An Adaptive Diagnosis Method in PMC Model

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ABSTRACT

Recently, researchers have begun studying adaptive approaches to the diagnosable system. Hakimi et al. [4,6] assume that each unit is capable of testing every other units, with the tests being conducted one at a time in such a way that the choice of the next test to be performed depends on the results of the previous tests, rather than on a preselected pattern of test studied earlier. Hakimi and Nakajima [6] proposed an algorithm which identifies a fault-free unit after application of at most 2t-1 tests. Then they used the fault-free unit as a tester to identify all the fault units, thus using at most n+2t-2 tests to identify all the faulty units in a t diagnosable system with n units. But in general, it is not true in practice. To the best of our knowledge, there does not exist any result related to the adaptive diagnosis problem when the PMC model representation of the system is not a completely connected digraph. In this paper, we will investigate the application of adaptive diagnosis approach to a D(n,t,X) system which belongs to the PMC model.

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最近，自己診断可能なシステムの設計へのアダプティブな手法が研究されだしている．Hakimi等は，各ユニットが他の全てのユニットの正常性をテストできる(すなわち，グラフが完全有向グラフ)と仮定している．テストの順番は予め決められたものではなく，過去のテストの結果によって診断中に決めることができる．このモデルの下で，HakimiとNakajimaは高々$2t-1$回のテストの後にfault-free unitを一つ確認するアルゴリズムを提案した．このfault-free unitを試験機として用いてfault unitsを確認すれば，n個のユニットからなり高々t個の故障ユニットを持つシステムの全ての故障ユニットを高々$n+2t-2$回の試験によって確認することができる．しかし，一般にこのモデルは現実的ではない．また，システムのPMCモデル表現が完全有向グラフでない場合の適応診断問題に関する結果は知られていない．本論文ではPMCモデルに属する$D(n,t,X)$システムに対する適応診断法の適用を検討する。

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I. Introduction

In the area of fault-tolerant computing, fault diagnosis plays an important role. Modeling a system as a number of interconnected subsystems or units having the feature of mutual testing has received considerable attention [1-9] from the diagnosis point of view.

The most well-studied model to represent a diagnosable system S is the model introduced by Preparata et al.[1] (henceforth referred to as the PMC model), in which the system is represented by n units and on the assumption that the fault is detected at the unit level, each unit is tested by several other units of the system.

This model can be represented by a digraph G = (V,E) where V is a set of nodes, |V| = n, each node representing a unit and E is the set of edges, each edge (u(i),u(j)) is labeled with a(i,j) to represent the result of testing : a(i,j) = 0 if u(i) evaluates u(j) to be fault-free and a(i,j) = 1 if u(i) evaluates u(j) faulty. In the PMC model, if u(i) is fault-free, a(i,j) reflects the correct status of u(j); however, if u(i) is faulty, a(i,j) might be 0 or 1 irrespective of whether u(j) is faulty or not.

A set of all a(i,j)'s is said to be a syndrome of S. S is called t-diagnosable if for every syndrome, the faulty units can be uniquely identified so long as the number of faulty units does not exceed t. Given a system in terms of the digraph G, the necessary and sufficient conditions to find if the system is t-diagnosable were presented in [10] and a polynomial time algorithm to verify if a system is t-diagnosable was presented in [11] based on different characterization.

A system S belongs to the class of D(n,t,X) systems [2,5] iff there exists some set X = {x(1),x(2),...,x(t)} of positive integers such that n $\geq$ 2t+1 and n $\geq$ 2x(i)+1, 1 $\leq$ i $\leq$ t and E = {(u(i),u(j)) : i-j (mod n) $\in$ X}. Chwa and Hakimi [2] have shown the characterization of t/t diagnosable systems and a class of t diagnosable system, denoted by D(n,t,X). An algorithm to identify all faulty units in O(|E|) is also given in [2].

A new approach for diagnosing the faulty units in a system has received attention in recent days. This is referred to as the adaptive approach. According to this approach, since all the test results might not be necessary to find the faulty units, tests are conducted as seem necessary. Informally it can be described as follows. To start with, some tests are carried out in steps and at each step what results are to be carried out depends on the results obtained so far. The number of steps needed and the total number of tests to be conducted depend on the testing feature of the different units. The t-adaptive diagnosis problem for a system is as follows. Given a system(modeled as the PMC model for example), assuming that the number of faulty units is at most t, how to conduct the tests adaptively and on the basis of the test results, how to identify the faulty units using fewest number of steps and fewest number of total tests?

