カニングアントシステムの収束特性について
On the Convergence Property of the Cunning Ant System

筒井　茂義

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アブストラクト：先の論文において，新しいアントコロ
ニー最適化（*Ant Colony Optimization*) アルゴリズム
として，カニングアントシステム（*cAS*）を提案し，良好
な性能を有することを明らかにした．本論文では，フェロ
モン濃度情報のエントロピーの変化を観察するすることに
より，カニングアントシステムの性能の解析を行う．

In a previous paper, we proposed a novel ACO algorithm named the cunning Ant System and showed it to have better performance than existing ACO algorithms. In this article, we analyze the cunning Ant System focusing on the convergence process of searches based on entropy of the pheromone trail.

Keywords: ant colony optomization, cunning ant system, covergence analysis, entropy measure

1 Introduction

Ant colony optimization (ACO) has been applied with great success to a large number of combinatorial oprimization problems[1]. The first ACO algorithm was called the Ant System (AS) [2], and is applied to TSP. Since then, many advanced ACO algorithms were proposed as extensions of AS. These include Rank-Based Ant System (AS_{rank}) [3], Ant Colony System (ACS) [4], and MAX-MIN Ant System (MMAS) [5].

In the previous paper [6], we have proposed a variant of an ACO algorithm called the *cunning Ant System* (*cAS*) and evaluated the performance using TSP instances available at TSPLIB. The results showed that *cAS* worked well on the test instances and has performance that suggests it may be one of the most promising ACO algorithms. We also evaluate *cAS* when it is combined with Lin-Kernighan (LK) local search heuristics using larger sized TSP instances. The results also showed promising performance.

cAS introduced an important scheme which we call *cunning*. In constructing a new solution, *cAS* uses partly existing partial solutions. With this scheme, we may prevent premature stagnation by reducing strong positive feedback to the trail density. However, analytical study on the effectiveness of using this scheme remains for future work.

In this article, we analyze *cAS* focusing on the convergence process of searches based on entropy of the pheromone trail. The results showed clear evidence that using partial solutions with the cunning scheme can prevent premature stagnation in the search.

2 An Overview of Cunning Ant System [6]

cAS uses agents called cunning ants (*c-ants*), which differ from traditional ants in the manner of solution construction. A part of each new solution is taken from one of the previously generated solutions (also called a donor ant; *d-ant*) whereas the remainder of the solution is generated probabilistically from pheromone density $\tau_{ij}(t)$ as usual, where t is iteration counter of the algorithm. In a sense, since this agent in part appropriates the work of others to construct a solution, we named the agent a cunning ant after the metaphor of its cunning behavior.

In cAS, we maintain an archive consisting of m candidate solutions generated in the past; kth solution in the archive at iteration t is denoted by $s_{k,t}$ ($k \in \{1, 2, \ldots, m\}$). At iteration t, a new $c\text{-}ant_{k,t+1}$(solution) is generated for each position k in the archive using the current $s_{k,t}$ i.e., solution in this position, as the donor. Then, the newly generated $c\text{-}ant_{k,t+1}$ is compared with its donor $s_{k,t}$ with respect to the objective function, and the best of the two survives as the next solution in this position of the archive, $s_{k,t+1}$.

The pheromone density is updated with $s_{k,t+1}$ for $k=1, 2, \ldots, m$ and $\tau_{ij}(t+1)$ is obtained as in Ant System (AS) [2], keeping all pheromone densities within the interval $[\tau_{min}, \tau_{max}]$ as in MMAS [5].

Let us represent the number of nodes of partial solution that are constructed based on $\tau_{ij}(t)$, by l_s (i.e., l_c, the number of nodes of partial solutions from its donor, is $n - l_s$). Then cAS introduces the control parameter γ which can define $E(l_s)$ (the average of l_s) by $E(l_s) = n \times \gamma$. For each creation of a new candidate solution, we determine l_s so that $E(l_s) = n \times \gamma$ is satisfied. One simple approach is to determine value of l_s deterministically as $l_s = n \times \gamma$, where n is the problem size. In this research, we used a probabilistic function defined in [6] (please see reference [6] for details).

3   Definition of Entropy of Pheromone Density

In cAS, the γ is the most important parameter among others. As seen in [6], cAS has good performance with γ values of [0.2, 0.5] without heuristic and γ values of [0.3, 0.5] with heuristic. In cAS, we may prevent premature stagnation of the search, because only a part of the nodes ($n \times \gamma$ in average) in a string are newly generated, and this may prevent over exploitation caused by strong positive feedback to $\tau_{ij}(t)$.

In this paper, we analyze the convergence process of cAS by measuring the diversity of pheromone density. To measure this diversity, we use the measure of entropy. In this section, we define an entropy to measure the diversity of $\tau_{ij}(t)$ and prepare analysis for the convergence processes in section 4.

