On the Three-Dimensional Channel Routing

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Abstract—**The 3-D channel routing is a fundamental problem on the physical design of 3-D integrated circuits. The 3-D channel is a 3-D grid G and the terminals are vertices of G located in the top and bottom layers. A net is a set of terminals to be connected. The object of the 3-D channel routing problem is to connect the terminals in each net with a tree (wire) in G using as few layers as possible and as short wires as possible in such a way that wires for distinct nets are disjoint. This paper shows that any set of n 2-terminal nets can be routed in a 3-D channel with $O\left(\sqrt{n}\right)$ layers using wires of length $O\left(\sqrt{n}\right)$. We also show that there exists a set of n 2-terminal nets that requires a 3-D channel with $\Omega\left(\sqrt{n}\right)$ layers to be routed.**

I. Introduction

The three-dimensional (3-D) integration is an emerging technology to implement large circuits, and currently being extensively investigated. (See [1]–[6], [8], for example.) In this paper, we consider a problem on the physical design of 3-D integrated circuits.

The 3-D channel routing is a fundamental problem on the physical design of 3-D integrated circuits. In the 3-D channel routing, the channel is a 3-D grid G consisting of columns, rows, and layers which are planes defined by fixing x-, y-, and z-coordinates, respectively. (See Fig. 1.) A terminal is a vertex of G located in the top or bottom layer. A net is a set of terminals to be connected. A net containing k terminals is called a k-net. A tree connecting the terminals in a net is called a wire. The object of the 3-D channel routing problem is to connect the terminals in each net with a wire in G using as few layers as possible and as short wires as possible in such a way that wires for distinct nets are disjoint. The number of layers is called the height of the 3-D channel. The purpose of this paper is to show the following two theorems.

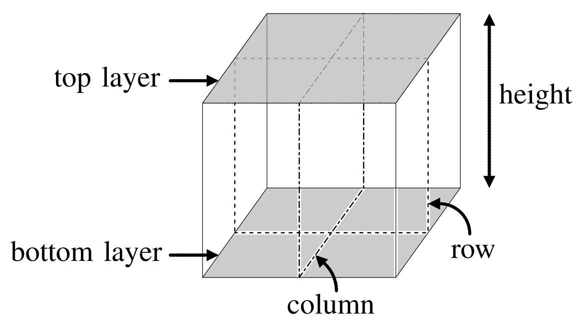

Fig. 1. The three-dimensional channel.

Theorem 1: If the layers are square 2-D grids, the terminals are located on vertices with even x- and y-coordinates, and each net has terminals both in top and bottom layers, then any set of n 2-nets can be routed in a 3-D channel of height $O(\sqrt{n})$ using wires of lenght $O(\sqrt{n})$.

Theorem 2: There exists a set of n 2-nets that requires a 3-D channel of height $\Omega\left(\sqrt{n}\right)$ to be routed.

Theorem 1 implies that any set of n 2-nets can be routed in a 3-D channel of volume $O(n^{3/2})$. It should be noted that for the ordinary 2-D channel routing there exists a set of n 2-nets requiring a 2-D channel of area $\Omega(n^2)$ to be routed [7].

Other models for the 3-D channel routing can be found in the literature [3], [5], [8].

II. Preliminalies

We consider a 3-D channel of height h, which is a $2\sqrt{n} \times 2\sqrt{n} \times h$ 3-D grid. Each grid point is denoted by (x, y, z) with $0 \leq x, y \leq 2\sqrt{n} - 1$ and $0 \leq z \leq h - 1$. The column, row, and layer defined by $x = i$, $y = j$, and $z = k$ are called the i-column, j-row, and k-layer, respectively. The $(h-1)$-layer and 0-layer are corresponding to the top and bottom layers, respectively. Let

$\mathcal{N} = \{N_i | 0 \le i \le n-1\}$ be a set of n 2-nets, and let $(X_i^{(h-1)}, Y_i^{(h-1)}, h-1)$ and $(X_i^{(0)}, Y_i^{(0)}, 0)$ be the terminals of N_i $(0 \le i \le n-1)$, where $X_i^{(h-1)}$, $Y_i^{(h-1)}$, $X_i^{(0)}$, and $Y_i^{(0)}$ are even, and $(X_i^{(h-1)}, Y_i^{(h-1)}, h-1) \ne (X_j^{(h-1)}, Y_j^{(h-1)}, h-1)$ and $(X_i^{(0)}, Y_i^{(0)}, 0) \ne (X_j^{(0)}, Y_j^{(0)}, 0)$ if $i \ne j$.

