 20)\}$.

Theorem 8. K_n has a balanced C_{102} - t -foil design if and only if $n \equiv 1 \pmod{204t}$.

Theorem 9. K_n has a balanced C_{119} - t -foil design if and only if $n \equiv 1 \pmod{238t}$.

Theorem 10. K_n has a balanced C_{136} - t -foil design if and only if $n \equiv 1 \pmod{272t}$.

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