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Inverse Composite Alignment of a sphere under orthogonal projection for ball spin estimation

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In this technical report, we propose a method for measuring the spin of a table tennis ball in the framework of Inverse Composite Alignment (ICA). We assume that the projection of the ball onto the image plane is orthographic, and the shape of the ball is a rigid sphere. Under these assumptions we derive an update rule for the motion parameters. Because of the precomputation of the Hessian matrix in ICA and the simplifying assumptions, the motion parameters estimation at each frame is very fast. We show experimental results obtained by our prototype system for measuring spins in real image sequences of table tennis rallies.

1. Introduction

In this technical report, we propose a method for measuring the spin of a table tennis ball. In table tennis, the spin of the ball is one of the important indications used to evaluate the skills of a player. For a skillful player, the spin exceeds 5000 rotations per minute (rpm), while it is typically around 3000 rpm for a novice[3),4)]. Presently, the spin is measured either by using a spinometer or by performing a 2D image analysis with a high-speed camera. For such an application, the use of computer vision techniques can be very useful.

Tracking the motion of a rigid object, such as a ball, is a problem which has been studied since the 1980s. Early works were based on calculating optical flow[2)], while recent approaches seem to favor the use of local features[1)]. However, the small size of a table tennis ball, and motion blur (see Fig. 1), make it difficult to compute optical flow, or to detect reliably features in the ball region. An image registration-based method for spin measurement of a table tennis ball[5),6)] has been proposed to overcome this problem, however, it is not very efficient, so that computing time can take up to several minutes for each frame.

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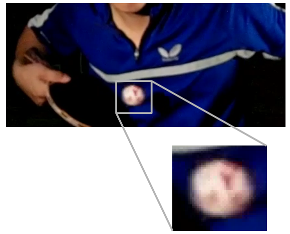


Fig. 1 An image of a table tennis player and a ball (with enlarged image).

Here we propose a novel image registration-based method for measuring the spin of a ball. The main contribution of this report lies in developing a spin measurement method based on Inverse Composite Alignment (ICA)[7),8)] that accelerates computation by precomputing the Hessian matrix. Moreover, we employ several assumptions to simplify the problem formulation: the camera is orthographic and ball's radius and location at each frame are given. Also, the 3D shape of the ball is used to obtain depth information. This is similar to 2.5D ICA[9)], but specialized to a sphere.

The organization of the report is as follows. In section 2, we briefly review ICA. Then, we introduce several assumptions necessary to simplify the problem formulation, define the motion parameters, and derive an update rule for their estimation. In section 4, we show some experimental results obtained by our prototype system for measuring spins in real image sequences of table tennis rallies.

2. Inverse Composite Alignment

In this section, we describe ICA in brief. We have two successive images I_1 and I_2, and let $I_1(\boldsymbol{x})$ be the intensity value at location $\boldsymbol{x} = (x, y)^T$ in image I_1. A registration minimizes the sum of squared difference between corresponding intensities in I_1 and I_2.

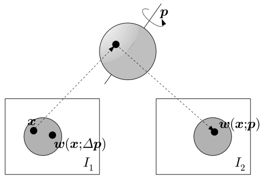


Fig. 2 Two frames relation.

In ICA, the objective function f to be minimized is given in the following form:

$$f = \sum_{\boldsymbol{x}} \left| I_1(\boldsymbol{w}(\boldsymbol{x}; \Delta\boldsymbol{p})) - I_2(\boldsymbol{w}(\boldsymbol{x}; \boldsymbol{p})) \right|^2 , \quad (1)$$

where $\boldsymbol{w}(\boldsymbol{x}; \boldsymbol{p})$ is a warping function that gives the location where a point $\boldsymbol{x}$ is moved by a motion parameter $\boldsymbol{p}$. Note that $\boldsymbol{w}$ should be an identity mapping when the parameter is zero: $\boldsymbol{w}(\boldsymbol{x}; \boldsymbol{0}) = \boldsymbol{x}$. $\Delta\boldsymbol{p}$ is obtained at each iteration step, then an update rule for ICA $\boldsymbol{w}(\boldsymbol{x}; \boldsymbol{p}) \circ \boldsymbol{w}(\boldsymbol{x}; \Delta\boldsymbol{p})^{-1} \rightarrow \boldsymbol{w}(\boldsymbol{x}; \boldsymbol{p})$ is used. Here $\circ$ denotes a composition of two warps that means that the warping functions should form a group: if $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ are valid warp functions, so is $\boldsymbol{w}_1 \circ \boldsymbol{w}_2$.

