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エラー抑制法を組み込んだ量子計測

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概要：Quantum metrology has a potential for achieving high sensitivity by harnessing quantum effects and can be applied in various areas ranging from material science to biology. Because the sensitivity is reduced by decoherence, many efforts have been made to recover the sensitivity under the effect of the decoherence. However, most of the researches have considered only statistical errors: reducing general systematic errors remains almost unexplored. Actually, systematic errors could nullify the advantage of the quantum strategy over the classical one. Here, we propose error-mitigated quantum metrology, a protocol to mitigate systematic errors by combining quantum metrology with quantum error mitigation. We demonstrate that our protocol mitigates systematic errors and recovers the superclassical scaling in a practical situation under time-inhomogeneous noise. Our results pave the way for hybrid structures with quantum computing and quantum metrology.

Error-mitigated quantum metrology

1. Introduction

Quantum metrology aims to improve the sensitivity to estimate unknown parameters by harnessing quantum effects such as entanglement [1–6]. Such precise sensing technology has various applications such as a measurement of atomic frequency [7, 8], magnetometry [9–16], thermometry [17–19], an electrometer [20,21],and electron spin resonance [22–24].

A qubit based sensing is particularly promising for practical applications. A typical protocol is as follows: (1) preparing the probe qubits in a specific quantum state: (2) exposing the state to the target magnetic field: (3) performing a measurement for the readout: (4): Repeating (1)-(3) many times: (5) post-processing the measurement data with a theoretical model to estimate the target magnetic field.

A quantum strategy has a scaling advantage over the classical strategy for sensing. For given L qubits, a variance of the estimation scales as $\mathcal{O}(L^{-1})$ with separable states for the probe qubits, which is called the standard quantum limit (SQL). On the other hand, a variance of the estimation can scale as $\mathcal{O}(L^{-2})$ with entangled qubits, and this scaling is called the Heisenberg limit.

Since an entangled state is fragile against decoherence, it is not trivial whether the entangled sensors can surpass the separable sensors under realistic conditions. For specific noise such as local Markovian dephasing, the variance shows the same scaling as the SQL even when we use entanglement for the probe qubits. To surpass the SQL under the decoherence, several efforts have been devoted: using quantum superclassical effect [25, 26], implementing quantum teleportation [27, 28], and applying quantum error correction [29–38]. However, most of the previous works have considered only reducing statistical errors; techniques to suppress *systematic errors* remain almost unexplored.

Systematic errors are fatal for quantum metrology. Un-

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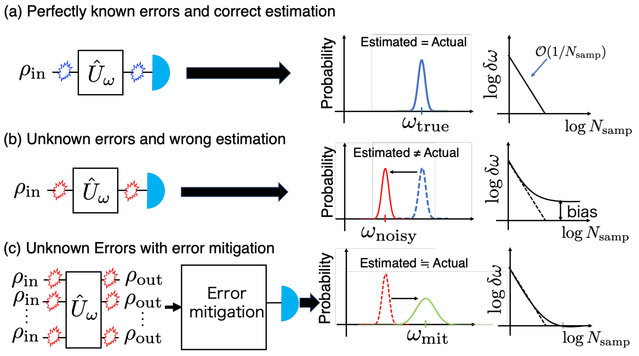

図 1 A schematic illustration on the present work. (a) No systematic noise happens with a correct estimation. (b) Systematic noises happen with a wrong estimation. (c) Error-mitigated quantum metrology exponentially reduces the systematic noise and recovers the estimation uncertainty of the target value ω.

like statistical errors, systematic errors are not reduced even when the number of samples increases, thus seriously limiting the sensitivity [22, 39] [Fig. 1(b)]. In quantum metrology, systematic errors usually result from a difference between a theoretical model adopted by the experimentalists and the actual one. To estimate the target parameter, we fit the experimental data with a theoretical model, and an inaccuracy of the theoretical model induces a bias in the estimation. If we had perfect knowledge about the noise model by using process tomography, systematic errors would not occur [Fig. 1(a)]. Such an inaccuracy of the theoretical model typically comes from drift noise; for example, a fluctuation of the coherence time has been observed [40, 41]. Such fluctuations prevent us to estimate the correct amplitude of the noise. Although some experimental works have been carried out to reduce specific systematic errors, a general approach is missing.