If $f : A \to B$ is a mapping, $f(A') = \{f(a) | a \in A'\}$ is the image of $A' \subseteq A$ and $f^{-1}(B') = \{a | f(a) = B'\}$ is the pre-image of $B' \subseteq B$. We denote by $f|_{A'}$ the restriction of f to A'. That is, $f|_{A'} : A' \to B$ and $f|_{A'}(a') = f(a')$ for $\forall a' \in A'$. If $g : B \to C$ is also a mapping, $g \circ f$ is a composite mapping from A to C defined as $g \circ f(a) = g(f(a))$ for $\forall a \in A$. A bijection $\pi : A \to A$ is called *a permutation on* A.

For a positive integer I, let $[I] = \{0, 1, \ldots, I-1\}$.

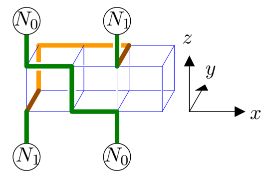

Fig. 2. A routing for a set of two 2-nets.

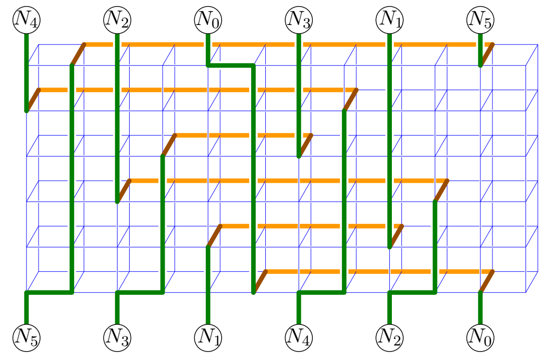

Fig. 3. A τ-routing for a set of six 2-nets.

III. 2-D Channel Routing

We consider in this section a 2-D channel of height m, which is a $2m \times 2 \times m$ 3-D grid G'. Let $\mathcal{N}' = \{N'_i | 0 \le i \le m-1\}$ be a set of m 2-nets, and let $(X_i^{(m-1)}, 0, m-1)$ and $(X_i^{(0)}, 0, 0)$ be the terminals of N'_i $(0 \le i \le m-1)$, where $X_i^{(m-1)}$ and $X_i^{(0)}$ are even, and $(X_i^{(m-1)}, 0, m-1) \ne (X_j^{(m-1)}, 0, m-1)$ and $(X_i^{(0)}, 0, 0) \ne (X_j^{(0)}, 0, 0)$ if $i \ne j$.

Lemma 1: $\mathcal{N}'$ can be routed in G'.

Proof: Let $p_0, p_1, \ldots, p_k$ be grid points of G' such that p_i and p_{i+1} differ in just one coordinate, $0 \le i \le k-1$. Then, we denote by $[p_0, p_1, \ldots, p_k]$ a wire connecting p_0 and p_k obtained by connecting p_i and p_{i+1} by an axis-parallel line segment, $0 \le i \le k-1$. Suppose without loss of generality that $X_0^{(m-1)} = X_1^{(0)}$. Then, if $m \ge 3$, $\mathcal{N}'$ can be routed in G' using a wire defined by (1) for N'_0, a wire defined by (2) for N'_1, and wires defined by (3) for N'_i, $2 \le i \le m-1$. It is not difficult to see that the wires defined above are disjoint. If $m = 2$, $\mathcal{N}'$ can be routed in G' as shown in Fig. 2. ■

The routing defined in the proof of Lemma 1 is called *a τ-routing for $\mathcal{N}'$*. An example of τ-routing is shown in Fig. 3.