We define $I(t)$, entropy of pheromone density $\tau_{ij}(t)$ as follows:

$$I(t) = -\frac{1}{n} \sum_{i=1}^{n} \sum_{j \neq i} p_{ij}(t) \log p_{ij}(t) \quad (1)$$

where, $p_{ij}(t)$ is defined for $i \neq j$ and is

$$p_{ij}(t) = \frac{\tau_{ij}(t)}{\sum_{j \neq i} \tau_{ij}(t)} \quad (2)$$

As is understood from the definition of entropy, the upper bound of $I(t)$ is obtained when all elements of $\tau_{ij}(t)$ have the same values as found during the initialization stage ($t=0$). This value is calculated as

$$\overline{I} = \log(n - 1). \quad (3)$$

The lower bound of $I(t)$ is obtained when all elements of $\tau_{ij}(t)$ have values of τ_{min} or τ_{max}. Please note here that how this occurs is different between symmetric and asymmetric $\tau_{ij}(t)$ as is true for symmetric TSP and asymmetric TSP, respectively. Let's consider an extreme case that all strings have the same set of edges and put the pheromone on the set. If this iteration continues for a long time, all elements of $\tau_{ij}(t)$ converge to τ_{min} or τ_{max}.

In a symmetric case, a node is connected to two nodes and has two non-directional edges with the same magnitude of pheromone density. On the other hand, in an asymmetric case, although a node is connected to two nodes, they are directional. Therefore, for the symmetric case, τ_{ij} ($j=1, 2, \ldots, n$) converges when the value of the two elements equal τ_{max} and the other $n-3$ elements equal τ_{min} for each i ($i = 1, 2, \ldots, n$). On the other hand, for the asymmetric case, τ_{ij} ($j=1, 2, \ldots, n$) converges when one element of τ_{ij} is τ_{max} and the other $n-2$ elements are τ_{min} for each i ($i= 1, 2, \ldots, n$).

Thus, the lower bounds of $I(t)$ for symmetric and asymmetric cases are obtained as

$$\underline{I_S} = \log(2r + n - 3) - \frac{2r \log(r)}{2r + n - 3}, \quad (4)$$

$$\underline{I_A} = \log(r + n - 2) - \frac{r \log(r)}{r + n - 2}, \quad (5)$$

where $r = \tau_{max}/\tau_{min}$.

In the following analysis, we use the normalized $I_N(t)$ which is defined with $I(t)$, $\overline{I}$, and $\underline{I}$ as

$$I_N(t) = \frac{I(t) - \underline{I}}{\overline{I} - \underline{I}} \quad (6)$$

Then, $I_N(t)$ takes values in [0.0, 1.0].

4   Results

Figures 1–7 show the convergence processes of cAS on the test problems in QAPLIB using $I_N(t)$ and

Error. In each figure, the left is change of I_N and right is change of *Error*. The γ values were tested for 0.1, 0.3, 0.5, 0.7, and 0.9.

From Figures 1–7, we can clearly observe that with smaller values of γ, values of entropy I_N defined by Eq. 6 decrease slower than with larger values of γ as tour construction numbers increase, resulting in successful searches in performance measure *Error*. With larger values of γ, values of entropy decrease faster than with smaller values of γ, and we can observe stagnation of the search.

Thus, we can see the usefulness of the cunning scheme with smaller values of γ. That is, on average, taking the rate of $(1-\gamma)$ partial solution from existing solutions, and having the rate of γ partial solution being generated anew from the pheromone density can maintain diversity of the system, resulting in good balance between exploration and exploitation of the search. However, with extreme smaller values of γ, i.e., $\gamma \leq 0.1$, the search processes become much slower, though the diversity of pheromone density can be maintained. Thus, to choose appropriate smaller γ values is important.
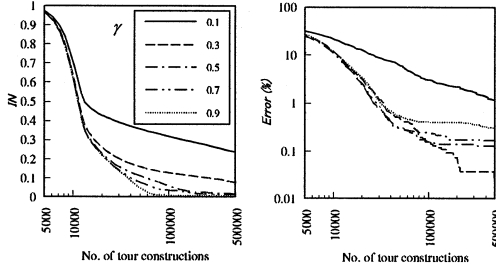

Figure 1: Convergence process on eil51

Figure 8 shows the convergence process with LK heuristic on rl5934 instance for cAS with γ value of 0.4 and γ vale of 1.0 ($\gamma = 1$ means that no cunning action is apllied). We can see a clear difference between convergence processes of two systems observed in *Error*. However, we cannot observe clear difference in entropy measure until around 300 tour constructions. Around this point cAS with $\gamma = 0.4$ found optimal solutions (*Error* $= 0$). Beyond this point, the entropy of cAS with $\gamma = 0.4$ is even smaller than cAS with $\gamma = 1$. Thus, usefulness of
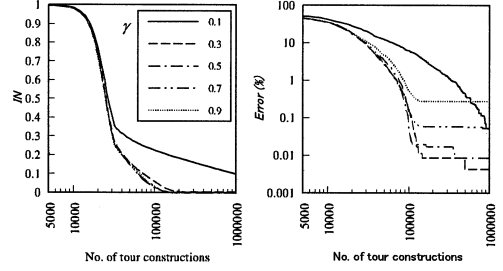