Quantum error mitigation (QEM) [42], which is developed for noisy intermediate-scale quantum computers [43], can be a solution to reduce systematic noises. QEM uses post-processing of the measurement outcome to recover the error-free expectation value at the expense of the sampling cost, but requires less qubit overhead than the quantum error correction. Such an increasing sampling cost allows QEM to reduce not statistical errors but systematic errors. Many kinds of error-mitigation techniques has been proposed such as extrapolation or probabilistic error cancellation [44, 45]. However, most of the methods are useless for quantum metrology due to the noise fluctuation.

In the present letter, we propose error-mitigated quantum metrology, a protocol to exponentially mitigate systematic errors and improve the sensitivity even under fluctuating noise [Fig. 1(c)]. Our protocol includes the idea of a recent-proposed error-mitigation technique, the virtual distillation [46, 47], to mitigate unknown stochastic errors. We carefully extend this idea in constructing our protocol to deal with fluctuating noise. We apply our protocol to entanglement sensor under Markovian and time-inhomogeneous noise and demonstrate that our protocol exponentially reduces the systematic error and 'recover' the scaling without systematic errors. Our result paves the way for applying error mitigation to quantum metrology to enhance the sensitivity.

2. Quantum metrology with systematic errors

Here, we describe a general theory for systematic errors in quantum metrology [48]. In a typical quantum-metrology setup, we prepare an initial state, expose this to the target fields characterized by a parameter ω, and obtain a state ρ_{c}. Then, on this state, we implement a measurement described by a projection operator $\hat{P}$ to provide a binary outcome. The measurement probability is given as

$$P_{\mathrm{c}} = \mathrm{Tr}[\hat{P}\rho_{\mathrm{c}}] = x_{\mathrm{c}} + y_{\mathrm{c}}\omega, \tag{1}$$

where x_{c} and y_{c} are some scalars. Here, we assume that ω is small, and we can ignore higher order terms. Based on the probability Eq. (1), we obtain the measurement outcome $m_j \in \{-1, 1\}$. Repeating the measurement N_{samp} times, we obtain the average value, $S_N = \sum_{j=1}^{N_{\mathrm{samp}}} m_j / N_{\mathrm{samp}}$. To estimate the parameter ω, we need to fit this experimental data with a theoretical model. If we have imperfect knowledge about the probe qubits, the theoretical model that we have should be different from the true one. We assume that the probability based on such an inaccurate theoretical model is given as

$$P_{\mathrm{e}} = \mathrm{Tr}[\hat{P}\rho_{\mathrm{e}}] = x_{\mathrm{e}} + y_{\mathrm{e}}\omega, \tag{2}$$

where ρ_{e}, x_{e}, and y_{e} denote the estimated values of ρ_{c}, x_{c}, and y_{c}, respectively: they can be different from the actual values. The average values of measurement outcome S_N and Eq. (2) give the estimated ω as $\omega_{\mathrm{est}} = (S_N - x_{\mathrm{e}})/y_{\mathrm{e}}$.

Systematic errors require us to consider the estimation uncertainty of the target quantity [48]. The estimation uncertainty of ω is defined as $\delta^2\omega = \left\langle (\omega_{\mathrm{est}} - \omega)^2 \right\rangle$ with the bracket denoting the ensemble average, and calculated as

$$\delta^2\omega = \frac{1}{y_{\mathrm{e}}^2}\left[\delta^2 P_{\mathrm{c}} + (x_{\mathrm{e}} - x_{\mathrm{c}})^2\right], \tag{3}$$