Hakimi and Schmeichel [4] assume that each unit is capable of testing every other unit, with the tests being conducted one at a time in such a way that the choice of the next test to be performed depends on the results of the previous tests, rather than on a preselected pattern of tests studied in earlier papers. Hakimi and Nakajima [6] proposed that one adaptively chooses the tests and obtains their results until one can identify a fault-free unit. Then one uses this unit as a tester to identify all the faulty units, thus using at most n+2t-2 tests to identify all units. Blecher [9] improved this bound by using a different procedure to find a fault-free unit in at most 2t-1 tests which enables one to identify the status of all units in at most n+t-1 tests. Schmeichel et al. [7] presented an adaptive algorithm as proceeding in "round". Informally, a round of testing is a set of tests where for each tests the testing and tested units are distinct. A parallel algorithm which identifies the faulty units within $O(\log_{\lfloor n/t \rfloor} t)$ round of testing, and which uses at most 4n tests was presented [7].

All the results available in the literature [4-8] for the adaptive diagnosis problem assume a simplified model of the system. They assume that in the system, every unit is capable of testing every other unit of the system. In short, representing the system in PMC model, every node has an outgoing edge to every other node of the system. Evidently this is far from a practical situatiuon. To the best of our knowledge, there does not exist any result related to the adaptive diagnosis problem when the PMC model representation of the system is not a completely connected digraph. This paper presents a first attempt to solve this problem in this direction. We assume that the system is represented as a D(n,t,X) system and present an adaptive diagnosis algorithm which finds all the faulty units in $O(t)$ rounds needing at most nt tests.

II. An adaptive Diagnosis Algorithm for D(n,t,X) system

Given positive integers n and t and a set of integers X such that $n \geq 2t+1$ and $X = \{x(1), x(2), \ldots, x(t)\}$ and $x(1) < x(2) < \cdots < x(t)$ where $n \geq 2x(t)+1$, a D(n,t,X) system is a system with n nodes and represented by the PMC model with the set of nodes given by $\{u(0), u(1), \cdots, u(n-1)\}$ and the set of directed edges is given as follows. $(u(i), u(j))$ is an edge iff $(i-j) \bmod n \in X$. Such a system has been shown to be t-diagnosable [2].

Formally, a round of testing is defined as a set $Y \subseteq E$ such that if $(u(\alpha), u(\beta)), (u(\gamma), u(\delta)) \in Y$, then $\alpha, \beta, \gamma, \delta$ are all distinct. It is simple to see that all the tests given by the edges in a round can be conducted in parallel. For each $x(i) \in X$, we define a partition on the set of nodes as follows. Two nodes u(j) and u(k) belong to the same block of $\pi(i)$ iff $\lfloor j/x(i) \rfloor = \lfloor k/x(i) \rfloor$. Let $b(\alpha i)$ denote the αth block of $\pi(i)$, $0 \leq \alpha \leq \lceil n/x(i) \rceil - 1$, such that if $u(j) \in b(\alpha i)$, then $\alpha = \lfloor j/x(i) \rfloor$. Let $p(i) = \lceil n/x(i) \rceil - 1$. Note that if n is not a multiple of x(i) then b(p(i),i) has fewer than x(i) nodes in it. Also since $n \geq 2x(t)+1$, there are at least three blocks in each $\pi(i)$. In the following, given any x(i), we will denote by $b(\alpha i) \rightarrow b(\beta i)$ the following set of edges.

$$b(\alpha i) \rightarrow b(\beta i) = \{(u(j), u(k)) \mid (j-k) \bmod n = x(i) \text{ and } u(j) \in b(\alpha i) \text{ and } u(k) \in b(\beta i)\}$$

From the construction of the blocks of the partition, it is simple to see that for some given i, all the tests given by $b(\alpha i) \rightarrow b(\beta i)$ are a round of testing. Our diagnosis algorithm has two steps. In the first step, we define how the tests are conducted in an adaptive fashion. In the second step, we define how the test results are used to identify the faulty units.