$\Delta\boldsymbol{p}$ is calculated at each step of iterations. By using the first order Taylor expansion of the term I_1, f can be approximated as follows:

$$f = \sum_{\boldsymbol{x}} \left| I_1(\boldsymbol{x}) + \nabla I_1(\boldsymbol{x}) \frac{\partial \boldsymbol{w}(\boldsymbol{x}; \boldsymbol{0})}{\partial \boldsymbol{p}} \Delta\boldsymbol{p} - I_2(\boldsymbol{w}(\boldsymbol{x}; \boldsymbol{p})) \right|^2 . \quad (2)$$

To obtain the best update $\Delta\boldsymbol{p}$ of the parameter that gives the minimum of the function, we solve $\frac{\partial f}{\partial \Delta\boldsymbol{p}} = \boldsymbol{0}$. Then, we have

$$\Delta\boldsymbol{p} = H^{-1} \sum_{\boldsymbol{x}} \left[\nabla I_1(\boldsymbol{x}) \frac{\partial \boldsymbol{w}(\boldsymbol{x}; \boldsymbol{0})}{\partial \boldsymbol{p}} \right]^T \left[I_2(\boldsymbol{w}(\boldsymbol{x}; \boldsymbol{p})) - I_1(\boldsymbol{x}) \right] , \quad (3)$$

where the Hessian H is given by

$$H = \sum_{\boldsymbol{x}} \left[\nabla I_1(\boldsymbol{x}) \frac{\partial \boldsymbol{w}(\boldsymbol{x}; \boldsymbol{0})}{\partial \boldsymbol{p}} \right]^T \left[\nabla I_1(\boldsymbol{x}) \frac{\partial \boldsymbol{w}(\boldsymbol{x}; \boldsymbol{0})}{\partial \boldsymbol{p}} \right] . \quad (4)$$

The advantage of ICA is to avoid the computation of H in each iteration step. Because computing H is very time consuming, precomputation and reuse of H make the algorithm very fast.

3. Design of the System

In this section, we describe the assumptions, motion parameters, and the warping function used in our system for measuring the table tennis ball's spin.

3.1 Assumptions

We assume that the camera is orthographic. This is a reasonable assumption because the ball is much smaller (40 mm in diameter) than the distance to the camera. In the experiments, the images are taken at a distance of 3 to 5 meters from a player. Therefore, the depth of the ball can be ignored.

The radius r of the ball is assumed to be given. In the experimental setup, the direction of the motion of the ball between the players is usually perpendicular to the optical axis of the camera. Hence, the change of appearance of the ball is negligible, and it is reasonable to assume that r is constant for all frames and given in advance.

Also we assume that the center location $\boldsymbol{c} = (c_x, c_y)^T$ of the ball in every frame is given by some other source: e.g. by using circle detection with the Hough transform, or simply by user interaction.

3.2 Motion parameters

We choose the angular velocities about three axes to parameterize the spin of the ball. Angular velocities are represented by the Euler angles α, β, γ between successive frames. Then, a rotation matrix R is constructed by the angles: $R = R_\alpha R_\beta R_\gamma$. However, note that the angles are not accumulated over frames, but reset every frame to represent angular velocities (not angles) in order to avoid error accumulation.

As we consider the translation of the ball only in the image plane, translation is represented by a vector $\boldsymbol{t} = (t_x, t_y)^T$. Because the direction of the ball's motion is approximately perpendicular to the optical axis and the depth of the ball is relatively small as mentioned above, we can ignore the translation along the depth direction.

Therefore, the motion parameter vector $\boldsymbol{p}$ we use includes the following five parameters: $\boldsymbol{p} = (\alpha, \beta, \gamma, t_x, t_y)^T$.

3.3 Ball shape and depth

Since the motion parameters represent three-dimensional rigid motion, we also use the depth of the ball to register the images. The depth is obtained as follows. We assume that the target is a sphere with radius r: $x^2 + y^2 + z^2 = r^2$. Rewriting this equation, we obtain an equation for the depth at location x on the ball:

$$z(\boldsymbol{x}) = \pm\sqrt{r^2 - (x^2 + y^2)}. \tag{5}$$

We simply use the positive values of the above equation. This enables a visibility test easy because if the depth of a transformed point becomes negative, it is not visible to the camera.

3.4 The warping function

We define the warping function as follows:

$$\boldsymbol{w}(\boldsymbol{x};\boldsymbol{p}) = P_o \left[R \begin{pmatrix} \boldsymbol{x} - \boldsymbol{c} \\ z(\boldsymbol{x} - \boldsymbol{c}) \end{pmatrix} + \begin{pmatrix} \boldsymbol{t} \\ 0 \end{pmatrix} \right] + \boldsymbol{c}, \tag{6}$$

where z is obtained from Eq. (5), and P_o is an orthographic projection matrix:

$$P_o = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{7}$$

The warp includes a 3D rigid transformation (rotation and translation) of a 2D point $\boldsymbol{x}$. First, $\boldsymbol{x}$ is centered at $\boldsymbol{c}$ (i.e., $\boldsymbol{x} - \boldsymbol{c}$), and the z component is added to make a 3D vector. Next, it is rotated by R and translated by $\boldsymbol{t}$. Then, the 3D vector is projected to 2D by P_o. Finally, $\boldsymbol{c}$ is added to move it back.