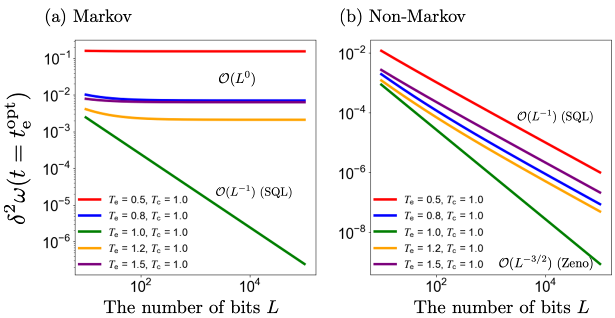

図 2 The estimation uncertainty of ω as a function of the number of qubits L for several combinations of T_{e} and T_{c}, where the total measurement time is set to be $T = 100$. When we estimate a coherence time correctly, $T_{\mathrm{e}} = T_{\mathrm{c}}$, $\delta^2\omega$ follows the standard quantum limit for the Markovian noise in (a), while it follows the superclassical scaling for the time-inhomogeneous noise in (b). When $T_{\mathrm{e}} \neq T_{\mathrm{c}}$, $\delta^2\omega$ is constant for the Markovian noise in (a), while it follows the standard quantum limit for the time-inhomogeneous noise in (b).

where $\delta^2 P_{\mathrm{c}}$ is the variance of P_{c}, which is typically $\delta^2 P_{\mathrm{c}} \simeq P_{\mathrm{c}}(1-P_{\mathrm{c}})/N_{\mathrm{samp}}$, and we neglected $(y_{\mathrm{e}}-y_{\mathrm{c}})^2\omega^2 + 2(x_{\mathrm{e}}-x_{\mathrm{c}})(y_{\mathrm{e}}-y_{\mathrm{c}})\omega$ because ω is small. The second term in Eq. (3) comes from a systematic error $x_{\mathrm{e}} - x_{\mathrm{c}}$ induced by the wrong estimation of the probability $P_{\mathrm{c}} \neq P_{\mathrm{e}}$. As N_{samp} increases, the first term in Eq. (3), which comes from a statistical error, decreases whereas the second term remains. Most of the previous theoretical studies have focused on the statistical error by assuming $P_{\mathrm{c}} = P_{\mathrm{e}}$. In the following, we investigate how the systematic error $x_{\mathrm{e}} - x_{\mathrm{c}}$ affects the scaling of the variance $\delta^2\omega$ against L when we adopt the Ramsey interferometry measurement under the effect of Markovian and time-inhomogeneous noise.

3. Ramsey interferometry measurement under Markovian and time-inhomogeneous noise

Here, we investigate how systematic errors affect the estimation uncertainty of ω using Ramsey interferometry measurement [5] under the Markovian and time-inhomogeneous local amplitude damping. We assume that we do not precisely know the coherence time of the system, and this induces the systematic error. The measurement protocol is as follows. Firstly, we prepare the L-qubits Greenberger–Horne–Zeilinger (GHZ) state: $\rho_0 = |\mathrm{GHZ}\rangle\langle\mathrm{GHZ}|$ where $|\mathrm{GHZ}\rangle = (|0...0\rangle + |1...1\rangle)/\sqrt{2}$. Subsequently, the initial state undergoes local amplitude damping as follows: $\mathcal{E}^{(L)} \circ \mathcal{E}^{(L-1)} \circ \cdots \mathcal{E}^{(1)}(\rho_0)$, where $\mathcal{E}^{(i)}[\rho] = K_0^{(i)}\rho K_0^{(i)} + K_1^{(i)}\rho K_1^{(i)}$ is the error map of the amplitude damping on ith qubit with

$$K_0^{(i)} = \begin{pmatrix} \sqrt{1-\epsilon} & 0 \\ 0 & 1 \end{pmatrix}, K_1 = \begin{pmatrix} 0 & 0 \\ \sqrt{\epsilon} & 0 \end{pmatrix} \tag{4}$$

being Klaus maps on the ith qubit and ϵ being an error rate. The resulted initial density matrix is

$$\begin{aligned}\mathcal{E}(\rho) = &\frac{1}{2}\Big[(1-\epsilon)^L |1...1\rangle\langle 1...1| + (1+\epsilon^L)|0...0\rangle\langle 0...0| \\ &+(1-\epsilon)^{L/2}(|1...1\rangle\langle 0...0| + |0...0\rangle\langle 1...1|)\Big] \\ &+\frac{1}{2}\sum_{k=1}^{2^L-1} p_k|\psi_k\rangle \qquad (5)\\ = &\frac{1}{2}[\lambda_+|\lambda_+\rangle\langle\lambda_+| + \lambda_-|\lambda_-\rangle\langle\lambda_-|] \\ &+\frac{1}{2}\sum_{k=1}^{2^L-1} p_k|\psi_k\rangle, \qquad (6)\end{aligned}$$

