異種タスク集合に対する複数電圧レベルDVFSに関する一考察

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概要：省電力化のためのＤＶＦＳを考慮した単一プロセッサ上での複数タスク実行は，ＤＶＦＳにおける基本問題の一つと考えることができ，その特徴理解は種々の現実的ＤＶＦＳ問題を考える際の基礎となると考えられる．本稿では，制限された電圧レベル数の下でのＤＶＦＳにおける電圧レベル決定とタスクの電圧レベルへの割り当ての同時最適化について考察する．Karush-Kuhn-Tucker 条件は非線形最適化問題の解に対する必要条件であるが，これを当該問題に適用することで，最適解が持つ特徴を明らかにすると共に，効率の良い求解アルゴリズムを導出する．

A Study on Multi-Level DVFS for Heterogeneous Task Set

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Abstract: This paper discusses DVFS with a limited number of voltage levels (ML-DVFS), especially concurrent optimization of voltage levels and voltage assignment is investigated. Based on the KKT conditions for the optimum solution of a nonlinear optimization problem, several properties of the optimum solution of ML-DVFS problem are revealed. Proposed solution algorithm consists of the enumeration of partitioning of a task set and two-level bisection search on a voltage level and an auxiliary parameter which is one of Lagrangian multipliers in KKT conditions.

1. Introduction

Power consumption is one of the major concerns for wide range of computing systems from high performance VLSI chip to highly parallel computer system. For CMOS LSIs, there is well known power-performance trade-off, and the efforts to find out a best power supply voltage for minimizing consuming power (energy) under a given performance constraint have been made extensively. Voltage-Frequency Scaling (VFS) [1]-[5] would be one of the most popular techniques for this purpose. The concept of VFS is simple enough to be applied to wide range of LSI/computer systems, which implies that there are many variants in terms of target system model, target task model, constraints, objectives, etc.

In this paper, we will discuss a very basic issue of VFS, that is, how supply voltage levels are determined in the VFS environment. We will focus mainly on DVFS for multiple heterogeneous tasks to be processed on a single processors with a single overall deadline. When each task can take its own voltage level, the solution of DVFS is computed with linear time complexity with respect to the number of tasks.[6]. However, the number of available supply voltage levels is limited in major practical systems.

This paper discusses DVFS with a limited number of voltage levels, but the aim of this paper is not only to provide the optimum solution of voltage schedule in DVFS, but also to afford valuable insights into DVFS. Our discussion is mainly based on Karush-Kuhn-Tucker conditions (KKT conditions), and shows how KKT conditions characterize the optimum solution of our DVFS problem.

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2. Problem Formulation

2.1 Basic notations

Let V_i and m_i be the supply voltage and the number of cycles, respectively, of a task t_i, $i \in \{1, 2, \cdots, n\}$.

Operating (clock) frequency of a task t_i is given as;

$$f(V_i)$$

with a function $f(voltage)$ which is common for all tasks.

Dynamic energy consumption of a task t_i is given with a dynamic energy per cycle $e_i^{(D)}(V_i)$ as follows.

$$e_i^{(D)}(V_i) \cdot m_i$$

Static energy consumption of a task t_i is given as;

$$P^{(S)}(V_i) \times \frac{m_i}{f(V_i)}$$

with a static power function $P^{(S)}(voltage)$ which is common for all tasks.

Our objective is;

$$E_{total}(\boldsymbol{V}) \ = \ \sum_{i=1}^{n} \left(e_i^{(D)}(V_i) + \frac{P^{(S)}(V_i)}{f(V_i)} \right) \cdot m_i \ \to \min$$

$$\text{subject to} \ \ T_{total}(\boldsymbol{V}) \ = \ \sum_{i=1}^{n} \frac{m_i}{f(V_i)} \ \le \ T_{max}$$

where T_{max} is a deadline specified as a part of the input description to the problem.

Task ID numbering is assumed to follow the decreasing order of the dynamic energy consumption under the same supply voltage. That is,

$$e_1^{(D)}(V) > e_2^{(D)}(V) > \cdots > e_n^{(D)}(V)$$

2.2 Multi-level DVFS

Now we consider that each task can take one of L supply voltages, $\mathcal{V}_1, \mathcal{V}_2, \cdots, \mathcal{V}_L$, where $L < n$. With out loss of generality, we assume;

$$\mathcal{V}_1 < \mathcal{V}_2 < \cdots < \mathcal{V}_L$$

Note that $\mathcal{V}_1, \mathcal{V}_2, \cdots, \mathcal{V}_L$ are unknown variables to be determined in our problem formulation.