The components of the Jacobian $\frac{\partial \boldsymbol{w}(\boldsymbol{x};\boldsymbol{p})}{\partial \boldsymbol{p}}$ are as follows:

$$\frac{\boldsymbol{w}(\boldsymbol{x};\boldsymbol{p})}{\partial \alpha} = P_o \left[\frac{\partial R_\alpha}{\partial \alpha} R_\beta R_\gamma \begin{pmatrix} \boldsymbol{x} - \boldsymbol{c} \\ z(\boldsymbol{x} - \boldsymbol{c}) \end{pmatrix} \right], \tag{8}$$

$$\frac{\boldsymbol{w}(\boldsymbol{x};\boldsymbol{p})}{\partial \beta} = P_o \left[R_\alpha \frac{\partial R_\beta}{\partial \beta} R_\gamma \begin{pmatrix} \boldsymbol{x} - \boldsymbol{c} \\ z(\boldsymbol{x} - \boldsymbol{c}) \end{pmatrix} \right], \tag{9}$$

$$\frac{\boldsymbol{w}(\boldsymbol{x};\boldsymbol{p})}{\partial \gamma} = P_o \left[R_\alpha R_\beta \frac{\partial R_\gamma}{\partial \gamma} \begin{pmatrix} \boldsymbol{x} - \boldsymbol{c} \\ z(\boldsymbol{x} - \boldsymbol{c}) \end{pmatrix} \right], \tag{10}$$

$$\frac{\boldsymbol{w}(\boldsymbol{x};\boldsymbol{p})}{\partial \boldsymbol{t}_x} = (1, 0)^T, \tag{11}$$

$$\frac{\boldsymbol{w}(\boldsymbol{x};\boldsymbol{p})}{\partial \boldsymbol{t}_y} = (0, 1)^T. \tag{12}$$

3.5 Update rule for the motion parameters

An update rule in the form $\boldsymbol{w}(\boldsymbol{x};\boldsymbol{p}) \circ \boldsymbol{w}(\boldsymbol{x};\Delta\boldsymbol{p})^{-1} \to \boldsymbol{w}(\boldsymbol{x};\boldsymbol{p})$ is derived as follows.

Let two 2D points and their corresponding 3D points be:

$$\boldsymbol{x}_1 = \boldsymbol{w}(\boldsymbol{x};\Delta\boldsymbol{p}) = P_o \boldsymbol{X}_1, \tag{13}$$

$$\boldsymbol{x}_2 = \boldsymbol{w}(\boldsymbol{x};\boldsymbol{p}) \quad = P_o \boldsymbol{X}_2. \tag{14}$$

Now the composition of two warps is obtained by writing $\boldsymbol{x}_2$ in terms of $\boldsymbol{x}_1$ by traversing $\boldsymbol{x}_1$, $\boldsymbol{x}$, and then $\boldsymbol{x}_2$.

Here $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are as follows:

$$\boldsymbol{X}_1 = \Delta R(\boldsymbol{X} - \boldsymbol{C}) + \Delta \boldsymbol{T} + \boldsymbol{C}, \tag{15}$$

$$\boldsymbol{X}_2 = R(\boldsymbol{X} - \boldsymbol{C}) + \boldsymbol{T} + \boldsymbol{C}, \tag{16}$$

where

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{x} \\ z(\boldsymbol{x} - \boldsymbol{c}) \end{pmatrix}, \ \boldsymbol{C} = \begin{pmatrix} \boldsymbol{c} \\ \boldsymbol{0} \end{pmatrix}, \ \boldsymbol{T} = \begin{pmatrix} \boldsymbol{t} \\ \boldsymbol{0} \end{pmatrix}. \tag{17}$$

ΔR and $\Delta \boldsymbol{T}$ are parameters corresponding to $\Delta \boldsymbol{p}$.

