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# ピッチ・クラス集合論に基づく楽曲分析プログラムの実装と問題点について

高岡 明

東京工科大学

現在までに書かれたピッチ・クラス集合論に基づく楽曲分析プログラムは、そのほとんどが $T_n/T_nI$ type の導出に機能が限定されている。"Dubiel" は、より多くの機能を持つ分析プログラムである。楽曲分析においてピッチ・クラス集合の特定を自動化するためには、pc-set の分割基準が明確でなければならない。ピッチ・クラス集合論は、ピッチ集合の抽象化によって、記述の一般性に優れるが、それ自体は分割の基準を提供することはできない。無調音楽における pc-set の分割に際しては、Butler の "Intervallic Rivalry Model" が有効であると思われる。Butler のモデルに基づき、pc-set の分割を自動化するメッソドを Dubiel に実装した。

# On a Computer Program for Pitch-Class-Set Analysis: Its Implementation and Related Issues

Akira Takaoka

Tokyo University of Technology

Most computer programs for pitch-class-set analysis which have appeared to date are limited to finding the $T_n/T_nI$ type of a given pc-set. By contrast, "DUBIEL," written in Java, is currently the most feature-rich application for pc-set analysis. Although the implementation of most classes and methods is rather straightforward, that of the method of identifying pc-sets in a composition involves the thorny issue of segmentation in atonal theory. An automated segmentation of pc-sets seems possible by means of Butler's "Intervallic Rivalry Model" A method based on the model is implemented in Dubiel so that it can perform automated segmentation of pc-sets.

# 1 Introduction: Methods to be Implemented

Since pitch-class-set analysis ("pc-set analysis" for short) involves a number of tedious, mechanical arithmetic calculations, it seems quite natural that some computer programs have been written to assist analysts.[1] Brinkman (1990) provides us with various functions, which can be an integral part of a program for pc-set analysis as well as a complete program. His programs utilize bit-map operations, which run faster with a trade-off of a lack of portability because of their reliance on hardware-specific features. Recent advances in micro-processors seem to make such an approach unnecessary. In addition, Castine (1994) discusses object-oriented features of his program based on pc-set theory.

Most programs currently available are limited to finding the $T_n/T_nI$ type of a given pc-set. All of these programs miss some functions I would like to use. For this reason, I have written my own program called "DUBIEL." DUBIEL, written in Java, is currently the most feature-rich application for pc-set analysis. DUBIEL can tell, for example, whether or not a pc-set can be derived as a vertical or from a given superset; whether or not an ordered pc-set can be derived from a series form and, if so, by what operations.

The program offers many other functions, or methods to find the $T_n/T_nI$ type, the $I$-vector, and $M_7$ of a given pc-set; to find the $T_n/T_nI$ type through its name; to find the $T_n/T_nI$ through an $I$-vector; to find if a pc-set is a subset of or derived from another; to find the number of invariant pc's under a given operation; to find if an ordered pc-set is a subset of or derived from another; to list the $M_n$'s of a pc-set and its $T_n$, $T_nI$, and $T_n/T_nI$; to list all the $T_n$'s, $T_nI$'s, $RT_n$'s, $RT_nI$'s of an ordered pc-set; to list all the $r$'s, $r_nT_n$'s, $r_nT_nI$'s, $r_nRT_n$'s, $r_nRT_nI$'s of an ordered pc-set; to list all the subsets of a pc-set; and so on.

# 2 Toward Automated Segmentation of Pc-Sets

Pitch-class set theory indeed provides us with powerful tools to represent pitch relations. Because of its generality and formalism resulting from the abstraction of currently used particular music-theoretical notions, the theory can describe virtually any pitch contents and relations, regardless of stylistic characteristics, whether atonal or tonal.

The first step of abstraction, taken by Milton Babbitt, in the theory is, needless to say, the introduction of the notion "pitch-class," which stands for the generalization of particular "pitches." Further generalizations in terms of class-inclusion relationship have been proposed by some other theorists. Extensive use of symbols representing more inclusive classes of musical entities, properties and relations, is a natural consequence of the development of the theory. Forte's theory of set-complexes (Forte, 1973), for example, can be regarded as the abstractions and generalizations of particular relationships expressed in conventional music-theoretical metaphors such as "chord progressions."

In order to describe musical entities in atonal music using pitch-class set theory, some

[1] Please refer to Rahn (1980) for the basics of pc-set theory.

new subclassifications defined by the properties are inevitable. Therefore, theorists using pitch-class set theory need some way of classifying pc-sets. This is done by "similarity" or "relatedness" because similar or related pc-sets or interval-vectors should belong to the same classes, or musical kinds. For example, Forte's theory of set-complexes (Forte 1973) is in part based on this notion. Other theorists relying on this notion include Isaacson (1990), Lewin (1979–80), Lord (1981), Morris (1979–80), Rahn (1979–80), and Teitelbaum (1965).

Then, some theorists such as Forte and Morris (1979–80) have introduced ways of measuring the degrees of similarities among pitch-class sets, by which pitch-class sets are classified into collections in terms of some commonly shared properties such as the number of common pitch-classes between "two non-equivalent sets of the same cardinal number," particular interval contents, combinatoriality, and so on.

These approaches are based on the same assumption as the following: Natural kinds such as *Man* and *Wolf* can be represented as sets of particular entities.[2] However, since any things can be brought together into a set, there must be some conditions or criteria which all the members of a set must satisfy so that the set will be a kind. Such conditions are particular properties that all the members of a set possess. Thus natural kinds can be represented by listing all the and only the properties, which all the members of a set possess.