In our formulation, initially, we assume that each task is splittable into several subtasks (without execution-cycle overhead) so that each subtask can be driven by a different supply voltage with other subtasks. We will introduce a variable $x_{i\ell}$ for a task t_i, which denotes the ratio of execution cycles in subtasks driven by $\mathcal{V}_\ell$ or a lower voltage than $\mathcal{V}_\ell$ over all execution cycles of t_i. The following is trivial from the definition.

$$0 = x_{i0} \le x_{i1} \le x_{i2} \le \cdots \le x_{i(L-1)} \le x_{iL} = 1$$

In the following, $\boldsymbol{x}$ denotes the vector of variables $x_{i\ell}$, $i \in \{1, \cdots, n\}$, $\ell \in \{1, \cdots, L-1\}$, and $\boldsymbol{\mathcal{V}}$ denotes the vector of variables $\mathcal{V}_\ell$, $\ell \in \{1, \cdots, L\}$.

The overall energy consumption and the overall execution time for the given set of tasks are given as follows.

$$E_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) = \sum_{i=1}^{n} \sum_{\ell=1}^{L} \left(e_i^{(D)}(\mathcal{V}_\ell) + \frac{P^{(S)}(\mathcal{V}_\ell)}{f(\mathcal{V}_\ell)} \right) \cdot m_i \cdot \left(x_{i\ell} - x_{i(\ell-1)} \right)$$

$$T_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) = \sum_{i=1}^{n} \sum_{\ell=1}^{L} \frac{m_i}{f(\mathcal{V}_\ell)} \cdot \left(x_{i\ell} - x_{i(\ell-1)} \right).$$

Hence our multi-level DVFS problem (ML-DVFS) is formulated as follows.

Problem ML-DVFS:

$$\begin{aligned} &\text{Minimize} \ \ E_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}), \\ &\text{subject to} \ T_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) - T_{max} \le 0 \\ &\qquad x_{i(\ell-1)} - x_{i\ell} \le 0, \quad i \in \{1, \cdots, n\}, \\ &\qquad\qquad \ell \in \{1, \cdots, L\} \end{aligned}$$

where $x_{i0} = 0$ and $x_{iL} = 1$.

3. KKT Conditions and Discussions

In KKT conditions for a non-linear optimization problem, each constraint is associated with one Lagrangian multiplier. According to our problem formulation, the following Lagrangian multipliers are reserved.

$$\begin{aligned} T_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) - T_{max} \le 0 \quad &: \quad \lambda \ge 0 \\ x_{i(\ell-1)} - x_{i\ell} \le 0 \quad &: \quad \mu_{i\ell} \ge 0 \end{aligned}$$

At first, a general form (a vector form) of KKT conditions is shown below.

$$\begin{aligned} &\nabla_x(E_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}})) \\ &\quad + \lambda \cdot \nabla_x(T_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) - T_{max}) \\ &\quad + \sum_{i=1}^{n} \sum_{\ell=1}^{L} \mu_{i\ell} \cdot \nabla_x(x_{i(\ell-1)} - x_{i\ell}) \ = \ \mathbf{0} \qquad (1) \end{aligned}$$

$$\begin{aligned} &\nabla_{\mathcal{V}}(E_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}})) \\ &\quad + \lambda \cdot \nabla_{\mathcal{V}}(T_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) - T_{max}) \ = \ \mathbf{0} \qquad (2) \end{aligned}$$

$$(T_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) - T_{max}) \cdot \lambda \ = \ 0 \qquad (3)$$

$$\left(x_{i(\ell-1)} - x_{i\ell} \right) \cdot \mu_{i\ell} \ = \ 0, \ \ 1 \le i \le n, \ \ 1 \le \ell \le L \qquad (4)$$

where $\nabla_x(g)$ denotes the gradient vector of g with respect to the variable vector $\boldsymbol{x}$, and $\nabla_{\mathcal{V}}(g)$ does with respect to $\boldsymbol{\mathcal{V}}$.