IV. Proof of Theorem 1

A. Technical Lemmas

For positive integers I and J, we define that $M = \{m_{i,j} | i \in [I], j \in [J]\}$, and $M_{*j} = \{m_{i,j} | i \in [I]\}$, and $M_{i*} = \{m_{i,j} | j \in [J]\}$. Let D be a set with $|D| = J$ and $f : M \to D$ be a mapping such that

$$|f^{-1}(d)| = I \quad \text{for } \forall d \in D. \tag{4}$$

Let π_j be a permutation on M_{*j} for $\forall j \in [J]$, and $\Pi = \{\pi_j | j \in [J]\}$. Define that $R_\Pi(i) = \bigcup_{j \in [J]} \pi_j^{-1}(m_{i,j})$. $|R_\Pi(i)| = J$, by definition. For such Π and each $i \in [I]$, we define that

$$\begin{aligned} W_\Pi(d,i) &= \begin{cases} 1 & \text{if } d \in f(R_\Pi(i)), \\ 0 & \text{if } d \notin f(R_\Pi(i)), \end{cases} \\ W_\Pi(i) &= \sum_{d \in D} W_\Pi(d,i), \text{ and} \\ W(\Pi) &= \sum_{i=0}^{I-1} W_\Pi(i). \end{aligned}$$

By definition, $1 \le W_\Pi(i) \le J$ and $W_\Pi(i) = J$ if and only if $f|_{R_\Pi(i)}$ is a bijection, that is, $|f(R_\Pi(i))| = J$. Thus we have the following lemma.

Lemma 2: If $W_\Pi(i) < J$, there exists $d \in D$ such that $\left| f|_{R_\Pi(i)}^{-1}(d) \right| \ge 2$, and there exists an integer $i' \in [I]$ suth that $d \notin f(R_\Pi(i'))$.

$$\left[\left(X_0^{(m-1)},0,m-1\right),\left(X_0^{(m-1)}+1,0,m-1\right),\left(X_0^{(m-1)}+1,0,0\right),\left(X_0^{(m-1)}+1,1,0\right),\left(X_0^{(0)},1,0\right),\left(X_0^{(0)},0,0\right)\right] \quad (1)$$

$$\left[\left(X_1^{(m-1)},0,m-1\right),\left(X_1^{(m-1)},0,1\right),\left(X_1^{(m-1)},1,1\right),\left(X_1^{(0)},1,1\right),\left(X_1^{(0)},0,1\right),\left(X_1^{(0)},0,0\right)\right] \quad (2)$$

$$\left[\left(X_i^{(m-1)},0,m-1\right),\left(X_i^{(m-1)},0,i\right),\left(X_i^{(m-1)},1,i\right),\left(X_i^{(0)}+1,1,i\right),\left(X_i^{(0)}+1,1,0\right),\left(X_i^{(0)}+1,0,0\right),\left(X_i^{(0)},0,0\right)\right] \quad (3)$$

Proof: If $W_\Pi(i) < J$ then $f|_{R_\Pi(i)}$ is not a bijection, and so $\left|f|_{R_\Pi(i)}\right| \geq 2$ for some $d \in D$. It follows that $d \notin f(R_\Pi(i'))$ for some $i' \in [I]$ by (4). ∎

We need the following easy lemma on directed multi-graphs.

Lemma 3: For a directed multi-graph H with the vertex set D, if there exists a vertex $d_0 \in D$ with $\deg_{\text{out}}(d_0) \geq \deg_{\text{in}}(d_0) + 1$ then there exists a vertex $d_p \in D$ such that $\deg_{\text{in}}(d_p) \geq \deg_{\text{out}}(d_p)+1$ and there exists a directed path $(d_0, d_1, \ldots, d_p)$ in H, where $\deg_{\text{in}}(d)$ and $\deg_{\text{out}}(d)$ is the in- and out-degrees of d in H.

