Automatic Design of Optimum Train Diagram

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Introduction

The problem of designing train diagrams is important, because it is related to the efficiency of train systems. It is necessary to introduce electronic information processing technique into train diagram design replacing human experience.

As too many factors must be considered in the estimation of an optimum diagram, it is theoretically impossible. One of the difficult points in searching for optimum diagrams is non-linearity of the estimating function. In the method described below, the question is solved by dividing the variable space into subspaces in which the linearity holds.

Although the new Tokaido trunk line is taken as an example, this method is applicable also to other systems with minor modifications.

1. *Railway System Model*

Railway systems are complicated and their operation has to be determined by various conditions and restrictions, which make the analysis thereof difficult. Therefore, a simplified system has been used and restrictions have been clarified.

(a) Diagrams of up-trains of the new Tokaido trunk line will be designed. The railway system includes 12 stations, i.e. Tokyo, Yokohama, Odawara, Atami, Shizuoka, Hamamatsu, Toyohashi, Nagoya, Hajima, Maibara, Kyoto and Osaka.

(b) Trains are divided into two classes—limited express trains (abbreviated as LET) which stop only at Nagoya except Tokyo and Osaka, and express trains (abbreviated as ET) which stop at every station.

(c) The priority of LETs is higher than ETs, and when the two classes of trains come too close, ET must wait at some station. This waiting, however, does not occur between two trains of the same class.

(d) Constants such as traveling time, stopping time, spacing and minimum time between two ETs are determined by the class of trains and the stations.

(e) Above constants do not vary either by accidents or operational errors.

(f) Diagrams are periodical and only one LET starts in one period.

(g) Criterion for estimating diagrams is the sum total of waiting times, which does not include the normal stopping time of ETs included in a single periodic cycle. Therefore, the optimum diagrams constitute those with minimum waiting time.

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2. *Analysis*

2.1. *Data for train operation*

Data for LETs are shown in Table 1 while those for ETs are shown in Table 2. The followings represent definitions of terms:

Table 1 Data for express trains.

	Arrive	Leave	Travelling Time	Stopping Time	Spacing
Tokyo	—	0.00.00	17.15	—	4.57
Yokohama	0.17.15	0.19.15	20.39	2.00	3.39
Odawara	0.39.54	0.41.54	10.34	2.00	3.35
Atami	0.52.28	0.54.28	28.15	2.00	3.34
Shizuoka	1.22.43	1.24.43	26.43	2.00	3.30
Hamamatsu	1.51.26	1.53.26	16.07	2.00	3.31
Toyohashi	2.09.33	2.11.33	25.42	2.00	3.29
Nagoya	2.37.15	2.40.15	12.25	3.00	3.34
Hajima	2.52.40	2.54.40	18.03	2.00	3.36
Maibara	3.12.43	3.14.43	26.44	2.00	3.42
Kyoto	3.41.27	3.43.27	16.33	2.00	3.29
Osaka	4.00.00	—		—	—

Table 2 Data for limited express trains.

	Arrive	Leave	Travelling Time	Forbidden Band
Tokyo	—	0.00.00	14.05	—0.04.47—0.04.55
Yokohama	0.14.29	0.14.29	16.05	0.08.00—0.15.23
Odawara	0.30.34	0.30.34	6.10	0.24.03—0.31.26
Atami	0.36.44	0.36.44	23.03	0.29.36—0.37.43
Shizuoka	0.59.47	0.59.47	22.14	0.53.30—1.00.42
Hamamatsu	1.22.01	1.22.01	11.01	1.14.04—1.22.58
Toyohashi	1.33.02	1.33.02	23.57	1.26.57—1.33.54
Nagoya	1.56.59	1.58.59	9.31	1.54.18—2.00.41
Hajima	2.08.30	2.08.30	13.15	2.01.50—2.09.27
Maibara	2.21.45	2.21.45	22.42	2.14.38—2.22.42
Kyoto	2.44.27	2.44.27	15.33	2.39.05—2.45.55
Osaka	3.00.00	—		—

*Leaving and arriving time: time required for passengers to get on and off.

*Spacing: the manimum time interval to be reserved when two ETs successively leave a station.

*Forbidden band: ETs must not leave a station at any time in this band, including leaving time of an LET.

