Mathematical Model of Permanent Magnet Synchronous Motor

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Abstract: The mathematical model of a permanent magnet synchronous motor (PMSM) is necessary to design the control of PMSM. The mathematical model on three-phase system is not commonly used for the control design. This approach is the time-varying model. Therefore, the control strategy design becomes even more difficult. Owing to this problem, this paper presents the dynamic model of the PMSM using the DQ modeling method. In addition, the proposed model has been validated with the exact topology model in MATLAB/Simulink program.

Keywords: permanent magnet synchronous motor (PMSM); DQ modeling method; model validation

1. Introduction

Nowadays, the PMSM is widely used for many applications, especially for electric vehicle (EV). In order to achieve a high performance PMSM drives, the control of the PMSM drives should be suitably designed. From literature reviews, there are many control techniques (field-oriented control, predictive DTC scheme, nonlinear torque control scheme, etc.) and many estimation techniques (flux vector, speed) [1-3]. The control design of PMSM drives is based on the mathematical model of PMSM. Therefore, the aim of this work is to derive the mathematical model of PMSM. The DQ modeling approach is selected to derive the mathematical model of the studied PMSM.

This paper is structured as follows. The principle of the PMSM is briefly explained in Section 2. The mathematical model of the PMSM are presented in Section 3. The model validation is expressed in Section 4. Finally, Section 5 concludes the purpose of this paper.

2. The Principle of the PMSM

The operation of a PMSM is similar to a three-phase induction motor. The three-phase voltage source connected with the stator winding produces a rotating magnetic field (RMF). The RMF cause the rotor to turn. The power losses in the rotor side do not occur because the rotor of PMSM is a permanent magnet. Moreover, this machine can provide a constant torque. The structure and equivalent circuit of the PMSM are shown in Fig. 1.

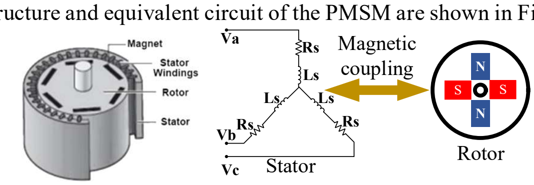

Fig. 1 The structure and equivalent circuit of the PMSM

3. Mathematical Model of the PMSM

The DQ modeling method is applied to derive a mathematical model of the system as depicted in Fig. 2. The DQ–axis in Fig. 3 is rotated with the angular speed (ω_r) by phase shift (θ_r). The stator voltages (v_{abc}) can be written for three-phase system as follows:

$$v_{abc} = R_s i_{abc} + \frac{d}{dt}(L_s\, i_{abc} + \lambda_{pm}(\theta)) \qquad (1)$$

$$\frac{d}{dt}\lambda_{pm}(\theta) = -\omega_r \lambda_{pm} \begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - 2\pi/3) \\ \sin(\theta_r + 2\pi/3) \end{bmatrix} \qquad (2)$$

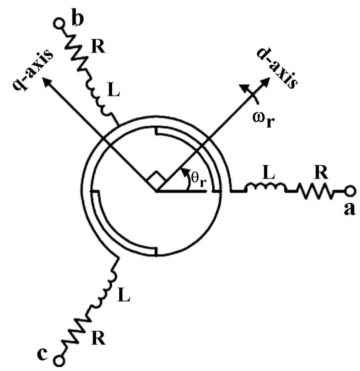

Fig. 3 The vector diagram for DQ transformation

$$\left.\begin{aligned} v_a &= R_s i_a + L_s \frac{d}{dt} i_a - \omega_r \lambda_{pm} \sin(\theta_r) \\ v_b &= R_s i_b + L_s \frac{d}{dt} i_b - \omega_r \lambda_{pm} \sin(\theta_r - 2\pi/3) \\ v_c &= R_s i_c + L_s \frac{d}{dt} i_c - \omega_r \lambda_{pm} \sin(\theta_r + 2\pi/3) \end{aligned}\right\} \qquad (3)$$

where the $\frac{d}{dt}\lambda_{pm}(\theta)$ in (2) is the back EMF. Then, the mathematical model on three-phase system in (3) can be transformed into the DQ-axis. The dynamic equation of the PMSM on DQ-axis can be written in (4) - (5).

$$v_d = R_s i_d - \omega_r \lambda_q + \frac{d}{dt}\lambda_d = R_s i_d + L_d \frac{d}{dt} i_d - \omega_r L_q i_q + \frac{d}{dt}\lambda_{pm} \qquad (4)$$

$$v_q = R_s i_q + \omega_r \lambda_d + \frac{d}{dt}\lambda_q = R_s i_q + L_q \frac{d}{dt} i_q + \omega_r L_d i_d + \omega_r \lambda_{pm} \qquad (5)$$

As a result, the equivalent circuit of the PMSM in DQ-axis derived by using DQ modeling method is shown in Fig. 4.

