Vol.2013-MPS-92 No.29
2013/2/28

A new Differential Evolution using Pairwise Exclusive Hypervolume for Many-objective Optimization

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Abstract: Many-objective Optimization Problem (MOP) has a number of objectives considerably larger than two or three. In order to obtain a good approximation of the Pareto-optimal solution set for MOP, this paper presents a new evolutionary algorithm based on Differential Evolution (DE), which uses Pairwise Exclusive Hypervolume (PEH) as the secondary criterion for sorting non-dominated solutions. Through the numerical experiment and the statistical test conducted on test problems, the effect of PEH is evaluated in a comparison with conventional secondary criteria.

1. Introduction

Multi-objective optimization problems with a number of objectives considerably larger than two or three are often referred to as Many-objective Optimization Problems (MOPs). Conventional Multi-Objective Evolutionary Algorithms (MOEAs) such as well-known NSGA-II [1] can't be applied successfully to MOPs. This is because there is a tendency towards a higher proportion of non-dominated points, and under certain conditions these points dominate quickly as the number of objectives rises.

One of the most promising approaches for handling MOPs is to use a proper secondary criterion for sorting non-dominated solutions. Therefore, some secondary criteria that can replace the crowding-distance [1] have been proposed [2]. This paper presents a new MOEA based on Differential Evolution (DE) [3], which uses Pairwise Exclusive Hypervolume (PEH) [4] as the secondary criterion. Through the numerical experiment and the statistical test conducted on scalable test problems, the effect of PEH is evaluated in a comparison with other secondary criteria.

2. Formulation of MOP

This paper defines MOP on a continuous space. A decision vector $\vec{x} = (x_1, \cdots, x_D)$ is composed of D decision variables $x_j \in \Re$, $j \in \mathcal{I}_D = \{1, \cdots, D\}$. Each decision variable $x_j \in \Re$ is limited by lower $\underline{x}_j$ and upper $\overline{x}_j$ bounds. Thereby, decision space $\mathcal{X} \subseteq \Re^D$ is defined as $\mathcal{X} = \{\vec{x} \in \Re^D \mid \forall j \in \mathcal{I}_D;\ \underline{x}_j \le x_j \le \overline{x}_j\}$. Now, MOP with M $(M \ge 4)$ objectives is formulated as

$$\left[\begin{array}{ll} \text{minimize} & \vec{f}(\vec{x}) = (f_1(\vec{x}), \cdots, f_M(\vec{x})), \\ \text{subject to} & \vec{x} = (x_1, \cdots, x_D) \in \mathcal{X}. \end{array}\right. \tag{1}$$

An objective vector $\vec{f}(\vec{x})$ is composed of M objective values $f_m(\vec{x}) \in \Re$, $m \in \mathcal{I}_M = \{1, \cdots, M\}$. The objective space $\mathcal{F} \subseteq \Re^M$ is defined as $\mathcal{F} = \{\vec{f}(\vec{x}) \in \Re^M \mid \vec{x} \in \mathcal{X}\}$. In order to simplify the notation, we will use an objective vector $\vec{f} = (f_1, \cdots, f_M) \in \mathcal{F}$ to represent a corresponding decision vector or a solution $\vec{x} \in \mathcal{X}$ where $\vec{f} = \vec{f}(\vec{x})$ holds. A solution $\vec{f} \in \mathcal{F}$ is said to dominate $\vec{h} \in \mathcal{F}$ and denoted as $\vec{f} \succ \vec{h}$, if the following condition is true:

$$(\forall m \in \mathcal{I}_M;\ f_m \le h_m) \wedge (\exists n \in \mathcal{I}_M;\ f_n < h_n). \tag{2}$$

Similarly, a solution $\vec{f} \in \mathcal{F}$ is said to weakly dominate $\vec{h} \in \mathcal{F}$ and denoted as $\vec{f} \succeq \vec{h}$, if the following condition is true:

$$\forall m \in \mathcal{I}_M;\ f_m \le h_m. \tag{3}$$

3. Proposed PEH

Let consider a set of solutions $\mathcal{P} \subset \mathcal{F}$. A set of non-dominated solutions in $\mathcal{P}$ is denoted by $\widetilde{\mathcal{P}} \subseteq \mathcal{P}$. We can make a rectangular region $B(\{\vec{f}\}, \vec{r}) \subset \Re^M$ by using a non-dominated solution $\vec{f} \in \widetilde{\mathcal{P}}$ and a reference point $\vec{r} \in \Re^M$ where $\vec{f} \succ \vec{r}$ holds. Furthermore, a region $B(\widetilde{\mathcal{P}}, \vec{r}) \subset \Re^M$ of the set $\widetilde{\mathcal{P}} \subseteq \mathcal{P}$ is defined as

$$B(\widetilde{\mathcal{P}}, \vec{r}) = \bigcup_{\vec{f} \in \widetilde{\mathcal{P}}} B(\{\vec{f}\}, \vec{r}). \tag{4}$$

The hypervolume of $\widetilde{\mathcal{P}} \subseteq \mathcal{P}$ is defined completely as

$$H(\widetilde{\mathcal{P}}, \vec{r}) = vol(B(\widetilde{\mathcal{P}}, \vec{r})), \tag{5}$$

where $vol(B) \in \Re$ denotes the volume of a region $B \subset \Re^M$.

