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完全多重グラフのハミルトン C_k-Sevenfoil 分解アルゴリズム

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_k を k 点を通るサイクルとする。1点を共有する辺素な7個の C_k からなるグラフを C_k-sevenfoil という。本研究では、完全多重グラフ λK_n をハミルトン C_k-sevenfoil 部分グラフに分解する分解アルゴリズムについて述べる。

Hamilton C_k-Sevenfoil Decomposition Algorithm of Complete Multi-Graphs

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In graph theory, the decomposition problem of graphs is a very important topic. Various types of decompositions of many graphs can be seen in the literature of graph theory. This paper gives a Hamilton C_k-sevenfoil decomposition of the complete multi-graph λK_n.

1. Introduction

Let K_n denote *the complete graph* of n vertices. *The complete multi-graph* λK_n is the complete graph K_n in which every edge is taken λ times. Let C_k be *the k-cycle* (or *the cycle* on k vertices). *The C_k-sevenfoil* is a graph of 7 edge-disjoint C_k's with a common vertex and the common vertex is called *the center of the C_k-sevenfoil*. In particular, a C_k-sevenfoil satisfying $n = 7(k-1)+1$ is called *the Hamilton C_k-sevenfoil* because the C_k-sevenfoil spans λK_n.

When λK_n is decomposed into edge-disjoint sum of Hamilton C_k-sevenfoils, we say that λK_n *has a Hamilton C_k-sevenfoil decomposition*. This decomposition is called *a Hamilton C_k-sevenfoil design*.

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2. Hamilton C_k-sevenfoil decomposition of λK_n

Notation. We consider the vertex set V of λK_n as $V = \{1, 2, ..., n\}$. We denote a Hamilton C_k-sevenfoil passing through $v_1 - v_2 - v_3 - ... - v_k - v_1$, $v_1 - v_{k+1} - v_{k+2} - ... - v_{2k-1} - v_1$, $v_1 - v_{2k} - v_{2k+1} - ... - v_{3k-2} - v_1$, $v_1 - v_{3k-1} - v_{3k} - ... - v_{4k-3} - v_1$, $v_1 - v_{4k-2} - v_{4k-1} - ... - v_{5k-4} - v_1$, $v_1 - v_{5k-3} - v_{5k-2} - ... - v_{6k-5} - v_1$, $v_1 - v_{6k-4} - v_{6k-3} - ... - v_{7k-6} - v_1$ by $(v_1, v_2, v_3, ..., v_k) \cup (v_1, v_{k+1}, v_{k+2}, ..., v_{2k-1}) \cup (v_1, v_{2k}, v_{2k+1}, ..., v_{3k-2}) \cup (v_1, v_{3k-1}, v_{3k}, ..., v_{4k-3}) \cup (v_1, v_{4k-2}, v_{4k-1}, ..., v_{5k-4}) \cup (v_1, v_{5k-3}, v_{5k-2}, ..., v_{6k-5}) \cup (v_1, v_{6k-4}, v_{6k-3}, ..., v_{7k-6})$.

Theorem 1. If λK_n has a Hamilton C_k-sevenfoil decomposition, then (i) $n = 7(k-1)+1$ and (ii) $\lambda \equiv 0 \pmod k$ for $k \equiv 1, 5, 7, 11 \pmod{12}$, $\lambda \equiv 0 \pmod k$ for $k \equiv 4, 8 \pmod{12}$, $\lambda \equiv 0 \pmod{k/2}$ for $k \equiv 2, 10 \pmod{12}$, $\lambda \equiv 0 \pmod{k/3}$ for $k \equiv 0, 3, 9 \pmod{12}$, $\lambda \equiv 0 \pmod{k/6}$ for $k \equiv 6 \pmod{12}$.
Proof. When $n = 7(k-1)+1$, suppose that λK_n is decomposed into b Hamilton C_k-sevenfoils. Then $b = \lambda n(n-1)/14k = \lambda(7k-6)(k-1)/2k$. Thus, (i), (ii) hold.

Theorem 2. If λK_n has a Hamilton C_k-sevenfoil decomposition, then $(s\lambda)K_n$ has a Hamilton C_k-sevenfoil decomposition for every s.
Proof. Obvious. Repeat s times the Hamilton C_k-sevenfoil decomposition of λK_n.