Proof: Let $D' \subseteq D$ be a set of vertices d' such that there exists a directed path from d_0 to d' in H, and let $H[D']$ be the induced subgraph of H on D'. Let $\deg'_{\text{in}}(d')$ and $\deg'_{\text{out}}(d')$ be the in- and out-degrees of $d' \in D'$ in $H[D']$, respectively. Notice that $\deg'_{\text{out}}(d') = \deg_{\text{out}}(d')$ and $\deg'_{\text{in}}(d') \leq \deg_{\text{in}}(d')$ for every $d' \in D'$. Since $\deg'_{\text{out}}(d_0) = \deg_{\text{out}}(d_0) \geq \deg_{\text{in}}(d_0) + 1 \geq \deg'_{\text{in}}(d_0) + 1$, there exists a vertex $d' \in D'$ such that $\deg'_{\text{out}}(d') \leq \deg'_{\text{in}}(d') - 1$, which follows from the fact that $\sum_{d' \in D'} \deg_{\text{out}}(d') = \sum_{d' \in D'} \deg_{\text{in}}(d')$. Since $\deg'_{\text{out}}(d') = \deg_{\text{out}}(d')$ and $\deg'_{\text{in}}(d') \leq \deg_{\text{in}}(d')$, we have $\deg_{\text{out}}(d') \leq \deg_{\text{in}}(d') - 1$. By the definition of D', there exists a directed path from d_0 to d'. Thus, we have the lemma. ∎

Lemma 4: There exists a set Π of permutations π_j on M_{*j} ($j \in [J]$) such that for every $i \in [I]$, $f|_{R_\Pi(i)} \circ \pi_j^{-1}(m_{i,j}) \neq f|_{R_\Pi(i)} \circ \pi_{j'}^{-1}(m_{i,j'})$ if $j \neq j'$.

Proof: By definition, $J \leq W(\Pi) \leq IJ$, and $W(\Pi) = IJ$ if and only if Π satisfies the condition in the lemma. In order to prove the lemma, it suffices to show the following.

Claim 1: Let Σ be a set of permutations σ_j on M_{*j} ($j \in [J]$) with $W(\Sigma) \leq IJ - 1$. Then, there exists a set Π of permutations π_j on M_{*j} ($j \in [J]$) such that $W(\Pi) \geq W(\Sigma) + 1$.

Proof of Claim 1: Since $W(\Sigma) \leq IJ - 1$, there exists $i_0 \in [I]$ such that $W_\Sigma(i_0) \leq J - 1$. By Lemma 2, there exists $d_0 \in D$ such that

$$\left|f|^{-1}_{R_\Sigma(i_0)}(d_0)\right| \geq 2 \quad (5)$$

and there exists an integer $i_1 \in [I]$ with

$$d_0 \notin f(R_\Sigma(i_1)). \quad (6)$$

Consider a directed multi-graph H with vertex set D which has an arc $a_j = (f(\sigma_j(m_{i_0,j})), f(\sigma_j(m_{i_1,j})))$ for each $j \in [J]$. From (5) and (6), we have

$$\deg_{\text{out}}(d_0) \geq 2, \quad (7)$$
$$\deg_{\text{in}}(d_0) = 0, \quad (8)$$

respectively, where $\deg_{\text{in}}(d)$ and $\deg_{\text{out}}(d)$ is the in- and out-degrees of d in H, respectively. From Lemma 3, there exists a vertex $d_p \in D$ with

$$\deg_{\text{in}}(d_p) \geq \deg_{\text{out}}(d_p) + 1, \quad (9)$$

and there exists a directed path $P = (d_0, d_1, \ldots, d_p)$. Let a_{j_l} be an arc (d_l, d_{l+1}), $l \in [p]$. Notice that $f(\sigma_{j_l}(m_{i_0,j_l})) = d_l$ and $f(\sigma_{j_l}(m_{i_1,j_l})) = d_{l+1}$ for $\forall l \in [p]$. Therefore, $f(\sigma_{j_1}(m_{i_1,j_l})) = f(\sigma_{j_0}(m_{i_0,j_{l+1}}))$ for $l \in [p]$. Let $\mathcal{J}' = \{j_0, j_1, \ldots, j_{p-1}\}$, and $D' = \{d_0, d_1, \ldots, d_p\}$. For each $j \in [J]$, define that

$$\rho_j(m_{i,j}) = \begin{cases} m_{i,j} & \text{if } i \notin \{i_0, i_1\} \text{ or } j \notin \mathcal{J}', \\ m_{i_1,j} & \text{if } i = i_0 \text{ and } j \in \mathcal{J}', \\ m_{i_0,j} & \text{if } i = i_1 \text{ and } j \in \mathcal{J}'. \end{cases}$$