where $\lambda_{\pm} = \frac{1+\epsilon^L+(1-\epsilon)^L \pm E}{2}$ are eigenvalues with $E = \sqrt{[1+\epsilon^L-(1-\epsilon)^L]^2 + 4(1-\epsilon)^L}$, and $|\lambda_+\rangle = \cos\theta|1...1\rangle + \sin\theta|0...0\rangle$ and $|\lambda_-\rangle = -\sin\theta|1...1\rangle + \cos\theta|0...0\rangle$ are corresponding eigenstates with $\cos 2\theta = [(1-\epsilon)^L - (1+\epsilon^L)]/E$ and $\sin 2\theta = 2(1-\epsilon)^{L/2}/E$. Here $|\psi_k\rangle$ is the computational basis that is orthogonal to $|0...0\rangle$ and $|1...1\rangle$ (e.g. $|010...0\rangle, |011...0\rangle$) and $p_k = \epsilon^k(1-\epsilon)^{L-k}$ with k being the number of 0 in $|\psi_k\rangle$. Then the GHZ state evolves under the Hamiltonian $\hat{H} = \sum_{j=1}^{L}\omega\hat{\sigma}_z^{(j)}/2$ for a time t, where ω is the energy difference between $|0...0\rangle$ and $|1...1\rangle$. Throughout of our paper, we set $\hbar = 1$. Since the unitary time-evolution commutes with the noise process, we can consider the case that the noise occurs before the unitary evolution without loss of generality. Finally, we implement a projective measurement with $\hat{P}_y = |\mathrm{GHZ}_y\rangle\langle\mathrm{GHZ}_y|$, where $|\mathrm{GHZ}_y\rangle = (|0...0\rangle - i|1...1\rangle)/\sqrt{2}$. The probability of the measurement is

$$\begin{aligned}P = \mathrm{Tr}[\rho(t)\hat{P}_y] &= \frac{1+\epsilon^L+(1-\epsilon)^L}{4} + \frac{(1-\epsilon)^{L/2}}{2}Lt\omega \\ &\equiv x + y\omega \qquad (7)\end{aligned}$$

where we used $\sin\omega t \simeq \omega t$ due to small ωt.

We now turn our attention to the uncertainty of ω. In what follows, the quantities with the subscript c(e) denote the actual (estimated) values and are calculated with the actual (estimated) error rate $\epsilon_{\mathrm{c(e)}}$. In quantum metrology, we usually choose the interaction time to minimize the uncertainty. However, since we do not know the actual error rate, we can only estimate the variance, $\delta^2\omega_{\mathrm{e}} = P_{\mathrm{e}}(1-P_{\mathrm{e}})/(y_{\mathrm{e}}^2 N_{\mathrm{samp}})$. Then we minimizes the estimated variance, $\delta^2\omega_{\mathrm{e}}$, with respect to t under the total measurement time $T = N_{\mathrm{samp}}t$ and obtain the 'optimal' time $t_{\mathrm{e}}^{\mathrm{opt}}$. In fact, the estimation uncertainty of ω is calculated as $\delta^2\omega_{\mathrm{c}}(t = t_{\mathrm{e}}^{\mathrm{opt}})$.