Theorem 3. Let n be prime. When $n = 7(k-1)+1$, $\lambda \equiv 0 \pmod k$, and $k \equiv 1, 5, 7, 11 \pmod{12}$, λK_n has a Hamilton C_k-sevenfoil decomposition.

Example 3.1. Hamilton C_5-sevenfoil of $5K_{29}$.
$(n, g) = (29, 2)$ n-orbit : $1, 2, 4, 8, 16, 3, 6, 12, 24, 19, 9, 18, 7, 14, 28, 27, 25, 21, 13, 26, 23, 17,$

$5, 10, 20, 11, 22, 15, 1.$

Hamilton C_5-sevenfoil $= (29, 1, 2, 4, 8) \cup (29, 16, 3, 6, 12) \cup (29, 24, 19, 9, 18) \cup (29, 7, 14, 28, 27) \cup (29, 25, 21, 13, 26) \cup (29, 23, 17, 5, 10) \cup (29, 20, 11, 22, 15)$

Hamilton C_5-sevenfoil $= (29, 2, 4, 8, 16) \cup (29, 3, 6, 12, 24) \cup (29, 19, 9, 18, 7) \cup (29, 14, 28, 27, 25) \cup (29, 21, 13, 26, 23) \cup (29, 17, 5, 10, 20) \cup (29, 11, 22, 15, 1)$.

These 2 starters comprise a Hamilton C_5-sevenfoil decomposition of $5K_{29}$.

Example 3.2. Hamilton C_7-sevenfoil of $7K_{43}$.

$(n, g) = (43, 3)$ n-orbit : $1, 3, 9, 27, 38, 28, 41, 37, 25, 32, 10, 30, 4, 12, 36, 22, 23, 26, 35, 19, 14, 42, 40, 34, 16, 5, 15, 2, 6, 18, 11, 33, 13, 39, 31, 7, 21, 20, 17, 8, 24, 29, 1.$

Hamilton C_7-sevenfoil $= (43, 1, 3, 9, 27, 38, 28) \cup (43, 41, 37, 25, 32, 10, 30) \cup (43, 4, 12, 36, 22, 23, 26) \cup (43, 35, 19, 14, 42, 40, 34) \cup (43, 16, 5, 15, 2, 6, 18) \cup (43, 11, 33, 13, 39, 31, 7) \cup (43, 21, 20, 17, 8, 24, 29)$

Hamilton C_7-sevenfoil $= (43, 3, 9, 27, 38, 28, 41) \cup (43, 37, 25, 32, 10, 30, 4) \cup (43, 12, 36, 22, 23, 26, 35) \cup (43, 19, 14, 42, 40, 34, 16) \cup (43, 5, 15, 2, 6, 18, 11) \cup (43, 33, 13, 39, 31, 7, 21) \cup (43, 20, 17, 8, 24, 29, 1)$

Hamilton C_7-sevenfoil $= (43, 9, 27, 38, 28, 41, 37) \cup (43, 25, 32, 10, 30, 4, 12) \cup (43, 36, 22, 23, 26, 35, 19) \cup (43, 14, 42, 40, 34, 16, 5) \cup (43, 15, 2, 6, 18, 11, 33) \cup (43, 13, 39, 31, 7, 21, 20) \cup (43, 17, 8, 24, 29, 1, 3)$.

These 3 starters comprise a Hamilton C_7-sevenfoil decomposition of $7K_{43}$.

Example 3.3. Hamilton C_{11}-sevenfoil of $11K_{71}$.

$(n, g) = (71, 7)$ n-orbit : $1, 7, 49, 59, 58, 51, 2, 14, 27, 47, 45, 31, 4, 28, 54, 23, 19, 62, 8, 56, 37, 46, 38, 53, 16, 41, 3, 21, 5, 35, 32, 11, 6, 42, 10, 70, 64, 22, 12, 13, 20, 69, 57, 44, 24, 26, 40, 67, 43, 17, 48, 52, 9, 63, 15, 34, 25, 33, 18, 55, 30, 68, 50, 66, 36, 39, 60, 65, 29, 61, 1.$