Let $\pi_j = \sigma_j \circ \rho_j$, and $\Pi = \{\pi_j | j \in [J]\}$. By definition, $R_\Sigma(i) = R_\Pi(i)$ if $i \notin \{i_0, i_1\}$ and so

$$W_\Sigma(j) = W_\Pi(j) \text{ if } i \notin \{i_0, i_1\}. \quad (10)$$

Also, for $j \notin \mathcal{J}'$, $f(\sigma_j(m_{i,j})) = f(\pi_j(m_{i,j}))$, i.e.,

$$W_\Sigma(d,i) = W_\Pi(d,i) \text{ for } \forall d \notin D'. \quad (11)$$

For $\forall l \in [p]$, $W_\Sigma(d_l, i_0) = 1$, since $f(\sigma_{j_0}(m_{i_0,j_l})) = d_l$. Thus,

$$\sum_{d\in D'} W_\Sigma(d,i_0) = \sum_{l=0}^{p-1} W_\Sigma(d_l,i_0) + W_\Sigma(d_p,i_0) = p + W_\Sigma(d_p,i_0). \quad (12)$$

For $\forall l \in [p]$, $W_\Sigma(d_{l+1}, i_1) = 1$, since $f(\sigma_{i_l}(m_{i_l,i_1})) = d_{l+1}$. On the other hand, $W_\Sigma(d_0, i_1) = 0$ from (6). Therefore,

$$\sum_{d\in D'} W_\Sigma(d,i_1) = p. \quad (13)$$

For $\forall l \in [p]$, $W_\Pi(d_{l+1}, i_0) = 1$, since $f(\pi_{j_{l+1}}(m_{i_1,j_{l+1}})) = f(\sigma_{j_{l+1}}(m_{i_0,j_{l+1}})) = d_{l+1}$ by the definitions of π_j and ρ_j. By the definition of d_0, there is an integer $j \notin \mathcal{J}'$ such that $f(\sigma_i(m_{i_0,j})) = d_0$. Since $f(\sigma_i(m_{i_0,j})) = f(\pi_j(m_{i_0,j}))$ for $j \notin \mathcal{J}'$, $W_\Pi(d_0,i_0) = 1$. Thus,

$$\sum_{d\in D'} W_\Pi(d,i_0) = p+1. \quad (14)$$

Since $f(\pi_{j_l}(m_{i_1,j_l})) = f(\sigma_{j_l}(m_{i_0,j_l})) = d_l$ for $\forall l \in [p]$, $W_\Pi(d_l, i_1) = 1$, and we have

$$\sum_{d\in D'} W_\Pi(d,i_1) = p + W_\Pi(d_p,i_1). \quad (15)$$

From (9), if $W_\Sigma(d_p,i_0) = 1$ then there exists $j \notin \mathcal{J}'$ such that $f(\sigma_j(m_{i_1,j})) = f(\pi_j(m_{i_1,j})) = d_p$. This implies that $W_\Pi(d_p,i_1) = 1$ if $W_\Sigma(d_p,i_0) = 1$, i.e.,

$$W_\Pi(d_p,i_1) \geq W_\Sigma(d_p,i_0). \quad (16)$$

From (11)–(16),

$$W_\Pi(i_0) + W_\Pi(i_1) \geq W_\Sigma(i_0) + W_\Sigma(i_1) + 1.$$

Thus from (10), we have $W(\Pi) \geq W(\Sigma) + 1$.

This completes the proof of the claim and the lemma. ■

A set of permutations Π satisfying the condition in Lemma 4 is called *a set of shuffle permutations*. It is easy to see that a set of shuffle permutations can be found in $O(|M|^2)$ time.