To demonstrate the effect of bias noise, we calculate $\delta^2\omega_{\rm c}(t = t_{\rm e}^{\rm opt})$ using the actual (estimated) error rate being $\epsilon_{\rm c(e)} = 1 - \exp(-t/T_{\rm c(e)})$ for Markovian noise and $\epsilon_{\rm c(e)} = 1 - \exp(-t^2/T_{\rm c(e)}^2)$ for time-inhomogeneous noise, where $T_{\rm c(e)}$ being the actual(estimated) coherence time of the noise. The 'optimal' measurement interval is numerically found to be $t = t_{\rm e}^{\rm opt} \sim \mathcal{O}(L^{-1})$ for Markovian noise and $t = t_{\rm e}^{\rm opt} \sim \mathcal{O}(L^{-1/2})$ for time-inhomogeneous noise. The result is shown in Fig. 2(a)[(b)] for Markovian (time-inhomogeneous) noise. For both the noises, since the correct estimation of the coherence time, $T_{\rm c} = T_{\rm e}$ [the green line in Fig. 2], provides the true uncertainty, the uncertainty of ω follows the conventional scaling: standard quantum limit, $\delta^2\omega \sim \mathcal{O}(L^{-1})$ for the Markovian noise and the superclassical scaling, $\delta^2\omega \sim \mathcal{O}(L^{-3/2})$, for the time-inhomogeneous noise [25, 26]. On the other hand, the wrong estimation of the coherence time ($T_{\rm c} \neq T_{\rm e}$) significantly limits the advantage of the quantum strategy; the estimation uncertainty scales as $\delta^2\omega \sim \mathcal{O}(L^0)$ for the Markovian noise, and $\delta^2\omega \sim \mathcal{O}(L^{-1})$ (SQL) for the time-inhomogeneous noise. This is explained as follows. The 'optimal' time $t_{\rm e}^{\rm opt}$ scales as $\mathcal{O}(L^{-1})$ $[\mathcal{O}(L^{-1/2})]$ for Markovian (time-inhomogeneous) noise. The term coming from the systematic noise in Eq. (3), $(x_{\rm e} - x_{\rm c})^2/y_{\rm e}^2$, scales as $\mathcal{O}(L^0)$ $[\mathcal{O}(L^{-1})]$ because $x_{\rm c(e)} \sim \mathcal{O}(L^0)$ for both the noises and $y_{\rm e} \sim \mathcal{O}(L^0)$ $[y_{\rm e} \sim \mathcal{O}(L^{-1})]$. Since this scaling is poorer than that comes from the statistical error (the first term in Eq. (3)), the systematic noise is dominant, resulting in the constant (the SQL) scaling of the estimation uncertainty. In general, the scaling of the bias term $(x_{\rm e} - x_{\rm c})^2$ is worse than that of $\delta^2 P_{\rm c}/N_{\rm samp}$, and therefore the former spoils the latter.

4. Error-mitigated quantum metrology

So far, we have demonstrated that systematic errors spoil the scaling relation between $\delta^2\omega$ and L. Here, to reduce systematic errors, we propose a protocol of error-mitigated quantum metrology [Fig. 3(a)]. In the following, we describe our protocol step by step: note that the notations are the same as those above without error mitigation. The protocol before the projective measurement is the same as the aforementioned protocol except for preparing n copies of the GHZ states as the initial state. After the time evolution, the states are put to a quantum circuit shown in the error-mitigation part in Fig. 3(a). This circuit enables us to obtain $\mathrm{Tr}[\rho^n]$ with $\hat{O} = \hat{I}$ and $\mathrm{Tr}[\rho^n \hat{P}_y]$ with $\hat{O} = \hat{Y}$ [46]; see App. A for the virtual distillation. The aforementioned Ramsey interferometry model enables the analytical calculation of $\mathrm{Tr}[\rho^n]$ and $\mathrm{Tr}[\rho^n \hat{P}_y]$ as follows:

$$\mathrm{Tr}[\rho^n] = \frac{1}{2^n}\left[\lambda_+^n + \lambda_-^n + \sum_{k=1}^{L-1}\binom{L}{k}\epsilon^{nk}(1-\epsilon)^{n(L-k)}\right] \tag{8}$$

and

$$\mathrm{Tr}[\rho^n \hat{P}_y] = \frac{1}{2}\frac{\lambda_+^n + \lambda_-^n}{2^n} + \frac{1}{2}\frac{\lambda_+^n - \lambda_-^n}{2^n}\sin 2\theta L\omega t. \tag{9}$$

