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On the Centroid and the Spread of a Character Figure

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1. *Introduction*

The subject concerning with the print quality has been gradually taken up with the progress of research and development on the character recognition by machine. Several fonts, which are designed considering the variety of printed characters and the deterioration of their quality, have been put into practical use. Most of these fonts, however, have not been examined from the qualitative point of view so far.

It is considered that the psychical and/or sensuous characteristics of a character figure consist of beautifulness, uniformity, concord, legibility, discrimination, and so forth. These characteristics may be estimated from the centroid, size, counter, density, stroke-width, and geometrical shape, etc. Most of these factors, though, hardly be measured or quantified. In this paper, it has been intended to get some of these factors, the centroid and the spread. The position of the centroid of a character is useful for the evaluation of the legibility and the stability of the characters in printed, and the spread of a character gives the sensuous character size. The uniformity of these quantities over all characters in a font would make us "agreeabl for eyes".

The main results derived in this paper are as follows:

A figure $f(\boldsymbol{r})$ is defined in the $x-y$ plane Ω, where $\boldsymbol{r}$ is the column vector which represents a point

$$\boldsymbol{r}=\boldsymbol{i}x+\boldsymbol{j}y \qquad (1)$$

$(\boldsymbol{i}, \boldsymbol{j}$: unit vector)

in the plane Ω, and the function f takes a positive value proportion to the dark part of the pattern, and is zero for a white part.

$u(\boldsymbol{r})$ is a weight function which satisfies the condition:

$$\iint u(\boldsymbol{r})dxdy=1, \quad u(\boldsymbol{r})\geq 0, \qquad (2)$$

and determines "a visual field".

For simplicity, we employ expression used in the vector analysis from now. Then, the centroids $\boldsymbol{a}_0's$ of "a visual field", of a general figure, and of an ideal black and white figure are respectively defined by

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$$\boldsymbol{a}_0=\iint_\Omega u(\boldsymbol{r})\boldsymbol{r}dxdy=\iint_\Omega u(\boldsymbol{r})\boldsymbol{r}f(\boldsymbol{r})dxdy\Big/\iint_\Omega u(\boldsymbol{r})f(\boldsymbol{r})dxdy \qquad (3.1),\ (3.2)$$

$$=\iint_\Omega u(\boldsymbol{r})\boldsymbol{r}\{f(\boldsymbol{r})-K_0\}^2dxdy\Big/\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K_0\}^2dxdy. \qquad (3.3)$$

The spreads $\sigma_0's$ are also defined as follows:

$$\sigma_0^2=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)dxdy \qquad (4.1)$$

$$=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)f(\boldsymbol{r})dxdy\Big/\iint_\Omega u(\boldsymbol{r})f(\boldsymbol{r})dxdy \qquad (4.2)$$

$$=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)\{f(\boldsymbol{r})-K_0\}^2dxdy\Big/\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K_0\}^2dxdy \qquad (4.3)$$

(T means transposition).

Where $K_0's$ are the basic brightness which depend on the darkness of a figure and the brightness of background paper, and $K_0's$ are respectively defined by

$$K_0=\iint_\Omega u(\boldsymbol{r})f(\boldsymbol{r})dxdy=\iint_\Omega u(\boldsymbol{r})\boldsymbol{r}f(\boldsymbol{r})dxdy\Big/\iint_\Omega u(\boldsymbol{r})\boldsymbol{r}dxdy \qquad (5.1),\ (5.2)$$

$$=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)f(\boldsymbol{r})dxdy\Big/\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)dxdy \qquad (5.3)$$

If a given figure is an ideal black and white one, all these three kinds of defining expressions take the same value.