B. 3-D Channel Routing Algorithm

In this section, we show a polynomial time algorithm for computing a routing of $\mathcal{N}$ in a 3-D channel with $h = 3\sqrt{n}$ layers. We use two virtual terminals $(X_i^{(l)}, Y_i^{(l)}, l)$ and $(X_i^{(m)}, Y_i^{(m)}, m)$ for each net N_i such that $X_i^{(h-1)} = X_i^{(l)}$, $Y_i^{(l)} = Y_i^{(m)}$, and $X_i^{(m)} = X_i^{(0)}$, where $l = 2\sqrt{n}$ and $m = \sqrt{n}$. The algorithm consists of three phases and each of which uses $\sqrt{n}$ layers. For each net N_i, we connect $(X_k^{(h-1)}, Y_k^{(h-1)}, h-1)$ with $(X_k^{(l)}, Y_k^{(l)}, l)$ in the first phase, $(X_k^{(l)}, Y_k^{(l)}, l-1)$ with $(X_k^{(m)}, Y_k^{(m)}, m)$ in the second phase, and $(X_k^{(m)}, Y_k^{(m)}, m-1)$ with $(X_k^{(0)}, Y_k^{(0)}, 0)$ in the last phase.

The virtual terminals can be cmoputed as follows. Let $I = J = \sqrt{n}$ and let $M = \{m_{i,j} | i \in [I], j \in [J]\}$ be the set defined as $m_{i,j} = N_k$ if $j = X_k^{(h-1)}/2$ and $i = Y_k^{(h-1)}/2$. Define that $D = \bigcup_{i\in[I]}\{X_i^{(0)}\} = \{0, 2, \ldots, 2\sqrt{n}-2\}$ and $f(N_k) = X_k^{(0)}/2$ for $\forall k \in [n]$. It is easy to see that f satisfies (4). By Lemma 4, we can obtain in polynomial time permutations π_j such that if $j \neq j'$ then $f \circ \pi_j^{-1}(m_{i,j}) \neq f \circ \pi_{j'}^{-1}(m_{i,j'})$. We define virtual terminals $(X_k^{(l)}, Y_k^{(l)}, l)$, $k \in [n]$, as $X_k^{(l)} = X_k^{(h-1)}$ and $Y_k^{(l)} = 2i$, where i is such an integer that $\pi_j(N_k) = m_{i,j}$ for $j = X_k^{(h-1)}/2$. It should be noted that if $Y_k^{(l)} = Y_{k'}^{(l)}$ then $X_k^{(0)} \neq X_{k'}^{(0)}$, since $f \circ \pi_j^{-1}(m_{i,j}) \neq f \circ \pi_{j'}^{-1}(m_{i,j'})$ if $j \neq j'$.

We define $(X_k^{(m)}, Y_k^{(m)}, m)$ as $X_k^{(m)} = X_k^{(0)}$ and $Y_k^{(m)} = Y_k^{(l)}$ for $\forall k \in [n]$.

Our 3-D channel routing algorithm is shown in Fig. 4. Since each of Steps 1–3 uses $\sqrt{n}$ layers, we can obtained a 3-D routing of $\mathcal{N}$ in a 3-D channel with height $3\sqrt{n}$. Since the length of every wire of a τ-routing is at most $2\sqrt{n} + O(1)$, the maximum length of a wire is at most $6\sqrt{n} + O(1)$. This completes the proof of Theorem 1.

V. Proof of Theorem 2

Let $\mathcal{N} = \{N_i | 0 \leq i \leq n-1\}$ be a set of n 2-nets such that $X_i^{(h-1)} \leq \sqrt{n} - 2$ and $X_i^{(0)} \geq \sqrt{n}$ if $i \leq n/2$, and $X_i^{(h-1)} \geq \sqrt{n}$ and $X_i^{(0)} \leq \sqrt{n} - 2$ if $i \geq n/2+1$. Consider an arbitrary rouiting of $\mathcal{N}$ on a 3-D channel G and let h be the height of G. Then a path for every net in $\mathcal{N}$ must go through

Input $\mathcal{N} = \{N_k | k \in [n]\}$ with terminals $(X_k^{(0)}, Y_k^{(0)}, 0)$ and $(X_k^{(h-1)}, Y_k^{(h-1)}, h-1)$ for $\forall k \in [n]$.

Output Routing of $\mathcal{N}$.