These quantities provide an error-mitigated probability as $P_n = \mathrm{Tr}[\rho^n \hat{P}_y]/\mathrm{Tr}[\rho^n] = x_n + y_n\omega$, where

$$x_n = \frac{1}{2}\frac{1 + \left(\frac{\lambda_-}{\lambda_+}\right)^n}{1 + \left(\frac{\lambda_-}{\lambda_+}\right)^n + \sum_{k=1}^{L-1}\binom{L}{k}\left(\frac{\epsilon^k(1-\epsilon)^{(L-k)}}{\lambda_+}\right)^n}, \tag{10}$$

$$y_n = \frac{1}{2}\frac{\left[1 - \left(\frac{\lambda_-}{\lambda_+}\right)^n\right]Lt\sin 2\theta}{1 + \left(\frac{\lambda_-}{\lambda_+}\right)^n + \sum_{k=1}^{L-1}\binom{L}{k}\left(\frac{\epsilon^k(1-\epsilon)^{(L-k)}}{\lambda_+}\right)^n}. \tag{11}$$

Here, x_n and y_n are error-mitigated values of x and y in Eq. (7), respectively. The estimation uncertainty in our protocol is calculated to be $\delta^2\omega = [\delta^2 P_n + (x_{\rm e} - x_{\rm c})^2]/y_{\rm e}^2$, where $\delta^2 P_n$ is the variance of P_n calculated using the propagation of error as [46]

$$\begin{aligned}\delta^2 P_n &= \delta^2\left(\frac{\mathrm{Tr}[\rho^n \hat{P}_y]}{\mathrm{Tr}[\rho^n]}\right) \\ &= \frac{1 - (\mathrm{Tr}[\rho^n \hat{P}_y])^2}{(\mathrm{Tr}[\rho^n])^2 N'_{\rm samp}} + \frac{(1 - (\mathrm{Tr}[\rho^n])^2)(\mathrm{Tr}[\rho^n \hat{P}_y])^2}{(\mathrm{Tr}[\rho^n])^4 N'_{\rm samp}},\end{aligned} \tag{12}$$

where we assumed the same number of sampling for the numerator and the denominator. Here, $N'_{\rm samp} = N_{\rm samp}/(2n)$ because our protocol requires $2n$ times more sampling than the standard protocol: twice for the numerator and denominator, and n times for the error mitigation. The optimization protocol for the estimation uncertainty is the same as that without error mitigation except for using error-mitigated quantities. We firstly find the 'optimal' time $t = t_{{\rm e},n}^{\rm opt}$ that minimize the estimated variance, $\delta^2\omega_{{\rm e},n} = \delta^2 P_{{\rm e},n}/y_{{\rm e},n}^2$ under the total measurement time $T = N_{\rm samp}t$. By substituting $t = t_{{\rm e},n}^{\rm opt}$ for $\delta^2\omega_n = [\delta^2 P_{{\rm c},n} + (x_{{\rm c},n} - x_{{\rm e},n})^2]/y_{{\rm e},n}^2$, we obtain the estimation uncertainty of ω for the error-mitigated protocol. Our protocol enables both $x_{{\rm e},n}$ and $x_{{\rm c},n}$ to exponentially converge to $1/2$ as n increases, and thus the systematic noise, $x_{{\rm e},n} - x_{{\rm c},n}$, exponentially approaches zero (see Eq. (**??**)). In this way, our protocol mitigates the systematic error.

To show the performance of our protocol, we numerically find the 'optimal' time $t = t_{{\rm e},n}^{\rm opt}$ and plot $\delta^2\omega_n(t = t_{{\rm e},n}^{\rm opt})$ as a function of L in Fig. 3(b) [Fig. 3(c)] for local Markovian (time-inhomogeneous) amplitude damping