Step 1 : This step is composed of t phases and each phase is composed of three rounds. Each phase is conducted for a distinct element of X. Each node u(j) is assumed to have a tag bit called TAG(j) and two counters called C0(j) and C1(j). When a set of tests are requested to be conducted, the test (u(j),u(k)) in the set is carried out if the TAG(j) bit is 0 and is not carried out if the TAG(j) is 1. If a test (u(j),u(k)) is carried out then the counter C0(k) is incremented if the test result was 0, otherwise the counter C1(k) is incremented.

Phase 1 : Initially all TAG bits, C0 and C1 are zeros. If p(t) is even then conduct the tests given by round E1, E2, and E3 for i=t in three rounds of testing. If p(t) is odd then conduct the tests given by rounds O1, O2 and O3 for i=t in three rounds of testing. In either case, for each test (u(j),u(k)) carried out in any round, the test result is assigned to be the new value of TAG(k). After all the rounds are completed in this phase, the test result obtained for each test (u(j),u(k)) might be used to modify C0(k) or C1(k). If TAG(j)=0 after all rounds are completed then the counter C0(k) (or C1(k)) is incremented if the test result of (u(j),u(k)) was 0 (respectively, 1). If, however, TAG(j)=1 then, neither C0(k) nor C1(k) is modified.

$$E1 = \{ b(1i)\rightarrow b(0i), b(3i)\rightarrow b(2i), \ldots, b((p(i)-1)i)\rightarrow b((p(i)-2)i) \}$$

$$E2 = \{ b(2i)\rightarrow b(1i), b(4i)\rightarrow b(3i), \ldots, b(p(i)i)\rightarrow b((p(i)-1)i,) \\ b(0i)\rightarrow b((p(i)-1)i) \}$$

$$E3 = \{ b(0i)\rightarrow b(p(i)i) \}$$

$$O1 = \{ b(1i)\rightarrow b(0i), b(3i)\rightarrow b(2i), \ldots, b(p(i)i)\rightarrow b((p(i)-1)i) \}$$

$$O2 = \{ b(2i)\rightarrow b(1i), b(4i)\rightarrow b(3i), \ldots, b((p(i)-1)i)\rightarrow b((p(i)-2)i), \\ b(0i)\rightarrow b(p(i)i) \}$$

$$O3 = \{ b(0i)\rightarrow b((p(i)-1)i \}$$

Phases 2 through t : For the jth phase, $2 \le j \le t$, if p(j-1) is even then conduct the tests given by rounds E1, E2 and E3 for i = j-1 in three rounds of testing. If p(j-1) is odd then conduct the tests given by rounds O1, O2 and O3 for i = j-1 in three rounds of testing. If a test (u(j),u(k)) was carried out then, C0(k) (or C1(k)) is incremented if the test result was 0 (respectively, 1). If TAG(j)=0 then C0(k) and C1(k) is modified.

It is simple to observe that if for some $x(i) \in X$, n is multiple of x(i) and p(i) is even then for that x(i), the corresponding phase needs only two rounds. In general therefore, 3t rounds of testing are sufficient to conduct all the tests mentioned above. Depending on the test results of phase 1, some of the tests in phases 2 through t might not be carried out based on the TAG bits. At worst the total number of tests carried out is nt.

Step 2 : From the test results, each unit is identified as fault-free or faulty as follows. If $C0(j) \geq C1(j)$ then $u(j)$ is identified as a fault-free unit, otherwise $u(j)$ is identified as a faulty uint.

III. Proof of Correctness of the Algorithm

We use the idea given in [2]. Let the digraph $G=(V,E)$ represent a $D(n,t,X)$ system. It is assumed that the number of faulty units is at most t. In [3], we have shown that the digraph G representing a $D(n,t,X)$ system has node connectivity t. Hence for every $u(i) \in V$, from the test results we can always determine whether $u(i)$ is faulty or not if all test results were available. We will see later on that some of the test results do not contribute to the diagnosis process and this is why some of the tests are not carried out in our algorithm.

