Application of Incentive Based Scoring Rule Deciding Pricing for Smart Houses

Shantanu Chakraborty $^{1,a)}$ Takayuki Ito $^{1,b)}$

Abstract: Defining appropriate pricing strategy for smart environment is important and complicated at the same time. In our work, we device an incentive based smart dynamic pricing scheme for consumers facilitating a hierarchical scoring mechanism. This mechanism is applied between consumer agents (CA) to electricity provider agent (EP) and EP to Generation Company (GENCO). Based on the Continuous Ranked Probability Score (CRPS), a hierarchical scoring system is formed among these entities, CA-EP-GENCO. As CA receives the dynamic day-ahead pricing signal from EP, it will schedule the household devices to lower price period and report the prediction in a form of a probability distribution function to EP. EP, in similar way reports the aggregated demand prediction to GENCO. Finally, GENCO computes the base discount after running a cost-optimization problem. GENCO will reward EP with a fraction of discount based on their prediction accuracy. EP will do the same to CA based on how truthful they were reporting their intentions on device scheduling. The method is tested on real data provided by Ontario Power Company and we show that this scheme is capable to reduce energy consumption and consumers' payment.

Keywords: Smart Grid, Pricing Scheme, Scoring Rule, Demand Response, Device Scheduling

1. Introduction

With the growing needs of environmental sustainability and continuing changes in electric power deregulation, smart grid becomes an inevitable choice for the society. While such grid infrastructure in mind, houses started to adopt devices which can be controlled, maintained, monitored and even scheduled as necessity calls. Smart house technology used to make all electronic devices around a house act "smart" or more autonomous. Nearly all major appliances in the future will take advantage of this technology through home networks and the Internet. Such feature of smart grid is a way for ordinary electronics and appliances to communicate with each other, consumers and even energy provider (EP). Recently, smart pricing has attracted much attention as one of the most important demand-side management (DSM) strategies to encourage users to consume electricity more wisely and efficiently [1].

On different note, in order to numerically measure up the actual realization of a probabilistic event which was forecasted ahead, *scoring rule* was defined [2], [3]. Moreover, it binds the assessor to make a careful prediction and hence truthfully elicit his/her private preferences. Which is why, *scoring rule* has been applied successfully while truthful incentive designing in diverse applications such as voting rules [4] and [5]. Strictly proper scoring rules can be employed by a mechanism designer to ascertain that agents accurately declare their privately calculated distributions, reflecting their confidence in their own forecast. The applicability of scoring rule is being investigated in field of smart-grid. For

^{b)} ito.takayuki@nitech.ac.jp

instance, [6] presented a methodology for predicting aggregated demand in smart-grid.

Household devices such as Roomba vacuum cleaners, LG Thinq smart oven [7] are some commercially available smart devices that can be controlled and monitored via smart-meter. Using such devices, consumers (actually a consumer agent, refereed as CA hereafter, will be responsible to take such decision in conjunction with smart-meter) can respond to day-ahead dynamic pricing signal by effectively and intelligently managing and scheduling devices, thereby flattening out peak demand and achieving better resource utilization.

This paper presents a hierarchical scoring rule based payment mechanism for CA provided by the EP and GENCO in response to the dynamic day-ahead time dependent pricing. The consumers will be rewarded a discount on the price to measure up how well they predict the shifting the devices/loads towards the lower demand (lower price as well) periods. These rewards are again a fraction of the discount which were provided by GENCO to the corresponding EP depending on EP's prediction of required energy demand. The reward mechanism is based on a strictly proper scoring rule. The scoring rule is chosen to reflect to work with continuous variable (the normal distribution, as in the proposed method) and measure up how accurate the prediction could be. The Continuous Ranked Probability Score [8] possess such characteristics. EP will formulate an optimization problem total energy demand for its consumers and reports to GENCO. GENCO then run an optimization algorithm that will minimize the cost of providing rewards to EPs while satisfying EPs energy demand. Therefore, the reward is actually dependent on both the consumers prediction and EP's optimization problem.