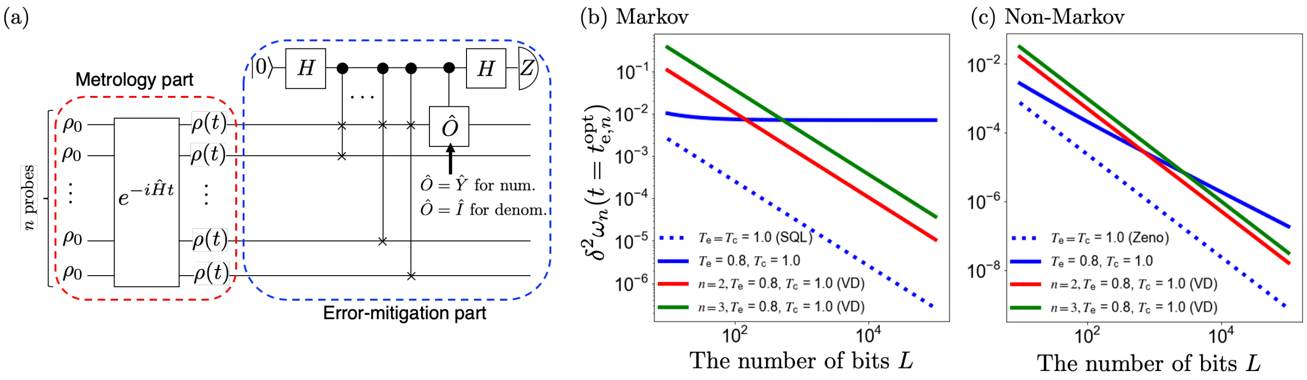

図 3 (a) A quantum circuit for the error-mitigated quantum metrology. (b)[(c)] The estimation uncertainty $\delta^2\omega_n$ for Markovian (time-inhomogeneous) local amplitude damping as a function of the number of qubits L with $T_{\rm e} = 0.8$ and $T_{\rm c} = 1.0$ for $n = 2, 3$, where the total measurement time is set to be $T = 100$. The red and green lines shows that the virtual distillation can recover standard quantum limit (the superclassical scaling) even when $T_{\rm e} \neq T_{\rm c}$.

noise. As shown with the blue line in Fig. 3(b) [Fig. 3(c)], $\delta^2\omega_n$ follows the constant (the SQL) for the Markov (time-inhomogeneous) noise without error mitigation due to the systematic error. Importantly, our protocol recovers the SQL (the superclassical) scaling, $\delta^2\omega \sim \mathcal{O}(L^{-1})$ [$\delta^2\omega \sim \mathcal{O}(L^{-3/2})$], as the red (for $n = 2$) and green (for $n = 3$) lines show. Although the performance of the virtual distillation under coherent errors seems limited [?], our demonstration suggests that our protocol is resilient for a local amplitude damping (T1 noise), which is a typical coherent error in quantum metrology.

5. Possible experimental realization

Finally, we propose a possible experimental realization of our scheme using a hybrid structure of quantum sensor and quantum computer. Superconducting flux qubits (FQs) are promising candidates for both quantum metrology and quantum computation. The FQs have a strong coupling with the magnetic fields, and are suitable for quantum magnetic field sensing. Moreover, since the FQs are artificial atoms, there are many degrees of freedom for the circuit design, and so they have advantage in the scalability, which is prerequisite for a quantum computer. Moreover, the long coherence time of the FQ are useful for both quantum metrology and quantum computation. Therefore, we could use the FQ not only for the conventional Ramsey measurements but also for our scheme to utilize the entanglement generation and virtual distillation where high-fidelity quantum operations are required.

6. Conclusions and discussions

In conclusion, we have pointed out that systematic errors spoil a scaling relation between $\delta^2\omega$ and L, and illustrated this by considering Ramsey measurements under Markovian and time-inhomogeneous local amplitude damping. To mitigate the systematic error, we have proposed error-mitigated quantum metrology, a protocol including error mitigation using the quantum circuit shown in Fig. 3(a). We have found that our protocol exponentially reduces the systematic errors and recovers the scaling without systematic errors. Our results have suggested that error mitigation can be useful for quantum metrology under systematic errors, which typically appear with estimation errors. Notably, our protocol can be applied to the case with any stochastic and fluctuating noise. In contrast, quantum metrology with quantum error correction can be applied to specific cases such as bit-flip errors. Thus, our protocol will pave the way to achieve high sensitivity in quantum metrology under general stochastic errors.