2. *Fundamental Quantity Comprising the Information of a Figure*

Let $f(\boldsymbol{r})-K$ be a relative brightness of a figure $f(\boldsymbol{r})$ measured from the basic brightness K. Then, the information of a figure per unit area at any point $\boldsymbol{r}$ observed from a restricted point $\boldsymbol{a}$ may be represented by $u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a})\cdot\{f(\boldsymbol{r})-K\}^2$, and the total information of a figure $I(\boldsymbol{a},K)$ is defined by

$$I(\boldsymbol{a},K)\equiv\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a})\{f(\boldsymbol{r})-K\}^2dxdy \qquad (6)$$

The quantity $I(\boldsymbol{a},K)$ is named the fundamental quantity comprising the information of a figure $f(\boldsymbol{r})$. Following theory shall be developed based on this fundamental quantity $I(\boldsymbol{a},K)$.

From the second derivatives of $I(\boldsymbol{a},K)$ with respect to $\boldsymbol{a}$ and K, two functions $J(K)$ and $L(\boldsymbol{a})$ are defined by

$$J(K)\equiv\frac{1}{2}\nabla_{\boldsymbol{a}}\cdot\nabla_{\boldsymbol{a}}I(\boldsymbol{a},K)=\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K\}^2dxdy \qquad (7)$$

$$L(\boldsymbol{a})\equiv\frac{1}{2}\frac{\partial^2}{\partial K^2}I(\boldsymbol{a},K)=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a})dxdy \qquad (8)$$

Then, the spread measure σ and the intensity measure A are derived from Eqs. (6), (7) and (8) as follows:

$$\sigma^2=\frac{I(\boldsymbol{a},K)}{J(K)}=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a})\{f(\boldsymbol{r})-K\}^2dxdy\Big/\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K\}^2dxdy \qquad (9)$$

$$A^2 \equiv \frac{I(\boldsymbol{a}, K)}{L(\boldsymbol{a})} = \frac{\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a})\{f(\boldsymbol{r})-K\}^2 dxdy}{\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a}) dxdy} \tag{10}$$

3. *The Condition of Stationarity*

The quantities σ and A given by the Eqs. (9) and (10) vary according to the values $\boldsymbol{a}$ and K. Now, we determine $\boldsymbol{a}$ and K so as to make both $\boldsymbol{a}$ and K stationary values, and denote these values $\boldsymbol{a}_0$ and K_0. In order to obtain these $\boldsymbol{a}_0$ and K_0, differentiating Eqs. (9) and (10) with respect to $\boldsymbol{a}$ and K respectively, and equalize to zero, the following equations are obtained:

$$\left\{\begin{array}{ll} \nabla_{\boldsymbol{a}}\sigma^2 = \nabla_{\boldsymbol{a}} I(\boldsymbol{a}, K)/J(K) = \boldsymbol{0} & (11) \\ \dfrac{\partial}{\partial K}\sigma^2 = \left\{J(K)\dfrac{\partial}{\partial K}I(\boldsymbol{a}, K) - I(\boldsymbol{a}, K)\dfrac{\partial}{\partial K}J(K)\right\}\Big/ J^2(K) = 0 & (12) \\ \nabla_{\boldsymbol{a}} A^2 = \{L(\boldsymbol{a})\nabla_{\boldsymbol{a}} I(\boldsymbol{a}, K) - I(\boldsymbol{a}, K)\nabla_{\boldsymbol{a}} L(\boldsymbol{a})\}/L^2(\boldsymbol{a}) = \boldsymbol{0} & (13) \\ \dfrac{\partial}{\partial K}A^2 = \dfrac{\partial}{\partial K} I(\boldsymbol{a}, K)\Big/ L(\boldsymbol{a}) = 0 & (14) \end{array}\right.$$

Rearranging these equations, we have

$$\left\{\begin{array}{ll} \nabla_{\boldsymbol{a}} I(\boldsymbol{a}, K) = (-)2\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})\{f(\boldsymbol{r})-K\}^2 dxdy = \boldsymbol{0} & (15) \\ \dfrac{\partial}{\partial K} I(\boldsymbol{a}, K) = (-)2\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a})^T(\boldsymbol{r}-\boldsymbol{a})\{f(\boldsymbol{r})-K\} dxdy = 0 & (16) \\ \nabla_{\boldsymbol{a}} L(\boldsymbol{a}) = (-)2\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}) dxdy = \boldsymbol{0} & (17) \\ \dfrac{\partial}{\partial K} J(K) = (-)2\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K\} dxdy = 0 & (18) \end{array}\right.$$