The rest of the paper is organized as follows. Section 2 intro-

¹ Nagoya Institute of Technology, Nagoya, Aichi, Japan

a) shantanu.chakraborty@nitech.ac.jp

duces the system model architecture while Section 3 describes the applied scoring rule function and associated key points. The cost optimization formulation and discount distributions are detailed in Section 4. Section 5 presents agent simulation based on Ontario Power System data [9]. Related work are elucited in Section 6. Finally, Section 7 concludes the paper with followup research goals.

2. Incentive Based Dynamic Pricing: System Model

GENCOs and EPs are responsible to determine electricity pricing. GENCOs make revenue by selling energy to the distributers (in our context, EP) based on their demand while distributers provide that energy to consumers.^{*1} A supply-demand chain is thus formed among these entities. Figure 1 shows the model outline architecture depicting the such major components. However, considering such model, it is critically important to have a sophisticated smart pricing scheme that will take advantage of the DSM technique as well as incentivise the CAs to schedule smart devices in order to reduce the total demand.



Fig. 1 GPC Model Architecture

As a mechanism deign to incentivize agents (both the CAs and EPs) for providing private probabilistic information accurately (truthfully) and to the best of their forecasting ability, scoring rule is being applied in this model. Interestingly, such scenario coincides with DSM strategy where consumer responses to demand by shifting their device to lower price periods. Therefore, EP incentivises consumers not only based on their prediction accuracy but also on the question of whether they shifted such loads to lower priced periods. Strictly proper scoring rules can be employed by a mechanism designer to ascertain that agents accurately declare their privately calculated distributions, reflecting their confidence in their own forecast. The details flow of information and task assignments are pointed in Figure 2. As we can see, GENCO will send the price information as a signal to EP. The price signal is typically determined based on the generation costs of electricity.*2 Although this model does not include the price determination mechanism, we assume that in dynamic pricing environment, the signal follows the demand. Which is, the price is higher when the demand is higher and its lower when demand is lower. The price signals are then conveyed to CAs via EPs. One thing can be noted that, one EP can provide energy to



*2 In our model, we assume that GENCOs operate on multiple plants of different types, such as coal, hydro and nuclear. Therefore, pricing signal could be a function of statistical forecast of historical price and the amount the EP pay to buy the energy from generation companies.



Fig. 2 Information Flow of the GPC Model

one or more consumers while one GENCO can also serve one or more EPs. Since, this model assumes a dynamic day-ahead pricing signal, CAs receive their prices one day in advance. Therefore, CAs can schedule their device usages for the upcoming day into the lower price periods. Lets say, the demand in each period *i* is D_i . The demand D_i in each period is assumed to be roughly the same each day due to repeated daily patterns in electricity demands (e.g. period 1 has the same demand on Monday, Tuesday, etc.). So, the aggregate demand over each day is usually constant. This assumption is verified using real traces from an Ontario operator of hourly demand data over seven years [9].

3. Continuous Ranked Probability Score (CRPS)

In order to rightfully incentivise the consumers on their prediction of device shifting; the continuous ranked probability score (CRPS) is applied [2]. CRPS is a strictly proper scoring rule that is used for continuous variable since, the traditional forms of proper and strictly proper scoring rules are usually not work with continuous variables. In the proposed method, Gaussian Distribution is used to model the consumers device shifting prediction and associated confidence. The usage of CRPS is investigated before in distributed power system operation to rightfully score the distributed energy resource(s) [10]. CRPS is able to measure the closeness of the prediction. Since, every device has different priority level of usage, we impose some weights over devices and calculate the actual weighted average of cumulative error as presented in Eq. (1)

$$\delta_{u} = \frac{\sum_{d=1}^{DV_{u}} \left(W_{d} \left| \frac{P_{d}^{u} - P_{d}^{p}}{P_{d}^{p}} \right| \right)}{\sum_{d=1}^{D_{u}}}$$
(1)