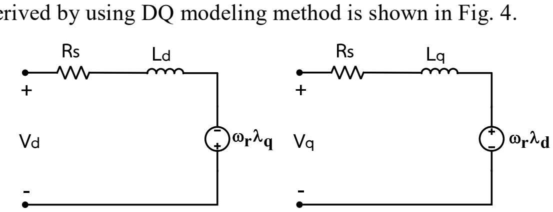

Fig. 4 The PMSM equivalent circuit in the DQ-axis

From the (4)-(5), the developed torque (T_e) and the angular motor speed (ω_m) can be calculated in (6)-(7).

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$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right)$$
$$= \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_{pm} i_q + \left(L_d - L_q\right) i_d i_q\right) \quad (6)$$

$$\left.\begin{aligned} \omega_m &= \int\left(\frac{T_e - T_L - B\omega_m}{J}\right)dt \\ \omega_r &= \frac{d}{dt}\theta_r = \left(\frac{P}{2}\right)\omega_m \Rightarrow \theta_m = \theta_r\left(\frac{2}{P}\right) \end{aligned}\right\} \quad (7)$$

4. Model Validation

The simulation for model validation uses the exact topology model in SimPowerSystem of MATLAB/Simulink called the benchmark model. The parameters of the PMSM in Fig. 1 are given in Table 1. The proposed model implemented by MATLAB/Simulink is illustrated in Fig. 5.

Table. 1 Parameters of the PMSM

Symbol	Description	Value
R_S	Stator resistance	0 55 Ω
L_d	D-axis inductance	16 61 mH
L_q	Q-axis inductance	16 22 mH
λ_{pm}	Permanent magnet flux	0 121 Vs
J	Rotor inertia	$7\ 2460 \times 10^{-3}$ kg m^2
P	Pole pair	4
P_S	Rated power	750 W
N_S	Rated speed	3000 rpm

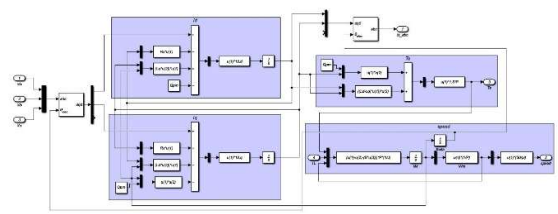
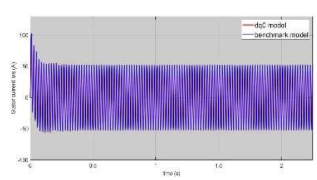
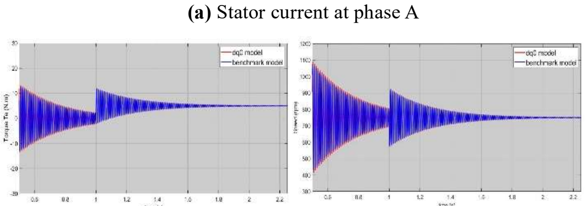


Fig. 5 The simulation model used to validate the PMSM model

Fig. 6 shows the response comparison of stator current, load torque, and motor speed between the DQ model and the benchmark model.



(a) Stator current at phase A



(b) Changing torque and speed

Fig. 6 Model verification

The testing condition for model validation consists of the load torque changing and frequency changing which are detailed as presented in Table 2 and 3, respectively.

Table. 2 The model validation results for load torque changing

Load torque (N.m)	Model	Measured value			
		V_{abc} (V)	I_{abc} (A)	Speed (rpm)	Torque (N m)
1	DQ	219 97	36 81	750	1
	Benchmark	219 97	36 82	750	1 039
3	DQ	219 97	36 80	750	3
	Benchmark	219 97	36 80	750	3 039
5	DQ	220 00	36 80	750	5
	Benchmark	220 00	36 80	750	5 039

Table. 3 The model validation results for frequency changing

Frequency (Hz)	Model	Measured value			
		V_{abc} (V)	I_{abc} (A)	Speed (rpm)	Torque (N m)
50	DQ	220 00	36 80	750	5
	Benchmark	220 00	36 80	750	5 039
45	DQ	199 93	37 17	675	5
	Benchmark	199 93	37 17	675	5 035
40	DQ	179 80	37 59	600	5
	Benchmark	179 80	37 60	600	5 031
35	DQ	159 77	38 16	525	5
	Benchmark	159 77	38 16	525	5 027
30	DQ	139 83	38 92	450	5
	Benchmark	139 83	38 92	450	5 023

5. Conclusion

This paper presents how to derive the DQ model of the PMSM by using the DQ modeling method. The results confirm that the mathematical model of the PMSM from DQ modeling method provides a good accuracy compared with the simulation results from the exact topology model. In the future work, the authors will use the proposed model of the PMSM on DQ-axis to design the field oriented control (FOC) for PMSM drives.

Acknowledgments

The authors express their thanks to Prince of Songkla University.

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