The Exclusive Hypervolume (EH) of $\vec{f} \in \widetilde{\mathcal{P}}$ is defined as

$$EH(\vec{f}, \widetilde{\mathcal{P}}, \vec{r}) = H(\widetilde{\mathcal{P}}, \vec{r}) - H(\widetilde{\mathcal{P}} \setminus \{\vec{f}\}, \vec{r}). \tag{6}$$

EH can be used to determine which solution $\vec{f} \in \widetilde{\mathcal{P}}$ contributes least to the hypervolume of $\widetilde{\mathcal{P}}$. However, the main drawback of EH is the computational cost. PEH is defined for $\vec{f} \in \widetilde{\mathcal{P}}$ as

$$PEH(\vec{f}, \widetilde{\mathcal{P}}, \vec{r}) = \min_{\vec{h} \in \widetilde{\mathcal{P}} \setminus \{\vec{f}\}} \{EH(\vec{f}, \{\vec{f}, \vec{h}\}, \vec{r})\}. \tag{7}$$

PEH can be calculated in polynomial complexity [4]. By the way, the proposed PEH gives an upper bound of EH as

$$PEH(\vec{f}, \widetilde{\mathcal{P}}, \vec{r}) \ge EH(\vec{f}, \widetilde{\mathcal{P}}, \vec{r}). \tag{8}$$

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4. Algorithm of DE for MOP

The proposed DE-based MOEA is called DEMO2. DEMO2 has a set of candidate solutions $\vec{x}_i = (x_{1,i}, \cdots, x_{D,i}) \in \mathcal{P} \subseteq \mathcal{X}$, $i = 1, \cdots, N_P$. In order to generate a new solution called the trial vector from $\vec{x}_i \in \mathcal{P}$, a basic strategy of DE named "rand/1/exp" [3] is used. By using the maximum number of generations N_G in the stopping condition, DEMO2 is described as follows:

Step 1 Randomly generate N_P decision vectors $\vec{x}_i \in \mathcal{X}$ as an initial population $\mathcal{P}$. Evaluate $\vec{f}(\vec{x}_i)$ for all $\vec{x}_i \in \mathcal{P}$. Set $g = 0$.

Step 2 If $g = N_G$ holds, output the set $\widetilde{\mathcal{P}} \subseteq \mathcal{P}$ and terminate.

Step 3 For each target vector $\vec{x}_i \in \mathcal{P}$ $(i = 1, \cdots, N_P)$, execute everything from Step 3.1 to Step 3.3.

Step 3.1 Generate a new trial vector $\vec{u} \in \mathcal{X}$.

Step 3.2 Evaluate the objective vector $\vec{f}(\vec{u})$ for $\vec{u}$.

Step 3.3 If $\vec{u} \succeq \vec{x}_i$ holds, $\vec{u}$ replaces $\vec{x}_i \in \mathcal{P}$. If $\vec{x}_i \succ \vec{u}$ holds, $\vec{u}$ is discarded. Otherwise, $\vec{u}$ is added in $\mathcal{P}$.

Step 4 If the population size exceeds N_P, do the truncation method to choose the best N_P solutions for $\mathcal{P}$.

Step 5 Update generation as $g = g + 1$, and return to Step 2.

The population size $\sharp\mathcal{P}$ comes in the range between N_P and $2\,N_P$ after N_P trial vectors are generated. Truncation method in Step 4 is used to return $\sharp\mathcal{P}$ to N_P. DEMO2 is much the same with DEMO [5] except the truncation method described as follows:

Step 1 By using the non-dominated sorting [1], decide the non-domination rank for each solution $\vec{f} \in \mathcal{P}$.

Step 2 Select N_P solutions $\vec{f}$ from $\mathcal{P}$ in the ascending order of the non-domination rank and preserve those solutions in $\mathcal{P}$.

Step 3 If some solutions need to be selected from $\widetilde{\mathcal{Q}} \subseteq \mathcal{P}$, where every solution $\vec{f} \in \widetilde{\mathcal{Q}} \subseteq \mathcal{P}$ has the same non-domination rank, execute everything from Step 3.1 to Step 3.3.