where P_d^a and P_d^p describe the prices of energy when the devices are operated at hours *a* (actual) and *p* (predicted). DV_u is the set of devices for CA, *u*. Lets assume, each CA, *u* reports its relative prediction error in a form of uncertainty over it, represented by *Gaussian Distribution Function* $\mathcal{N}(\mu = 0, \sigma_u^2)$. The reward score is therefore, generated by CRPS for that particular u is defined as Eq. (2)

$$CRPS\left(\mathcal{N}(\mu=0,\sigma_{\mu}^{2}),\delta_{u}\right) = \sigma_{u}\left[\frac{1}{\sqrt{\pi}} - 2\varphi\left(\frac{\delta_{u}}{\sigma_{u}}\right) - \frac{\delta_{u}}{\sigma_{u}}\left(2\Phi\left(\frac{\delta_{u}}{\sigma_{u}}\right) - 1\right)\right]$$
(2)

where the probability density function and cumulative distribution function for *Gaussian Distribution Function* are denoted as φ and Φ , respectively. The notation *CRPS* ($\mathcal{N}(\mu = 0, \sigma_u^2), \delta_u$) can be simplified using *CRPS* (σ_u^2, δ_u).

3.1 Truthfulness of Agents: CA and EP

However, predicting correctly about the device shifting schedule will not necessarily incentivize the CA to truthfully report its intentions regarding device shifting. For instance, a CA can misreport about shifting period of a particular device (or group of devices) to a higher priced period while in actual it shifts the device in a lower priced time. Therefore, although the CA will lose the some discount by incorrect prediction, it will gain benefit by shifting device(s) in lower priced period. In order to incorporate such scenario and strictly incentive the CA for reporting its true prediction, the scoring rule needs to be revised. Assuming the price curve follows the demand curve, the scoring rule (SR) is defined as following

$$SR = \begin{cases} CRPS & if (P_d^a - P_d^p) \ge 0 \ \forall d \in DV_u \\ 0 & Otherwise \end{cases}$$
(3)

This above function ensures that any misreporting by CA will generate a 0 score. For example, consider a case where a CA wants to use a device at period 1 and misreports that s/he will use it at period 2 (which is a higher priced period than period 1). Therefore, although s/he gets a less score for misreporting, it would appear that s/he will be compensated by lower price in period 1. But according to Eq. (3), it will get 0 discount. Hence, any CA who misreports of its true intention about shifting any device, will get no discount. CRPS is a strictly proper scoring rule that also ensures the truthfulness of the reporting [2]. The proposed scoring rule (Eq.(3)) also possesses the strictness property of CRPS since its internal mechanism also based on CRPS. Therefore, the proposed scoring rule is also truthful. Figure 3 shows the realization of scoring factors for different errors and confidence level. As pointed out before, the CAs will report their predictions of device usage in the mean of relative error (Eq. (1)) aggregated over all devices. Since, the CAs are aware of the scoring system used by the EPs, they have the liberty to choose associated confidence level (i.e. the sigma; σ). From the graph presented in Figure 3, it is important to notice that,

- a. when a CA is highly confident about its prediction (i.e. $\sigma_u = 0$); highest score is rewarded only when the realized absolute error is zero
- b. when the realized error is relatively higher, the CA will be benefitted to report lower confidence (i.e. higher values of σ)
- c. most importantly, CAs do not know the exact shape of the function when it declares the prediction, since the actual error only realized when the event occur



Fig. 3 CRPS scoring mechanism for different errors

However, the CA has ideas how it will be scored. For instance, if it is likely to make a larger error, it implicitly chooses the function that will penalize it lower by reporting larger σ . On the other hand, if it is confident of its prediction accuracy, it will report a higher σ . By this way, we ensure CAs to report truthfully about their prediction intentions. Moreover, our model assume no *collusion* between the participating agent devices (CA and EP). As we mentioned (and will show in later section), the prediction that EP makes regarding the aggregated energy requirement for its CAs, does not involve any imposition or disturbance towards CAs' device prediction. Rather, EP uses CAs' intentions as a base to report its prediction. Therefore, EP also exhibits the *truthfulness* property. So, the agent devices (CA and EP) used in this model are *truthful* and *non-collusive*.