Hamilton C_{11}-sevenfoil $= (71, 1, 7, 49, 59, 58, 51, 2, 14, 27, 47) \cup (71, 45, 31, 4, 28, 54, 23, 19, 62, 8, 56) \cup (71, 37, 46, 38, 53, 16, 41, 3, 21, 5, 35) \cup (71, 32, 11, 6, 42, 10, 70, 64, 22, 12, 13) \cup (71, 20, 69, 57, 44, 24, 26, 40, 67, 43, 17) \cup (71, 48, 52, 9, 63, 15, 34, 25, 33, 18, 55) \cup (71, 30, 68, 50, 66, 36, 39, 60, 65, 29, 61)$

Hamilton C_{11}-sevenfoil $= (71, 7, 49, 59, 58, 51, 2, 14, 27, 47, 45) \cup (71, 31, 4, 28, 54, 23, 19, 62, 8, 56, 37) \cup (71, 46, 38, 53, 16, 41, 3, 21, 5, 35, 32) \cup (71, 11, 6, 42, 10, 70, 64, 22, 12, 13, 20) \cup (71, 69, 57, 44, 24, 26, 40, 67, 43, 17, 48) \cup (71, 52, 9, 63, 15, 34, 25, 33, 18, 55, 30) \cup (71, 68, 50, 66, 36, 39, 60, 65, 29, 61, 1)$

Hamilton C_{11}-sevenfoil $= (71, 49, 59, 58, 51, 2, 14, 27, 47, 45, 31) \cup (71, 4, 28, 54, 23, 19, 62, 8, 56, 37, 46) \cup (71, 38, 53, 16, 41, 3, 21, 5, 35, 32, 11) \cup (71, 6, 42, 10, 70, 64, 22, 12, 13, 20, 69) \cup (71, 57, 44, 24, 26, 40, 67, 43, 17, 48, 52) \cup (71, 9, 63, 15, 34, 25, 33, 18, 55, 30, 68) \cup (71, 50, 66, 36, 39, 60, 65, 29, 61, 1, 7)$

Hamilton C_{11}-sevenfoil $= (71, 59, 58, 51, 2, 14, 27, 47, 45, 31, 4) \cup (71, 28, 54, 23, 19, 62, 8, 56, 37, 46, 38) \cup (71, 53, 16, 41, 3, 21, 5, 35, 32, 11, 6) \cup (71, 42, 10, 70, 64, 22, 12, 13, 20, 69, 57) \cup (71, 44, 24, 26, 40, 67, 43, 17, 48, 52, 9) \cup (71, 63, 15, 34, 25, 33, 18, 55, 30, 68, 50) \cup (71, 66, 36, 39, 60, 65, 29, 61, 1, 7, 49)$

Hamilton C_{11}-sevenfoil $= (71, 58, 51, 2, 14, 27, 47, 45, 31, 4, 28) \cup (71, 54, 23, 19, 62, 8, 56, 37, 46, 38, 53) \cup (71, 16, 41, 3, 21, 5, 35, 32, 11, 6, 42) \cup (71, 10, 70, 64, 22, 12, 13, 20, 69, 57, 44) \cup (71, 24, 26, 40, 67, 43, 17, 48, 52, 9, 63) \cup (71, 15, 34, 25, 33, 18, 55, 30, 68, 50, 66) \cup (71, 36, 39, 60, 65, 29, 61, 1, 7, 49, 59)$.

These 5 starters comprise a Hamilton C_{11}-sevenfoil decomposition of $11K_{71}$.

Example 3.4. Hamilton C_{17}-sevenfoil of $17K_{113}$.

$(n, g) = (113, 3)$ n-orbit : $1, 3, 9, 27, 81, 17, 51, 40, 7, 21, 63, 76, 2, 6, 18, 54, 49, 34, 102, 80, 14, 42, 13, 39, 4, 12, 36, 108, 98, 68, 91, 47, 28, 84, 26, 78, 8, 24, 72, 103, 83, 23, 69, 94, 56, 55, 52, 43, 16, 48, 31, 93, 53, 46, 25, 75, 112, 110, 104, 86, 32, 96, 62, 73, 106, 92, 50, 37, 111, 107, 95, 59, 64, 79, 11, 33, 99, 71, 100, 74, 109, 101, 77, 5, 15, 45, 22, 66, 85, 29, 87, 35, 105, 89, 41, 10, 30, 90, 44, 19, 57, 58, 61, 70, 97, 65, 82, 20, 60, 67, 88, 38, 1.$

Hamilton C_{17}-sevenfoil $= (113, 1, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...)$