Solving these four equations simultanously, following relations are obtained:

$$\left\{\begin{array}{ll} \boldsymbol{a}_0 = \iint_\Omega u(\boldsymbol{r})\boldsymbol{r} dxdy & (3.1) \\ \quad = \iint_\Omega u(\boldsymbol{r})\boldsymbol{r}\{f(\boldsymbol{r})-K_0\}^2 dxdy\Big/\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K_0\}^2 dxdy & (3.3) \\ K_0 = \iint_\Omega u(\boldsymbol{r})f(\boldsymbol{r}) dxdy & (5.1) \\ \quad = \iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)f(\boldsymbol{r}) dxdy\Big/\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0) dxdy & (5.3) \end{array}\right.$$

Eqs. (9) and (10) give the stationary value σ_0 and A_0:

$$\sigma_0{}^2 = \iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)\{f(\boldsymbol{r})-K_0\}^2 dxdy\Big/\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K_0\}^2 dxdy \tag{4.3}$$

$$A_0{}^2 = \iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)\{f(\boldsymbol{r})-K_0\}^2 dxdy\Big/\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0) dxdy \tag{19}$$

Four equations are used in order to determine the values $\boldsymbol{a}_0$ and K_0. This means that there should exist special relationship between the figure $f(\boldsymbol{r})$ and the weight function $u(\boldsymbol{r})$.

4. *In Case of Ideal Black and White Figure*

If a given figure $f(\boldsymbol{r})$ is an ideal black and white one, the figure $f(\boldsymbol{r})$ satisfies

$$f^2(\boldsymbol{r}) \equiv k f(\boldsymbol{r}), \quad k>0. \tag{20}$$

Indeed, Eq. (20) yields

$$f(\boldsymbol{r})=0 \quad \text{or} \quad k. \tag{21}$$

Then, following relation is obtained:

$$\iint_\Omega u(\boldsymbol{r}) f^2(\boldsymbol{r}) dxdy = k\iint_\Omega u(\boldsymbol{r}) f(\boldsymbol{r}) dxdy = kK_0 \tag{22}$$

from Eqs. (5.1) and (20), and considering Eq. (21) it is noticed that the value satisfies the following relation:

$$0<K_0<k. \tag{23}$$

Eqs. (7) and (22) give

$$J(K_0)=\iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K_0\}^2 dxdy = K_0(k-K_0) \tag{24}$$

and also, Eqs. (3.1) and (21) give

$$\iint_\Omega u(\boldsymbol{r})\boldsymbol{r}\{f(\boldsymbol{r})-K_0)\}^2 dxdy = (k-2K_0)\iint_\Omega u(\boldsymbol{r})\boldsymbol{r} f(\boldsymbol{r}) dxdy + K_0^2 \boldsymbol{a}_0. \tag{25}$$

Therefore, we obtain

$$K_0\boldsymbol{a}_0 = \iint_\Omega u(\boldsymbol{r})\boldsymbol{r} f(\boldsymbol{r}) dxdy. \tag{26}$$

Combining (26) to (5.1) yields

$$\boldsymbol{a}_0 = \iint_\Omega u(\boldsymbol{r})\boldsymbol{r} f(\boldsymbol{r}) dxdy \Big/ \iint_\Omega u(\boldsymbol{r}) f(\boldsymbol{r}) dxdy. \tag{3.2}$$

Combining Eqs. (3.1) to (26) yields

$$K_0 = \iint u(\boldsymbol{r})\boldsymbol{r} f(\boldsymbol{r}) dxdy \Big/ \iint_\Omega u(\boldsymbol{r})\boldsymbol{r} dxdy. \tag{5.2}$$