4. GENCO and EP: Cost Optimization

Based on the device shifting prediction of CAs, EP will try to produce a *potential reward* (pr) which is actually based on the *shifting probability* of a particular device by the amount of shifting. In ideal case, where CA's device commitment prediction coincides with the actual one, there will be no shifting. Taking such scenario in mind, the *shifting probability* (SP) function is chosen to be concave and assumed to be increasing in *pr* and decreasing in shifting time *t*. Thus *SP* is defined as,

$$SP(pr,t) = \frac{pr}{(1+t)^2}$$
 (4)

Given the rewards pr_i (i = 1...N) in each period, the amount of demand shifted out of each period *i* into each period $k \neq i$ is calculated first. Then take the amount of electricity required by each device originally in period *i*, multiply by the *SP*, and sum over all users and their devices to obtain the following

$$\sum_{u=1}^{U} \sum_{j \in DV_i^u} e_j S P_j(pr_k, |k-i|)$$
(5)

where e_j is the amount of energy required by device *j*. DV_i^u is the set of the devices to be committed to *u* at period *i*. The total cost of offering *potential reward* by summing up the demand shifted into period *i* is calculated as,

$$X_{ep} = \sum_{i=1}^{N} pr_i \sum_{u=1}^{U} \sum_{k \neq i} \sum_{j \in DV_u^k} e_j S P_j(pr_i, |k-i|)$$
(6)

EP's cost minimization function for providing *potential rewards* based on the CA's device shifting probablities

$$\begin{array}{l} \min_{pr} \quad X_{ep} \\ s.t. \quad pr \ge 0 \end{array} \tag{7}$$

The *potential reward* (pr) is actually given in every hour. Therefore, the reward will be distributed on the consumers who owns those devices at which are committed at that particular hour.

4.1 EP's Report of demand to GENCO

EP will be incentivized by GENCO using CRPS function. Which is why, they will report regarding their energy demand (which is in fact the predicted demand summing over all CA) in a form of *Gaussian Distribution*. Total predicted demand for user u is shown in Eq. (8)

$$D_{u}^{p} = \sum_{i=1}^{N} \sum_{u=1}^{U} \sum_{j \in DV_{u}^{k}} e_{j} pr_{i}$$
(8)

Therefore, the estimated demand for a particular EP, D_u^p is the summation of individual prediction of all CA. Taking account of this prediction, the EP will report the aggregated *relative error* to GENCO as

$$\sigma_{EP}^2 = \frac{\sum_{u \in EP} (D_u^p * \sigma_u)^2}{(\sum_{u \in EP} D_u^p)^2}$$
(9)

Since Eq. (9) is an affine transformation of aggregated weighted cumulative relative error (earlier reported by CA; which are truthful), EP is also truthful in reporting the prediction provided that CAs are truthful.

4.2 GENCOs cost minimization formulations

Upon receiving the EPs' prediction on demand, GENCO will try to minimize its production cost while satisfying EPs' demand. The GENCOs usually operate multiple plants of different types, e.g. gas, hydroelectric (hydro), renewables and coal [11]. These plants may be categorized as base, intermediate, and peak-load. The base-load plants generally have a higher capital cost but low operating cost, and thus run all of the time (e.g., hydro, nuclear). Intermediate load plants (e.g., coal) have a higher operating cost, and peak load plants (e.g., gas turbines) have the highest operating cost. In any given period, if EPs' demand exceeds the baseload capacity, the generator turns to the intermediate-load plants and then finally, to peak-load plants to generate additional electricity. Under such consideration, c_{i1} denotes the marginal additional cost of using intermediate- rather than base-load plants in period i, and c_{i2} denotes the marginal additional cost of using peak- rather than intermediate-load plants in period *i*. These marginal costs are instances of the random variables; assuming that their actual values are exogenously determined for use in the EPs optimization problem. Figure 4 shows the piecewise-linear cost structure for these load plants; c_{i0} denotes the slope of baseload electricity generation costs. C_{i1} and C_{i2} denote the base- and intermediate-load capacities respectively for period i. These capacities are instances of random variables drawn from exogenous (i.e., price-independent) distributions. Time-series prediction algorithms such as triple-exponential smoothing or auto-regression can be used to estimate the base- and intermediate-load capacities from historical data and exogenous factors. The amount of