2.2. *Algorithm to search for optimum diagrams*

Let t be a period of a diagram. Leaving times of LETs can be assumed without loss of generality as

$$\ldots,\ -2t,\ -t,\ 0,\ t,\ 2t,\ \ldots \tag{1}$$

t must have its upper limit, else at least one ET can reach Osaka without being overtaken by LET. This bound can be obtained from Tables 1 and 2.

$$t \leq 1 \text{ hour } 9 \text{ minutes } 16 \text{ seconds} \tag{2}$$

Let

$$S_1,\ S_2,\ \ldots,\ S_{10} \tag{3}$$

be Yokohama, Odawara, ..., Kyoto. Let m be the number of stations at which ETs can wait for the first time. This value is determined by t.

$$1 \leq m \leq 10 \tag{4}$$

Then the stations which can be taken as the first waiting stations are

$$S_1,\ S_2,\ \ldots,\ S_m \tag{5}$$

Let n be the number of ETs included in a period. The 1st ET is defined as one which leaves Tokyo immediately after LET does, and the 2nd ET is one which leaves Tokyo immediately after the 1st ET does and so on.

Consider now a vector,

$$K=(k_1,\ k_2,\ \ldots,\ k_n),\quad m \geqq k_1 \geqq k_2 \geqq k_3 \ldots \geqq k_n \geqq 1 \tag{6}$$

Let K correspond to the following combination of the first waiting stations:

$$(S_{k_1},\ \ldots,\ S_{k_n}) \tag{7}$$

This means that the i-th train waits at station S_{ki}. Then vector K corresponds to one of the diagrams in which the i-th train waits at Station S_{ki} $(1 \leq i \leq n)$. Although many diagrams which satisfy above condition are available, it is easy to decide on the diagram with minimum waiting time. Let z be total waiting time of such local optimum diagram.

$$z(n, t; K)=z(n, t; k_1, k_2, \ldots, k_n) \tag{8}$$

In order to obtain the local opitimum diagram for given K, leaving times at Tokyo are made later and arriving times at Osaka earlier, while the first waiting stations are fixed. This algorithm is illustrated in Fig. 1 and the diagram is designed in the numbered order.

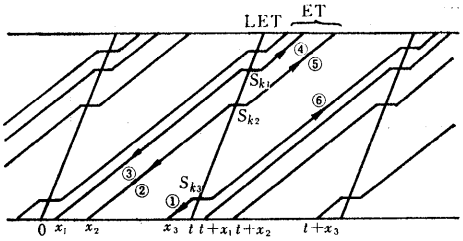

Fig. 1 A local optimum diagram.

Then the waiting time of the optimum diagram for given n and t must be

$$w(n, t) = \text{minimum for all } K \ \{z(n, t; K)\} \tag{9}$$

As the number of combinations of K is finite, it is possible for a computer to find the optimum diagram in finite time.

2.3. *Restriction of waiting line*

Only one waiting line is available at each station in real system (this is called restricted system). It is, however, theoretically interesting to solve the problem without such restriction (this is called non-restricted system). Then an attempt was made to find solutions for both cases. The number of combinations of K is much larger in non-restricted system than in restricted system.

2.4. *Program*

A program was fed into CDC G–20 computer. The whole program consisted of 1400 words. The flow chart is as shown in Fig. 2. The machine took 1 hour

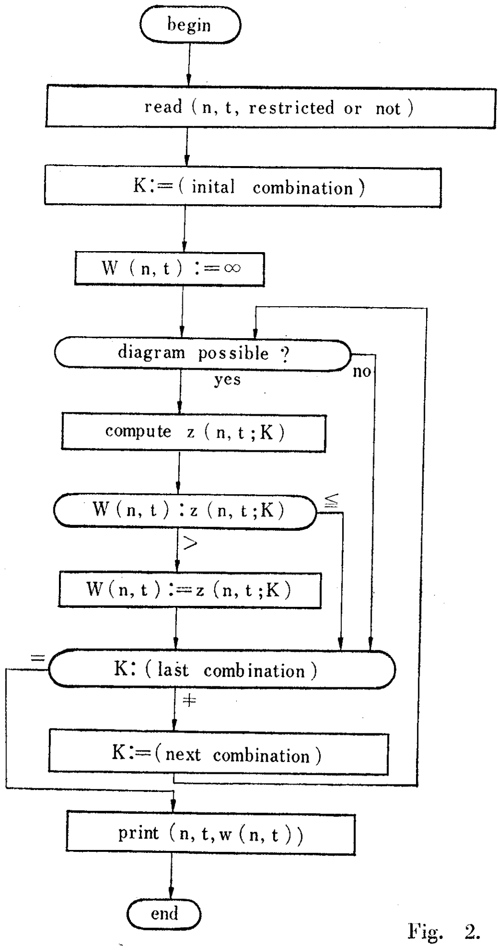

Fig. 2.

and 40 minutes to find solution to the restricted system and 15 hours for the non-restricted system.