As the next step, we have

$$I(\boldsymbol{a}_0, K_0) = \iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)\{f(\boldsymbol{r})-K_0\}^2 dxdy = (k-K_0)K_0 L(\boldsymbol{a}_0) \tag{27}$$

from Eqs. (6), (20), (5.3) and (8). Substituting this Eq. (27) to Eq. (10), we obtain

$$A_0^2 = K_0(k-K_0). \tag{28}$$

Comparing this Eq. (28) to Eq. (24), the following relation is obtained

$$A_0^2 = J(K_0) = \iint_\Omega u(\boldsymbol{r})\{f(\boldsymbol{r})-K_0\}^2 dxdy. \tag{29}$$

Combining Eqs. (29), (3.1) and (3.2), we have

$$A_0^2 = \iint_\Omega u(\boldsymbol{r})\boldsymbol{r}\{f(\boldsymbol{r})-K_0\}^2 dxdy \Big/ \iint_\Omega u(\boldsymbol{r})\boldsymbol{r} dxdy. \tag{30}$$

The following relation is given

$$I(\boldsymbol{a}_0, K_0)=L(\boldsymbol{a}_0)J(K_0) \tag{31}$$

from Eqs. (27), (28) and (29), consequently; Substituting Eq. (31) to Eq. (9) yields

$$\sigma_0^2=L(\boldsymbol{a}_0)=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)dxdy. \tag{4. 1}$$

Combining this Eq. (4. 1) to Eqs. (5. 1) and (5. 3) gives

$$\sigma_0^2=\iint_\Omega u(\boldsymbol{r})(\boldsymbol{r}-\boldsymbol{a}_0)^T(\boldsymbol{r}-\boldsymbol{a}_0)f(\boldsymbol{r})dxdy\Big/\iint_\Omega u(\boldsymbol{r})f(\boldsymbol{r})dxdy. \tag{4. 2}$$

It has been comprehended that the three kinds of defining expressions for the cenroid and for the spread of a figure described in Section 1 are reasonable.

Expanding $I(\boldsymbol{a}, K)$ arround $\boldsymbol{a}=\boldsymbol{a}_0$ and $K=K_0$, we obtain

$$I(\boldsymbol{a}, K)=\{A_0^2+(K-K_0)^2\}\{\sigma_0^2+(\boldsymbol{a}-\boldsymbol{a}_0)^T(\boldsymbol{a}-\boldsymbol{a}_0)\} \tag{32}$$

therefore, the following relation is obtained

$$I(\boldsymbol{a}, K)\geqq A_0^2\sigma_0^2=I(\boldsymbol{a}_0, K_0). \tag{33}$$

This Eq. (33) indicates that $I(\boldsymbol{a}_0, K_0)$ is the minimum value of $I(\boldsymbol{a}, K)$. It is noticed that the determining method of $\boldsymbol{a}_0$ and K_0 mentioned above is the process of eliminating the nonessential information of a figure.

5. *The Weight Function and the Defining Region of a Figure*

Now, we consider the problem to determine a finite region R from a given figure, when the weight function is given as a uniform value over the region.

The weight function $u(\boldsymbol{r})$ with uniform weight in a finite region R is given by

$$u(\boldsymbol{r})=\begin{cases}1/R: & \boldsymbol{r}\in R\\ 0: & \boldsymbol{r}\in R\end{cases} \tag{34}$$

where R also indicates the area of the region R. In this section, we make the assumption that a figure $f(\boldsymbol{r})$ is a ideal black and white one, and satisfies the following conditions:

$$\begin{cases}f^2(\boldsymbol{r})\equiv f(\boldsymbol{r}) & (35)\\ \iint_\Omega f(\boldsymbol{r})dxdy<\infty. & (36)\end{cases}$$

We take the basic region R_0 defined as follows:

$$\begin{cases}\dfrac{1}{R_0}\iint_{R_0}\boldsymbol{r}dxdy=0 & (37)\\ \dfrac{1}{R_0}\iint_{R_0}\boldsymbol{r}^T\boldsymbol{r}dxdy=\rho_0^2. & (38)\end{cases}$$

Now we assume the region $R\equiv R(\boldsymbol{a}, \lambda)$ is a similar figure with the basic region R_0 translated the position as $\boldsymbol{a}$ and expanded or contracted the area as λ times.