Fig. 4 Piecewise linear cost model for base-, intermediate- and peak- load demand

demand in each period *i* is

$$Y = D_i - \sum_{EP \in EPS} pr \sum_{u \in EP} D_u^p$$
(10)

Recall that, D_i is the total demand at period *i*. *EPS* is the set of EP registered to buy energy from that GENCO. The cost of meeting consumers demand at period *i*, therefore, is

$$X_{genco} = \sum_{m=1,2} c_{im} \left[Y - C_{im} \right]^+$$
(11)

Where $[Z]^+$ signifies maximum between 0 and Z. Therefore, the cost minimization problem becomes

$$\begin{array}{ll}
\min_{pr} & X_{genco} \\
s.t. & pr \ge 0
\end{array}$$
(12)

We see that, the optimization problems depicted in Eqs. (7) and (12) have the same control variables (which is the *potential reward*). Therefore, they can be combined as a single optimization problem. Recall that, the *shifting probability* function is a concave one which is increasing in pr and decreasing in time. Therefore, it can be proved that, the formulated optimization problem is a convex one. So, the final optimization problem is defined as

$$\min_{pr} \left[X_{ep} + X_{genco} \right]$$
s.t. $pr \ge 0$
(13)

The above equation ensures that, GENCO and EPs are able to minimize the cost of satisfying EPs' demand (which is actually aggregated demand from the registered CAs) and the generation cost, respectively. Now, the discount will be distributed among EPs. First, the contribution of each EPs is determined by the fraction of their CRPS score by normalizing the *total potential reward*. We recall that, EP provides its *relative error* as σ_{EP}^2 and upon realizing actual demand, its actual relative prediction error, say δ_{EP} . Therefore, the discount EP will receive for their truthful prediction is

$$Discount_{EP} = \frac{\left(\sum_{i=1}^{N} pr_i\right) * CRPS\left(\sigma_{EP}^2, \delta_{EP}\right)}{\sum_{EP \in EPS} CRPS\left(\sigma_{EP}^2, \delta_{EP}\right)}$$
(14)

Now EP's discount should be distributed among the consumers. This discount will be shared based on the scoring rule defined as Eq. (3). Note that, at the time of realization, EP already knows whether that CA is true to her device scheduling prediction. Thus, CRPS is determined according to Eq. (2) where actual weighted average of cumulative error, δ_u . However, as we pointed out,

CRPS itself is not good enough to prevent CA to misreport about its true intention, the scoring rule for CA is, therefore, further modified by using Eq. (3). The scoring factor for u at period i is therefore, defined as

$$sf_{u}^{i} = \frac{revenue(discount_{EP}) * SR(\sigma_{u}^{2}, \delta_{u})}{\sum_{u \in EP} SR(\sigma_{u}^{2}, \delta_{u})} \times pr_{i}$$
(15)

Note that, the EP does not necessarily always distribute the discount as a whole. Instead, it may choose (in most likely cases) to provide some fraction of it. In this way, EP generates some revenue. Such function is defined as *revenue(discount)*. In this model, we reduce the discount by 40% to keep the rest as revenue for EP. Finally, the consumer will be rewarded using sf_u^i . sf_u^i is scaled between 0 to 30 % since [9] set the maximum allowable discount is 30%.