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A.1 Virtual distillation

The virtual distillation is a recent-proposed error-mitigation technique [46,47] and can exponentially reduce stochastic unknown errors as follows. Suppose we want the expectation value of an observable $\hat{O}$ with an (error-free) ideal state $|psi\rangle_{\text{ideal}}$, but unexpected errors causes a noisy density matrices

$$\rho_{\text{noisy}} = (1-\epsilon)\left|\psi_{\text{ideal}}\right\rangle\left\langle\psi_{\text{ideal}}\right| + \epsilon\sum_{k=2}^{D} p_i \left|\psi_k\right\rangle\left\langle\psi_k\right|, \tag{A.1}$$

where ϵ is an error rate, D is a dimension of a system that we consider, $|\psi\rangle_k$ is an erroneous state with ϵp_k being a corresponding error probability. We assume that $(1-\epsilon) \gg \epsilon p_i$. We want $\left\langle\hat{O}\right\rangle_{\text{ideal}} = \left\langle\psi_{\text{ideal}}\middle|\hat{O}\middle|\psi_{\text{ideal}}\right\rangle$, but the noisy density matrix gives

$$\mathrm{Tr}[\rho_{\text{noisy}}\hat{O}] = (1-\epsilon)\langle\hat{O}\rangle_{\text{ideal}} + \epsilon\sum_{k=2}^{D} p_k \langle\hat{O}\rangle_k, \tag{A.2}$$

where $\langle\hat{O}\rangle_k = \langle\psi_k|\hat{O}|\psi_k\rangle$ is the expectation value of $\hat{O}$ with the erroneous state $|\psi_k\rangle$. In the virtual distillation, we calculate $\mathrm{Tr}[(\rho_{\text{noisy}})^n\hat{O}]/\mathrm{Tr}[(\rho_{\text{noisy}})^n]$ as the error-mitigated expectation value of $\hat{O}$:

$$\langle\hat{O}\rangle_{\text{mit}} = \frac{\mathrm{Tr}[(\rho_{\text{noisy}})^n\hat{O}]}{\mathrm{Tr}[(\rho_{\text{noisy}})^n]} = \frac{\langle O\rangle_{\text{ideal}} + \left(\frac{\epsilon}{1-\epsilon}\right)^n \sum_{k=2}^{D}(p_k)^n\langle\hat{O}\rangle_k}{1 + \left(\frac{\epsilon}{1-\epsilon}\right)^n \sum_{k=2}^{D}(p_k)^n} \tag{A.3}$$

$$= \langle\hat{O}\rangle_{\text{ideal}} + \left(\frac{\epsilon}{1-\epsilon}\right)^n \left[\sum_{k=2}^{D}(p_k)^n(\langle\hat{O}\rangle_k - \langle\hat{O}\rangle_{\text{ideal}})\right] + \mathcal{O}\left[\left(\frac{\epsilon}{1-\epsilon}\right)^{2n}\left(\sum_{k=2}^{D}(p_k)^n\right)^2\right] \tag{A.4}$$

This equation shows that $\langle\hat{O}\rangle_{\text{mit}}$ exponentially approaches

$\langle\hat{O}\rangle_{\mathrm{ideal}}$ as n increases. To calculate $\langle\hat{O}\rangle_{\mathrm{mit}}$, we prepare M copies of ρ_{noisy}, and implement a specific type of SWAP operations (derangement [46]) on them, and measure the ancilla qubit in the quantum circuit shown in Fig. 3(a) with $\hat{O}$ and without $\hat{O}$: the probabilities of getting 0 as the measurement outcome with $\hat{O}$ and without $\hat{O}$ are denoted as p_{num} and p_{denom}, respectively. These probabilities allows us to calculate $\langle\hat{O}\rangle_{\mathrm{mit}}$ as

$$\frac{2p_{\mathrm{num}}-1}{2p_{\mathrm{denom}}-1}=\frac{\mathrm{Tr}[(\rho_{\mathrm{noisy}})^n\hat{O}]}{\mathrm{Tr}[(\rho_{\mathrm{noisy}})^n]}=\langle\hat{O}\rangle_{\mathrm{mit}}\,. \tag{A.5}$$