If a figure is entirely contained in the region R, $\boldsymbol{a}_0$ and λ_0 can be determined by the following equations:

$$\boldsymbol{a}_0 \equiv \iint_R \boldsymbol{r} f(\boldsymbol{r}) dx dy \Big/ \iint_R f(\boldsymbol{r}) dx dy \tag{39}$$

$$\sigma_0{}^2 \equiv \iint_R (\boldsymbol{r}-\boldsymbol{a}_0)^T (\boldsymbol{r}-\boldsymbol{a}_0) f(\boldsymbol{r}) dx dy \Big/ \iint_R f(\boldsymbol{r}) dx dy \tag{40}$$

$$\lambda_0 = \sigma_0 / \rho_0, \tag{41}$$

therefore, the region R is defined. In the region R defined in this manner, as a matter of course, all equations described in Sections 2, 3 and 4 are valid.

Examples of the Basic Region R_0

i) A rectangular region with area $2b_0 \times 2c_0$

$$\rho_0{}^2 = \frac{1}{4b_0c_0}\int_{-b_0}^{b_0} dx \int_{-c_0}^{c_0} (x^2+y^2) dy = \frac{1}{3}(b_0{}^2 + c_0{}^2)$$

ii) An elliptic region with major axis $2b_0$ and minor axis $2c_0$

$$\rho_0{}^2 = \frac{4}{\pi b_0 c_0}\int_0^{b_0} dx \int_0^{\frac{c_0}{b_0}\sqrt{b_0{}^2-x^2}} (x^2+y^2) dy = \frac{1}{4}(b_0{}^2 + c_0{}^2)$$

iii) An equilateral trianguler region with the base $2b_0$, the height c_0

$$\rho_0{}^2 = \frac{2}{b_0c_0}\int_0^{b_0} dx \int_0^{c_0\left(1-\frac{x}{b_0}\right)} (x^2+y^2) dy = \frac{1}{6}\left(b_0{}^2 + \frac{c_0{}^2}{3}\right)$$

Fig. 1 shows circular, quadrirateral, and regular triangular basic regions with equal ρ_0.

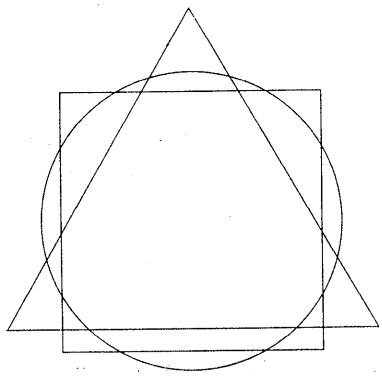

Fig. 1. Circular, quadrilateral, and regular triangular basic regions with equal ρ_0.

6. *An Example of Font Evaluation*

The theory described above is concerned with the centroid and the spread of one character figure. The legibility and the size of characters in a font would be measured by the uniformity of the centroids along the printing direction (a_x) and the uniformity of the spreads (σ_0).

Now, we evaluate the upper case alphabet in OCR-B font (recommended by the International Organization for Standardization). A character drawing on the $x-y$ plane is sampled by the matrix whose cell has 0.1×0.1 squre millimeters area. The value of a cell is one when a cell is contained in the stroke more than fifty percent, and otherwise the value is zero.

The basic region R_0 is determined by the mean of the character boundaries (the character boundary is the rectangle with sides parallel and perpendicular to the reference line which is tangental to the character out line and contains the character completely). The basic region R_0, in our case, is the rectangle and has 1.6×2.5 squrer millimeters area.