5. Agent Simulation

In this section, some data analysis and preliminary simulation results are presented in order to verify the feasibility of the hierarchical scoring rule based pricing mechanism as well as to demonstrate the ability to reduce GENCO's cost of electric generation and flatten electric usage over some periods. For scalability we assume 1 GENCO entity which supplies energy to 10 electricity providers (EP). Which in turn serves 100 consumers each. However, the initial simulation is limited to 1-GENCO, 1-EP and 1-consumer model to elaborate the models' validity. The real parameters are based on Ontario Independent Electric Operator ([9]). The base-load plant in Ontario is hydroelectric; (assume 60% are base-load plants). The production capacity of each plant is taken as constant across different periods of a day for the purposes of simulation. The intermediate-load consists of coal (operating at 20% efficiency, as is consistent with IESO) and the remaining hydroelectric plants. Finally, the peak plants are gas turbines, which are the most expensive to operate. The slopes of the cost functions for base-, intermediate- and peak-load plants (refer to Figure 4) are taken from the production estimates in [12]. The marginal costs of moving from intermediate- to peak load and base- to intermediate-load plants are calculated to be \$62.46/MWh and \$18.54/MWh respectively. Since its a GW based power system, the system data is effectively scaled down to provide simpler simulation conditions. The prices are also equivalently scaled into kWh level.



Fig. 5 Hourly active devices for a single consumer in a 24-hours period after running smart pricing scheme

Assuming the electricity usage of a single consumer, Figure 5 presents the periodically expected device scheduling after applying the smart pricing scheme. While the quantitative results of



Fig. 6 Discount provided to a single consumer for 96 hours. Total energy consumed 142.1 kWh.

these simulations will vary from market to market, the qualitative results suggest that smart pricing can indeed help GENCOs and EPs to even out consumption over the day and reduce the energy requirements from peak-load plants. Scrutinizing the cyclic electricity consumption pattern, it can be shown that for four consecutive days of energy consumption of 1 consumer reported is 142.1 kWh. However, before using the reward based pricing scheme, the total consumption recorded for a single consumer was 170.35 kWh (based on the single household consumption determined according to the data presented in [9]). Therefore, for a single consumer, the proposed scoring rule based reward scheme can reduce energy consumption down to 20%.

Figure 6 shows the rewards corresponding to the same energy consumption pattern as discussed in previous paragraph. It is noted that the rewards (discounts) are roughly cyclical, as might be expected. If we check the pattern, it is clearly seen that, in case of peak demand hour, the reward is minimum which states the fact that, it becomes difficult to make an accurate prediction in peak hour. Figure 7 depicts the effect of smart pricing scheme on pricing. We can see that, a CA can effectively reduce payment towards EP if it truthfully reports about its device scheduling on the basis of the day-ahead price signal and shifts them in lower priced period. To provide the scalability of the proposed method, Figure 8 is presented. It shows effect aggregated over 1000 consumers (1-GENCO, 10-EPs and 1000-consumers). It is noted that the peak-to-average (P2A) ratio of the consumption pattern before using scoring rule based smart pricing is 2.55 while it comes down to 1.91 when using the proposed pricing scheme. Qualitatively speaking, the P2A ratio is down by approximately 25%. Therefore, the proposed scheme works better when the number of consumers is higher. So, it can be said the method is practically viable and scalable. Moreover, such measure reflects the fact that the strictly proper scoring rule based reward mechanism is able to flatten the load demand.

6. Related Works

The earlier works that explored the potentials of such pricing schemes are mostly from the perspective of consumers on the light of scheduling devices according to the prediction of future prices. As for an instance, [13] proposes a mechanism for predicting prices one or two days in advance. The household devices/loads can be scheduled as to balance impatience with the will to save money. Here several users share a power source and simultaneously scheduling energy consumption in a distributed manner. EP's problem of determining prices based on the con-



Fig. 7 The price curve Vs. the payment which occurred before and after applying smart pricing scheme. This price signal is taken from 1st March. 2010.



Fig. 8 Energy usage before and after using of scoring rule based smart pricing; aggregated over 1000 consumers

sumer response has been studied in several literature. For example, [14] uses real data to quantitatively forecast consumers' scheduling of energy consumption. Although the other works do deal with aggregated demand across the consumers at different times, none of these ever take heterogeneity at the device level into account.