This assumption, however, is susceptible to several serious flaws. In the first place, although the set *Man* could somehow be defined with respect to its properties, they do not necessarily seem to correspond to our standard beliefs about man. *Man* can sufficiently be defined in any of but not limited to the following ways:

$$\begin{aligned}
&Man1 = \{x | Walking_Upright(x) \wedge Being_a_Mammal(x)\} \\
&Man2 = \{x | Having_a_Doubly_Articulated_Language(x)\} \\
&Man3 = \{x | Having_a_Human_DNA(x)\} \\
&\vdots
\end{aligned}$$

Obviously, not only *Man*2 and *Man*3 but also even *Man*1 would not typically represent our standard beliefs about man. That is, just specifying sufficient properties is inadequate to represent *Man*. Therefore, there must be a set of some other properties that play a major role to determine the class *Man*. So our task might be to find a set of adequate properties that do not conflict with our standard beliefs.

However, it might even be impossible to use properties as criteria that distinguish natural kinds from arbitrary sets, for properties themselves also can be put together in any arbitrary ways. We could conceive a set defined by two properties and represented, for example, as follows:

$$\{x | Having\_a\_Doubly\_Articulated\_Language(x) \wedge Having\_Four\_Feet(x)\}$$

[2] The following discussion owes much to Quine's famous paper "Natural Kinds" (Quine, 1969).

We know that this intelligent monster is not a natural kind simply because we already have knowledge that those two properties cannot be put together as those possessed by the members of a natural kind. In short, "Similarity cannot be equated with, or measured in terms of, possession of common characteristics." (Goodman, 1972: 443), and "The grouping of occurrences under a work or an experiment or an activity depends not upon a high degree of similarity but upon the possession of certain characteristics." (Goodman, 1972: 440)

Although, in principle, any kinds of infinite numbers of entities can be conceived, as just shown, by specifying any combinations of any properties, a so-called "ontological inflation," not all entities so conceived can be kinds. In order to avoid such a devastating ontological inflation, we usually impose some order on entities and conduct classification, or construct systems. But we have to know *in advance* what is the order and what properties must be chosen to conduct classifications. In short, it is in this sense trivial to say that natural kinds and standard beliefs associated with them are defined and represented by properties.

It should be noticed that similar arguments are commonly employed by the music theorists mentioned above to classify pitch-class sets. The steps of abstraction taken by those theorists have a serious side effect. Since any entities can be put together into a set and those properties that define a particular set cannot be uniquely determined, pc-set theory, because of its very generality and formalism, cannot by itself perform subclassifications that extract the properties, or the stylistic characteristics, of atonal music and thereby distinguish atonal music from other classes of musics. One must know in advance by some other means, in almost all cases just intuitively, what set of musical entities is a "kind," or the class of entities in atonal music, and what properties define and are relevant to the class. It is indeed this characteristic of pitch-class set theory that some theorists call "circularity" of the theory.

The grouping, or classification, of some musical entities, say, that of similar pc-sets, is not determined by the number of pitch- or interval-classes shared in common among its members. When we believe that some musical entities belong not to just a set, which might be a musical monster, but to a musical kind, we usually intuitively and often unconsciously exclude inadequate properties, and yet we do not know where adequacy comes from. Before representing musical kinds using pitch-class set theory, we have to detect, perhaps in an intuitive and not formal way, some qualities or adequate properties of them.

## 3 A Principle of Segmentation of Pc-Sets

Now, let me illustrate how connections between intuitively classified atonal kinds and a principle of particular pitch-organization can be established. Example 1 is the opening passage from Arnold Schoenberg's *Drei Klavierstücke*, op.11, no.1:

```
Measure 1      2          3      4
        B G#   G     A F  F   E
                 B        Db
                 F        A
                 Gb       Bb

        Example 1
```

```
Measure 1      2          3      4
        B G#   G     A F  F   E  D
                 B        Db
                 F        A      A
                 Gb       Bb     F

        Example 2
```

The passage sounds like implying, despite the awkward voice-leading, the chord D,A,F on the downbeat in m. 4 as shown in Example 2 (The reader is encouraged to actually hear it). In other words, measure 3 sounds like a half-cadence in d minor.

"Intervallic Rivalry Model" proposed by David Butler seems to explain why we have the sense of d-minorness. According to Butler (1989: 238), "The listener makes the perceptual choice of most-plausible tonic on the basis of style-bound conventions in the time ordering of intervals that occur only rarely in the diatonic set; that is, minor seconds (or enharmonics) and the tritone." Thus, since there is one tritone in m. 3 of Schoenberg's *Drei Klavierstücke*, $\{E, B^{\flat}\}$, as the circle of 5ths below shows, the passage is in $C$ scale on $F$ or on $B$. The reason why m. 3 sounds like $d$ minor is that, since there are two semitones, $\{B^{\flat}, A\}$ and $\{E, F\}$, the pc-set Bb,F,A,E is in $C$ scale on $F$ with $D^{\flat}$, or $C^{\sharp}$, as the leading tone in $d$ minor.

```
          *
       *     *
     F         A
   Bb------------E
     *         *
       *     *
        (Db)

     C scale on F
```

Based on the discussion above, Dubiel is implemented a method by which an atonal passage, or a stream of pitch classes, is automatically segmented into pc-sets so that each of them contains a single tritone.

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