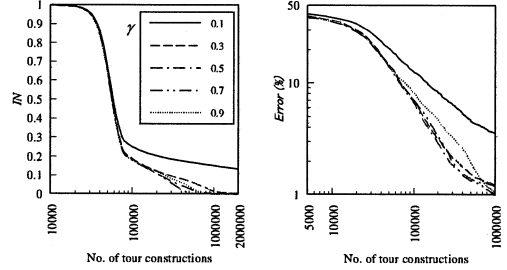Figure 2: Convergence process on kroA100

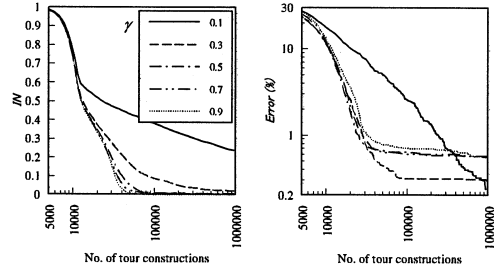Figure 3: Convergence process on d198
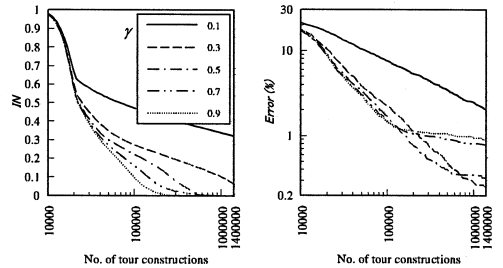

Figure 4: Convergence process on ry48p

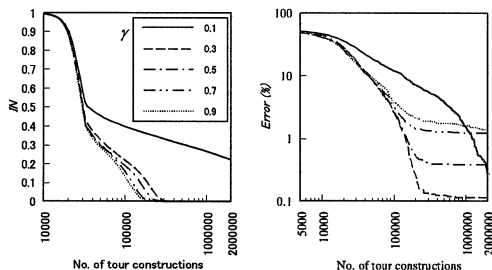Figure 5: Convergence process on ft70

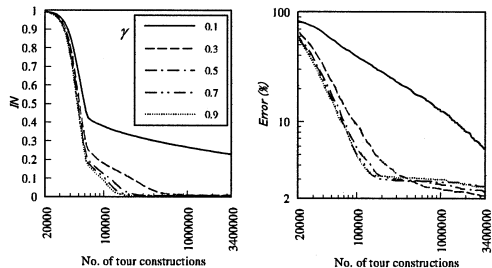Figure 6: Convergence process on kro124p
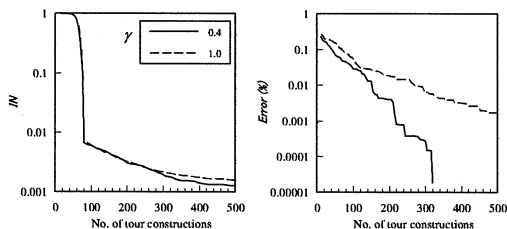

Figure 7: Convergence process on ftv170

Figure 8: Convergence process on rl5934

cAS when it is combined with LK heuristic can be observed, we cannot explain the evidence from entropy analysis. For analysis of cAS which is combined with powerful heuristics, we need to introduce other more finely-tuned measures.

5 Conclusion

Analysis of the cAS entropy measurement showed clear evidence that using partial solutions in generating new solutions in ACO algorithms is useful to maintain the diversity of the pheromone density, i.e., using partial solutions can prevent premature stagnation by reducing strong positive feedback to the trail density, resulting in higher performance of the algorithms.

Acknowledgements

References

[1] M. Dorigo and T. Stützle. *Ant Colony Optimization.* MIT Press, Massachusetts, 2004.

[2] M. Dorigo, V. Maniezzo, and A. Colorni. The ant system: Optimization by a colony of cooperating agents. *IEEE Trans. on SMC-Part B*, 26(1):29–41, 1996.

[3] B. Bullnheimer, R. F. Hartl, and C. Strauss. An improved ant system algorithm for the vehicle routing problem. *Annals of Operations Research*, 89:319–328, 1999.

[4] M. Dorigo and L. M. Gambardella. Ant colony system: A cooperative learning approach to the traveling salesman problem. *IEEE TEC*, 1(1):53–66, 1997.

[5] T. Stützle and H. Hoos. Max-min ant system. *Future Generation Computer Systems*, 16(9):889–914, 2000.

[6] S. Tsutsui. cAS: Ant colony optimization with cunning ants. *Proc. of the 9-th International Conference on Parallel Problem Solving from Nature (PPSN IX)*, pages 162–171, 2006.