Step 3.1 Decide a reference point $\vec{r} \in \Re^M$ for $\widetilde{\mathcal{Q}}$.

Step 3.2 Evaluate $PEH(\vec{f}, \widetilde{\mathcal{Q}}, \vec{r})$ for each $\vec{f} \in \widetilde{\mathcal{Q}}$.

Step 3.3 Select the necessary number of solutions $\vec{f}$ from $\widetilde{\mathcal{Q}}$ in the descending order of a secondary criterion, where $PEH(\vec{f}, \widetilde{\mathcal{Q}}, \vec{r})$ is proposed as the secondary criterion.

5. Experiment and Results

By using Crowding-Distance (CD) [1], ϵ-DOM [2] and PEH for the secondary criteria, respectively, three types of DEMO2s are applied to each of scalable test problems in [6] 50 times.

The program of DEMO2 was coded by the Java language and run on a personal computer (CPU: Intel® Core^{TM}i7 @3.33[GHz]; memory: 2[GB]; OS: Microsoft Windows XP). The control parameters of DEMO2 were chosen as $N_P = 100$ and $N_G = 400$ except DTLZ3 ($N_P = 200$ and $N_G = 600$).

Table 1 shows the run times averaged over 50 runs. The best result is highlighted by bold type in each problem. From Table 1, DEMO2 with CD is the fastest in all cases. DEMO2 with PEH is slightly faster than DEMO2 with ϵ-DOM. However, there is no significant difference among three DEMO2s in the run time.

All solutions obtained by DEMO2s became non-dominated ones in all cases. Table 2 compares the solution sets in Convergence Measure (CM) [6]. The solution sets obtained with CD have not converged in many cases. Table 3 compares PEH with CD and ϵ-DOM by using Wilcoxon test about the above CM, in which △ (▽) denotes PEH is significantly better (worse) than the other with risk $\alpha = 0.01$; ▲ (▼) denotes PEH is better (worse) than the other with risk $\alpha = 0.05$; and "—" means that there is no difference between two secondary criteria. From Table 3, we can confirm that PEH is better than the others in many cases.

Table 1 Run time of DEMO2 [ms]

criteria	M	DTLZ1	DTLZ2	DTLZ3	DTLZ4
CD	4	**404.3**	**407.1**	**2170.9**	**451.8**
	6	**536.8**	**523.1**	**2761.5**	**574.7**
	8	**666.9**	**645.3**	**3461.0**	**665.9**
ϵ-DOM	4	575.6	598.1	3109.0	625.9
	6	858.8	888.1	4888.3	899.1
	8	1120.3	1173.1	6550.6	1111.6
PEH	4	538.1	557.8	2920.2	586.8
	6	748.5	750.0	4269.7	774.6
	8	944.0	954.0	5401.2	924.7

Table 2 Convergence Measure (CM)

criteria	M	DTLZ1	DTLZ2	DTLZ3	DTLZ4
CD	4	**0.3625**	1.30E-7	0.3559	1.38E-7
	6	293.33	0.3448	1.16E+5	0.2564
	8	395.13	0.9373	6.09E+5	0.6372
ϵ-DOM	4	1.4020	1.67E-8	0.7194	1.05E-8
	6	**1.6572**	3.55E-7	4.8763	1.69E-8
	8	2.6893	1.41E-6	7.0184	8.36E-8
PEH	4	1.0982	**5.56E-9**	**0.1962**	**2.16E-9**
	6	2.3477	**3.57E-8**	**3.3888**	**6.21E-9**
	8	**2.6445**	**1.29E-7**	**6.6024**	**3.46E-8**

Table 3 Statistical test about CM

criteria	M	DTLZ1	DTLZ2	DTLZ3	DTLZ4
CD	4	—	△	△	△
	6	△	△	△	△
	8	△	△	△	△
ϵ-DOM	4	▲	△	△	△
	6	—	△	—	△
	8	—	△	—	—

Table 4 Statistical test about hypervolume

criteria	M	DTLZ1	DTLZ2	DTLZ3	DTLZ4
ϵ-DOM	4	△	△	△	△
	6	△	△	△	△
	8	△	△	△	△

Table 4 compares PEH with ϵ-DOM by using Wilcoxon test about the hypervolume. PEH is significantly better than ϵ-DOM in every case. Because the solution sets obtained with CD did not converge, the hypervolume could not be evaluated for CD.

6. Concluding Remarks

Through the experiment conducted on scalable test problems, the proposed secondary criterion PEH was compared with CD and ϵ-DOM respectively. As a result, it was observed that PEH was superior to counterparts in the quality of solution sets.

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