The equations for numerical calculation are as follows:
The horizontal coordinate of the centroid: a_x

$$a_x=\sum_{i,j\in R} i\cdot f(i,j)\Big/\sum_{i,j\in R} f(i,j)$$

The vertical coordinate of the controid: a_y

$$a_y=\sum_{i,j\in R} j\cdot f(i,j)\Big/\sum_{i,j\in R} f(i,j)$$

The spread: σ

$$\sigma=\sqrt{\sum_{i,j\in R}\{(i-a_x)^2+(j-a_y)^2\}\cdot f(i,j)\Big/\sum_{i,j\in R} f(i,j)}$$

The spread of the basic region: ρ_0

$$\rho_0=\sqrt{\sum_{i,j\in R_0}\{(i-\bar{a}_x)^2+(j-\bar{a}_y)^2\}\cdot 1\Big/\sum_{i,j\in R_0} 1}$$

Where

$f(i,j)=0$ or 1: binary matrix pattern
R: the region containing a figure
R_0: basic region
$\bar{a}_x, \bar{a}_y$: the position of the centroid of the basic region R_0.

Table 1 shows the mean values, standard deviations, and the ranges of the centroids and the spreads; calculated using above equations.

Table 1. The mean values, standard deviations, and ranges of the centroids and the spreads.

(in micron)

	mean value	standard deviation	range
a_x	−41	69	272
a_y	3	126	614
σ	877	46	175

It can not be expressed with certainty on the subjects that how to define the shape of the basic region, and how to restrict fluctuations of the values of the spreads and the centroids. The grade of "goodness" of the font, however, can be noticed by these results.

If the horizontal coordinate of the centroid coincide with the reference line for character spacing or is biased one direction, the font would be judged to be a legible one. The computed results of a_x exist with-in the reasonable range. Besides; the sensuous size of a character figure varies according to the value of the spread. Looking over the calculated results of σ, there is no value exceedingly deviate from the mean. Consequently, the OCR-B (upper casealphabet) font is judged to be a ligible one.

Fig. 2 shows the defining region of each character which is derived from varying the R_0 according to the spread value of each character. The frames indicating the spread quantities are drawn in the manner coinciding the centroids of the defining regions with those of characters.

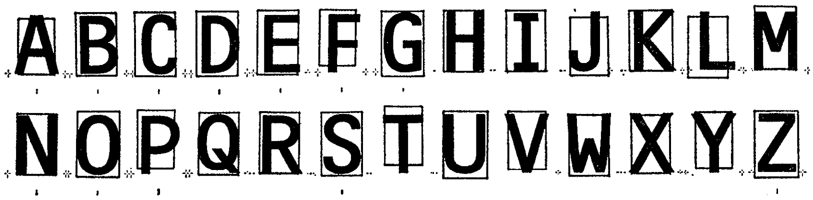

Fig. 2. The definition areas of the OCR-B font (upper case letters).

7. *Conclusion*

Generally it is very difficult to clarify what psychical and/or sensuous qualities of a character figure should be observed and how to be quantified them. The abovementioned matters are only a part of this quantification subject: the centroid and the spread of a character figure.

The experimental results for the upper case alphabet in the OCR-B font have comparatively good uniformity of the centroids and that of the spreads, moreover; the quantified centroid and the spread take similar quantity to the psychical and/or sensuous quantities (of the human race), and those facts have confirmed the validity of authors' theory.

The fundamental quantity comprising the information of a character figure may give an abrupt impression, however; the centroid $\boldsymbol{a}_0$ and the spread σ, which are defined by requiring the stationarity conditions of the fundamental quantity, correspond to psychical and/or sensuous quantities. This fact makes the fundamental quantity comprising the information of a figure reasonable.

The statistical consideration of the sensuous quality and authors' theory, and the discussion on the superiority or inferiority of fonts each other are next subjects. It is also important subject clarifying the other factors generally determining the psychical and/or sensuouas quantities, and quantifying them reasonably.