Additionally (and importantly), none of these works ever considered properly incentivizing the consumers for accurately scheduling their devices for future periods as well as making quality estimation of device scheduling for coming periods. Consumers reactions on dynamic pricing effect the scheduling of the household devices. Since there is no standard device commitment mechanism exists, the consumer may misreport their scheduling preferences which leads to an inefficient system design where the pricing is tend to be biased and manipulative. Again, EPs are also responsible for reporting GENCO about the total amount of energy they require for next day. In such case, correct and truthful prediction also provide them a way of being benefitted from GENCO in form of discount over buying price.

7. Conclusion

This paper introduces a new smart pricing scheme considering a model consists of generators, provider and consumers. The formulations are carried by devising a truthful mechanism for both consumer and provider entities where they will report their true intentions regarding device scheduling and energy demand, respectively. The scoring system (facilitating *Continuous Ranked Probability Score*) is designed such a way that, it will force consumer agents to report their true beliefs towards providers. On the other hand, provider agents themselves are incentivised to report the energy demand to generation companies (GENCO) truthfully to get a discount over price. The conducted simulation results show that, the proposed smart pricing scheme is able to reduce the total energy consumption as well as consumers payment towards providers. Therefore, consumers are benefitted since they paid less than the actual price and we have a cleaner environment with reduced energy production. As a future study, we will try to model the device sensitiveness towards scheduling and apply such mechanism for higher scaled smart power system.

Acknowledgments This work is partially supported by the Funding Program for Next Generation World-Leading Researchers (NEXT Program) of the Japan Cabinet Office.

References

- Samadi, P., Mohsenian-Rad, A., Schober, R., Wong, V. W. S. and Jatskevich, J.: Optimal real-time pricing algorithm based on utility maximization for smart grid, *Conf. on Smart Grid Communications*, IEEE (2010).
- [2] Gneiting, T. and Raftery, A.: Strictly proper scoring rules, prediction and estimation, *Journal of the American Statistical Association*, Vol. 102, No. 477, p. 359378 (2007).
- [3] Boutilier, C.: Eliciting forecasts from self-interested experts: scoring rules for decision makers, AAMAS '12: Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems (2012).
- [4] Xia, L. and Conitzer, V.: Finite local consistency characterizes generalized scoring rules, *IJCAI* (2009).
- [5] Ianovski, E., Yu, L., Elkind, E. and Wilson, M. C.: The Complexity of Safe Manipulation under Scoring Rules, *IJCAI* (2011).
- [6] Harry, R., R., A. and Gerding, E. H.: A scoring rule-based mechanism for aggregate demand prediction in the smart grid, AAMAS '12: Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems (2012).
- [7] LG: LG Smart Appliance with THINQ Technology, http://www.lgnewsroom.com/newsroom/contents/6741 (2011).
- [8] Matheson, J. E. and Winkler, R. L.: Scoring rules for continuous probability distributions, *Management Science*, Vol. 22, p. 10871095 (1976).
- [9] Ontario, I.: Market data, http://www.ieso.ca/imoweb/marketdata/marketData.asp, pp. 155–212 (2011).
- [10] Robu, V., K. R., C. G., R. A. and J. N. R.: Cooperative Virtual Power Plant Formation Using Scoring Rules, *Twenty-Sixth Conference on Artificial Intelligence*, AAAI (2012).
- [11] Masters, G.: *Renewable and Efficient Electric Power Systems*, Wiley Online Library (2004).
- [12] Morgan, J.: Comparing energy costs of nuclear, coal, gas, wind and solar, ttp://nuclearfissionary.com/2010/04/02/comparing-energycostsof-nuclear-coal-gas-wind-and-solar/, pp. 155–212 (2010).
- [13] Mohsenian-Rad, A. H., V. W. S. Wong, J. J. and Schober, R.: Optimal and autonomous incentive-based energy consumption scheduling algorithm for smart grid, *Innovative Smart Grid Technologies*, IEEE, pp. 1–6 (2010).
- [14] Faruqui, A. and Wood, L.: Quantifying the benefits of dynamic pricing in the mass market, *Edison Electric Institute*, *Tech. Rep* (2009).