3. *Results*

The train system is determined by fixing the number of ETs, n and cycle time t. t must satisfy the equation (2) and n must not be greater than 10 in restricted system and 13 in non-restricted system. Fig. 3 shows waiting time function $w(n, t)$ for all possible cases. Solid lines represent the former case and broken lines, the latter.

Cycle time t can be considered time band on a railway. An LET consumes some part of it which is equal to the forbidden bands in Table 2. On the other hand, an ET consumes some time band equal to the spacings in Table 1. When n ETs must be included in t, let t_m be the left time band for n-th train,

$$t_m = t - (\text{forbidden band}) - (n-1) \times (\text{spacing}) \tag{10}$$

Figs. 4 and 5 show $w\ (n, t)$ but in this case their abscissas represent t. In both cases the graphs show hyperbolical trends. It may, therefore, be said that t represents difficulty in the design of diagrams.

Conclusion

Minimum waiting time can be obtained from Fig. 3 for any cycle time and number of ETs. And Fig. 4 and 5 show that time band and maximum transporta-

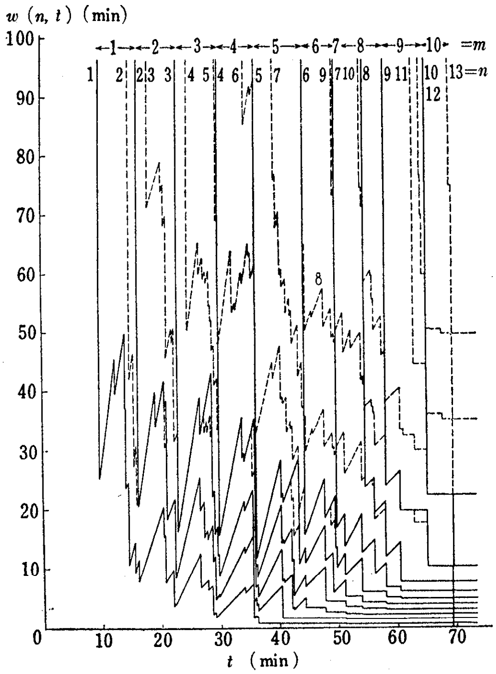

Fig. 3 Minimum total waiting time $W(n, t)$.

tion capacity are closely correlated. The method described in this report resembles the idea of human diagram designer. Therefore, it can be used in real diagram design and train control.

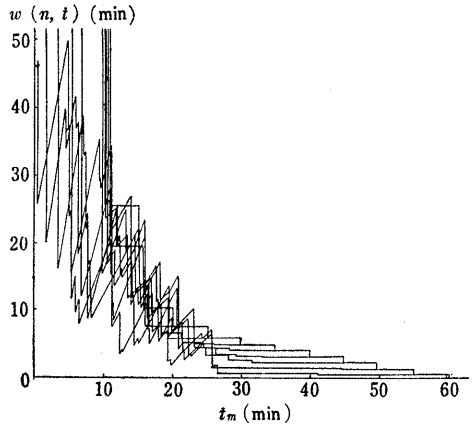

Fig. 4 $W(n, t)-t_m$ in restricted system.

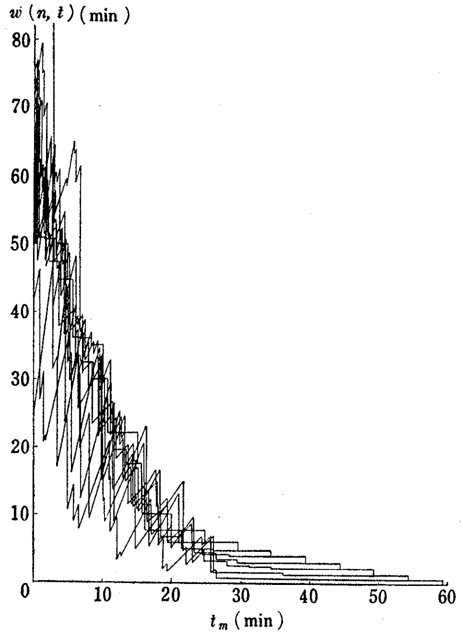

Fig. 5 $W(n, t)-t'_m$ in non-restricted system.