Hamilton C_{17}-sevenfoil $= (113, 3, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...)$

Hamilton C_{17}-sevenfoil $= (113, 9, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...)$

Hamilton C_{17}-sevenfoil $= (113, 27, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...)$

Hamilton C_{17}-sevenfoil $= (113, 81, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup$

$(113, ...) \cup (113, ...)$
Hamilton C_{17}-sevenfoil $= (113, 17, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...)$
Hamilton C_{17}-sevenfoil $= (113, 51, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...)$
Hamilton C_{17}-sevenfoil $= (113, 40, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...) \cup (113, ...)$.
These 8 starters comprise a Hamilton C_{17}-sevenfoil decomposition of $17K_{113}$.

Example 3.5. Hamilton C_{19}-sevenfoil of $19K_{127}$.
$(n, g) = (127, 3)$ n-orbit : $1, 3, 9, 27, 81, 116, 94, 28, 84, 125, 121, 109, 73, 92, 22, 66, 71, 86, 4, 12, 36, 108, 70, 83, 122, 112, 82, 119, 103, 55, 38, 114, 88, 10, 30, 90, 16, 48, 17, 51, 26, 78, 107, 67, 74, 95, 31, 93, 25, 75, 98, 40, 120, 106, 64, 65, 68, 77, 104, 58, 47, 14, 42, 126, 124, 118, 100, 46, 11, 33, 99, 43, 2, 6, 18, 54, 35, 105, 61, 56, 41, 123, 115, 91, 19, 57, 44, 5, 15, 45, 8, 24, 72, 89, 13, 39, 117, 97, 37, 111, 79, 110, 76, 101, 49, 20, 60, 53, 32, 96, 34, 102, 52, 29, 87, 7, 21, 63, 62, 59, 50, 23, 69, 80, 113, 85, 1$.
Hamilton C_{19}-sevenfoil $= (127, 1, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...)$
Hamilton C_{19}-sevenfoil $= (127, 3, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...)$
Hamilton C_{19}-sevennfoil $= (127, 9, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...)$
...
Hamilton C_{19}-sevenfoil $= (127, 84, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...) \cup (127, ...)$.
These 9 starters comprise a Hamilton C_{19}-sevenfoil decomposition of $19K_{127}$.

Example 3.6. Hamilton C_{29}-sevenfoil of $29K_{197}$.
$(n, g) = (197, 2)$ n-orbit : $1, 2, 4, 8, 16, 32, 64, 128, 59, 118, 39, 78, 156, 115, 33, 66, 132, 67, 134, 71, 142, 87, 174, 151, 105, 13, 26, 52, 104, 11, 22, 44, 88, 176, 155, 113, 29, 58, 116, 35, 70, 140, 83, 166, 135, 73, 146, 95, 190, 183, 169, 141, 85, 170, 143, 89, 178, 159, 121, 45, 90, 180, 163, 129, 61, 122, 47, 94, 188, 179, 161, 125, 53, 106, 15, 30, 60, 120, 43, 86, 172, 147, 97, 194, 191, 185, 173, 149, 101, 5, 10, 20, 40, 80, 160, 123, 49, 98, 196, 195, 193, 189, 181, 165, 133, 69, 138, 79, 158, 119, 41, 82, 164, 131, 65, 130, 63, 126, 55, 110, 23, 46, 92, 184, 171, 145, 93, 186, 175, 153, 109, 21, 42, 84, 168, 139, 81, 162, 127, 57, 114, 31, 62, 124, 51, 102, 7, 14, 28, 56, 112, 27, 54, 108, 19, 38, 76, 152, 107, 17, 34, 68, 136, 75, 150, 103, 9, 18, 36, 72, 144, 91, 182, 167, 137, 77, 154, 111, 25, 50, 100, 3, 6, 12, 24, 48, 96, 192, 187, 177, 157, 117, 37, 74, 148, 99, 1$.
Hamilton C_{29}-sevenfoil $= (197, 1, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...)$
Hamilton C_{29}-sevenfoil $= (197, 2, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...)$
Hamilton C_{29}-sevenfoil $= (197, 4, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...)$
...
Hamilton C_{29}-sevenfoil $= (197, 115, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...) \cup (197, ...)$.
These 14 starters comprise a Hamilton C_{29}-sevenfoil decomposition of $29K_{197}$.

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