Assuming $X = \{x(1), x(2), \ldots, x(t)\}$, for any node $u(i) \in V$, let us define $V_a=\{u(i+x(1)), u(i+x(2)), \ldots, u(i+x(t))\}$ and $V_b=\{u(i+x(1)+x(t)), u(i+x(2)+x(t)), \ldots, u(i+x(t)+x(t))\}$ where all additions are in modulo n. Let $G^i=(V^i,E^i)$ be a subgraph of G with respect to $u(i)$ such that $V^i=\{u(i)\} \cup V_a \cup V_b$ and $E^i=\{(u,u(i))$ for all $u \in V_a\} \cup \{(u(i+x(j)+x(t)), u(i+x(j))$ for all $1 \leq j \leq t\}$. It is simple to observe that since $n \geq 2x(t)+1$, $V_a \cap V_b=\phi$. Since G is a symmetric digraph, for each $u(i)$ the subgraph $G=(V^i,E^i)$ is defined and all such graphs have the same structure. Let $V_a(\alpha 1 \alpha 2)$ be the set of nodes of V_a which when testing $u(i)$ produces a test result $\alpha 2$ and which when tested by nodes of V_b produces a test result $\alpha 1$, where $\alpha 1, \alpha 2 \in \{0,1\}$. Note that in G^i, each node of V_a is tested by only one node of V_b. Let $V_b(\alpha 1 \alpha 2)=\Gamma^{-1}V_a(\alpha 1 \alpha 2)$ in $G^i=(V^i,E^i)$. Note that $|V_b(\alpha 1 \alpha 2)|=|V_a(\alpha 1 \alpha 2)|$. Then the following result is given in [2].

Lemma 1 : $u(i)$ is fault-free iff $|V_a(00)| \geq |V_a(01)|$.

Proof : Suppose $u(i)$ is faulty and $|V_a(00)| \geq |V_a(01)|$. Then all nodes in $\{u(i)\} \cup V_a(00) \cup V_b(00)$ must be a subset of the faulty set F and at least half of the vertices in $V_a(10) \cup V_a(11) \cup V_b(10) \cup V_b(11)$ must be faulty since a test result 1 for the test $(u(j),u(k))$ implies either $u(j)$ is faulty or $u(k)$ is faulty or possibly both. That is,
$|F| \geq 1 + 2|V_a(00)| + |V_a(10)| + |V_a(11)|$
$\geq 1 + |V_a(00)| + |V_a(01)| + |V_a(10)| + |V_a(11)| > t$, which is a contradiction.

From the above lemma, it follows directly that to find whether the unit $u(i)$ is faulty or not the set $V_a(1\alpha 2)$ does not contribute to the determination. The proposed algorithm does not carry out the testing of $u(i)$ by those nodes of V_a which are tested by the nodes of V_b to produce a test result 1. In the phase 1, all the tests $(u(j+x(t)), u(j))$ are carried out for all possible j. If a test result happens to be 1 then for every $u(i)$ having V_a containing $u(j)$, the test result of $(u(j), u(i))$ is useless for finding the faulty or fault-free status of $u(i)$. Hence by setting the TAG bit of $u(j)$ to 1, the test is not conducted. After all the phases are over, for any node $u(i)$, $C0(i)$ gives the value of $|V_a(00)|$ and $C1(i)$ gives the value of $|V_a(01)|$ and hence the inference made in the step 2 follows directly from the above lemma. The main purpose of arranging the nodes in different partitions for different values x's and to conduct the testing according to the specified rounds is simply to ensure that for any $x(i) \in X$, the tests $(u(j+x(i)), u(j))$ for all j can be conducted in a constant number of rounds.

IV. Conclusion

In [4-6] under assumptions of each unit being capable of testing every other unit, an adaptive diagnosis algorithm was presented. Because of the impracticality of this assumption, we considered a realistic situation of a $D(n,t,X)$ system to investigate the application of adaptive approach to the diagnosis problem. An algorithm which identifies all the faulty units in 3t rounds and $O(nt)$ time was given in this paper. Consider as an example, G to be any symmetric digraph having node connectivity t and which has diameter at least 3. Then for every node $u(i)$ there exists a node $u(\beta)$ such that there exist t node disjoint paths from $u(\beta)$ to $u(i)$ and all these paths must be of length at least 3. Hence for $u(i)$, a subgraph similar to G^i described above can be defined. It seems a simple extension of the algorithm presented here can be applied for this digraph G. In short, from the technique here, it is natural to conjecture that an adaptive diagnosis algorithm similar to the one presented here can be developed for a PMC model symmetric digraph having diameter at least 3 and node connectivity t.

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