Step 0 **for** $\forall k \in [n]$,
Compute virtual terminals $(X_k^{(l)}, Y_k^{(l)}, l)$ and $(X_k^{(m)}, Y_k^{(m)}, m)$.

Step 1 **for** $\forall i \in [\sqrt{n}]$,
Apply τ-routing to connect $(X_k^{(h-1)}, Y_k^{(h-1)}, h-1)$ and $(X_l^{(l)}, Y_l^{(l)}, l)$ with $X_k^{(h-1)} = X_k^{(l)} = 2i$ in a $(2 \times 2\sqrt{n} \times \sqrt{n})$-subgrid consisting of $2i$-column and $(2i+1)$-column, and z-layers $(l \leq z \leq h-1)$.

for $\forall k \in [n]$,
connect $(X_k^{(l)}, Y_k^{(l)}, l)$ with $(X_k^{(l)}, Y_k^{(l)}, l-1)$.

Step 2 **for** $\forall j \in [\sqrt{n}]$,
Apply τ-routing to connect $(X_k^{(l-1)}, Y_k^{(l-1)}, l-1)$ and $(X_l^{(m)}, Y_l^{(m)}, m)$ with $X_k^{(l-1)} = X_k^{(m)} = 2j$ in a $(2 \times 2\sqrt{n} \times \sqrt{n})$-subgrid consisting of $2j$-row and $(2j+1)$-row, and z-layers $(m \leq z \leq l-1)$.

for $\forall k \in [n]$,
connect $(X_k^{(m)}, Y_k^{(m)}, m)$ with $(X_k^{(m)}, Y_k^{(m)}, m-1)$

Step 3 **for** $\forall i \in [\sqrt{n}]$,
Apply τ-routing to connect $(X_k^{(m-1)}, Y_k^{(m-1)}, m-1)$ and $(X_l^{(0)}, Y_l^{(0)}, 0)$ with $X_k^{(m-1)} = X_k^{(0)} = 2i$ in a $(2 \times 2\sqrt{n} \times \sqrt{n})$-subgrid consisting of $2i$-column and $(2i+1)$-column, and z-layers $(0 \leq z \leq m-1)$.

Step 4 Output a wire for each 2-net by concatenating three paths above connecting terminals and virtual terminals.

Fig. 4. 3-D Channel Routing Algorithm

the column defined by $x = \sqrt{n} - 1$. Since the area of every column is $2\sqrt{n}h$, we have $2h\sqrt{n} \geq |\mathcal{N}| = n$. Thus, $h = \Omega(\sqrt{n})$.

References

[1] J. Baliga, "Chips go vertical," *IEEE Spectrum*, vol. 41, pp. 43–47, 2004.

[2] K. Banerjee, S. J. Souri, P. Kapur, and K. C. Saraswat, "3-D ICs: A novel chip design for improving deep-submicrometer interconnect performance and systems-on-chip integration," *Proc. of the IEEE*, vol. 89, pp. 602–633, 2001.

[3] M. L. Brady, D. J. Brown, and P. J. McGuinness, "The three-dimensional channel routing problem," in *Algorithmic Aspects of VLSI Layout*. World Scientific, 1993, pp. 213–244.

[4] S. Das, A. Fan, K.-N. Chen, C. S. Tan, N. Checka, and R. Reif, "Technology, performance, and computer-aided design of three-dimensional integrated circuits," *Proc. ISPD*, pp. 92–98, 2004.

[5] R. Enbody, G. Lynn, and K. Tan, "Routing the 3-D chip," in *Proc. the 28-th Design Automation Conference DAC'91*, 1991.

[6] S. T. Obenaus and T. H. Szymanski, "Gravity: Fast placement for 3-D VLSI," *ACM Trans. Design Automation of Electronic Systems*, pp. 298–315, 2003.

[7] R. Y. Pinter, "On rouing two-point nets across a channel," in *Proc. the 19-th Design Automation Conference DAC'82*, 1982, pp. 894–902.

[8] C. C. Tong and C.-L. Wu, "Routing in a three-dimensional chip," *IEEE Trans. Computers*, pp. 106–117, 1995.