First, we have

$$\boldsymbol{X} = \Delta R^{-1}(\boldsymbol{X}_1 - \boldsymbol{C} - \Delta \boldsymbol{T}) + \boldsymbol{C}, \tag{18}$$

then after substituting $\boldsymbol{X}$ in $\boldsymbol{X}_2$:

$$\boldsymbol{X}_2 = R((\Delta R^{-1}(\boldsymbol{X}_1 - \boldsymbol{C} - \Delta \boldsymbol{T}) + \boldsymbol{C}) - \boldsymbol{C}) + \boldsymbol{T} + \boldsymbol{C}, \tag{19}$$

$$= (R\Delta R^{-1})(\boldsymbol{X}_1 - \boldsymbol{C}) + (\boldsymbol{T} - R\Delta R^{-1}\Delta \boldsymbol{T}) + \boldsymbol{C}. \tag{20}$$

Thus, we obtain the updated motion parameters:

$$R \leftarrow R\Delta R^{-1} \tag{21}$$

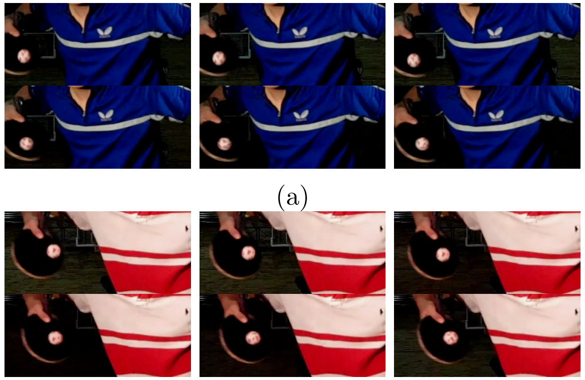
$$\boldsymbol{t} \leftarrow P_o(\boldsymbol{T} - R\Delta R^{-1}\Delta \boldsymbol{T}). \tag{22}$$

4. Experimental Results

Here we describe experimental results of spin estimation for real image sequences of table tennis rallies.

Fig. 3 shows two image sequences of different players. Images were taken at 600 fps by a fixed high-speed camera with halogen lamps mounted from the side of the player. An official 40 mm table tennis ball was randomly textured by marker pens. The angle parameters α, β, and γ were measured in radian per 1/600 second because they were estimated by using two successive frames. Then the angles were converted to rotation-per-minute (rpm). The radius and center locations were obtained by manually in these experiments.

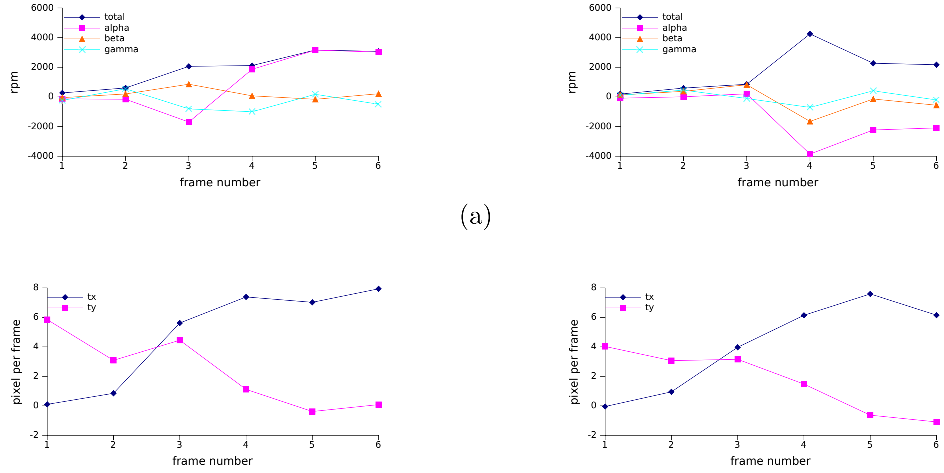
Fig. 4(a) shows the estimated spins in rpm. The spins α, β, and γ of are shown separately, while the total spin, $\sqrt{\alpha^2 + \beta^2 + \gamma^2}$, is also shown. Both sequences



(a)

(b)

Fig. 3 Sequences used in the experiments. (a) and (b) show two sequences for different players.



(a)

(b)

Fig. 4 Estimated RPM and translations. (a) Spins in rpm. (b) 2D translations in pixels at each frame.

start one or two frames before the racket hits the ball. As can be seen, the spins are small in the first two frames, then increase rapidly at the third frame.

2D translations are shown in Fig. 4(b). The vertical axis represents translations at each frame; e.g., velocities of the ball. At the beginning of the sequences, the balls are falling vertically, then hit by the racket. Therefore, t_y is large at first (because y axis is downward), then t_x increases (x axis is rightward) at the third frame.

The accuracy of the estimates has not been evaluated yet, however, the results are very useful for players and coaches who use the system for their training because they can see the ball spins of the players quantitatively instead of impact feelings or just watching the sequences.

5. Conclusions

We have proposed a method for measuring the spin of a table tennis ball with Inverse Composite Alignment under some assumptions that are practical for this application. The prototype system of the proposed method is useful and currently used for training of players. Although in this report we have focused specifically on table tennis, the proposed method has a large variety of potential applications in other sports too: football, baseball, golf, volleyball, and so on. Directions of future work to make the system more practical include automatic detection of the center and radius of the ball, use of information over three or more frames, and evaluation of the accuracy of the estimated spins.

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