3.1 $\nabla_{\boldsymbol{x}}$-relevant conditions

If we focus on $\partial/\partial x_{kp}$ in ∇_x, each scalar condition in (1) is given as;

$$\frac{\partial E_{total}}{\partial x_{kp}} + \lambda \cdot \frac{\partial (T_{total} - T_{max})}{\partial x_{kp}} + \sum_{i=1}^{n} \sum_{\ell=1}^{L} \mu_{i\ell} \cdot \frac{\partial (x_{i(\ell-1)} - x_{i\ell})}{\partial x_{kp}} = 0$$

which can be embodied as follows.

$$\left(e_k^{(D)}(\mathcal{V}_p) + \frac{P^{(S)}(\mathcal{V}_p)}{f(\mathcal{V}_p)} \right) \cdot m_k - \left(e_k^{(D)}(\mathcal{V}_{p+1}) + \frac{P^{(S)}(\mathcal{V}_{p+1})}{f(\mathcal{V}_{p+1})} \right) \cdot m_k + \lambda \cdot \left(\frac{m_k}{f(\mathcal{V}_p)} - \frac{m_k}{f(\mathcal{V}_{p+1})} \right) - \mu_{kp} + \mu_{k(p+1)} \ = \ 0$$

Since we are assuming $\mathcal{V}_p < \mathcal{V}_{p+1}$, and the clock frequency function $f(V)$ is assumed to be a monotonically increasing function of V, we have

$$\frac{1}{f(\mathcal{V}_p)} - \frac{1}{f(\mathcal{V}_{p+1})} > 0, \quad p \in \{1, \cdots, L-1\}.$$

Hence we can divide the above KKT condition by $(m_k/f(\mathcal{V}_p) - m_k/f(\mathcal{V}_{p+1}))$, and have the following.

$$\frac{\left(e_k^{(D)}(\mathcal{V}_p) + \frac{P^{(S)}(\mathcal{V}_p)}{f(\mathcal{V}_p)} \right) - \left(e_k^{(D)}(\mathcal{V}_{p+1}) + \frac{P^{(S)}(\mathcal{V}_{p+1})}{f(\mathcal{V}_{p+1})} \right)}{\left(\frac{1}{f(\mathcal{V}_p)} - \frac{1}{f(\mathcal{V}_{p+1})} \right)} + \lambda - \frac{\mu_{kp} - \mu_{k(p+1)}}{\left(\frac{m_k}{f(\mathcal{V}_p)} - \frac{m_k}{f(\mathcal{V}_{p+1})} \right)} \ = \ 0$$

Introducing a function EOT_k as;

$$EOT_k(\mathcal{V}_p, \mathcal{V}_{p+1}) \triangleq \frac{\left(e_k^{(D)}(\mathcal{V}_{p+1}) + \frac{P^{(S)}(\mathcal{V}_{p+1})}{f(\mathcal{V}_{p+1})} \right) - \left(e_k^{(D)}(\mathcal{V}_p) + \frac{P^{(S)}(\mathcal{V}_p)}{f(\mathcal{V}_p)} \right)}{\left(\frac{1}{f(\mathcal{V}_p)} - \frac{1}{f(\mathcal{V}_{p+1})} \right)},$$

the first KKT condition (1) can be written finally as follows.

$$EOT_k(\mathcal{V}_p, \mathcal{V}_{p+1}) + \left(\frac{\mu_{kp} - \mu_{k(p+1)}}{\frac{m_k}{f(\mathcal{V}_p)} - \frac{m_k}{f(\mathcal{V}_{p+1})}} \right) \ = \ \lambda, \quad k \in \{1, \cdots, n\}, \quad p \in \{1, \cdots, L-1\} \tag{5}$$

We will introduce several assumptions about EOT functions.

Assumption1 *When the inequalities*

$$EOT_k(\mathcal{V}_1, \mathcal{V}_2) < EOT_k(\mathcal{V}_2, \mathcal{V}_3) < \cdots < EOT_k(\mathcal{V}_{L-1}, \mathcal{V}_L)$$

hold, EOT function is said to be monotonic.

Assumption2 *From the task ID numbering rule, $e_k^{(D)}(V) > e_{k+1}^{(D)}(V)$. Under this task ID numbering rule, we assume*

$$EOT_k(\mathcal{V}_p, \mathcal{V}_{p+1}) > EOT_{k+1}(\mathcal{V}_p, \mathcal{V}_{p+1})$$

for all $1 \le k \le n-1$ and $1 \le p \le L-1$.

Lemma1 *Let $(\boldsymbol{x}, \boldsymbol{\mathcal{V}})$ be an optimum solution of ML-DVFS, and let λ and $\boldsymbol{\mu}$ be Lagrangian multipliers which satisfies KKT conditions.*

(1) If $EOT_k(\mathcal{V}_p, \mathcal{V}_{p+1}) < \lambda$ for some p, then $\mu_{kp} > \mu_{k(p+1)}$, which implies $\mu_{kp} > 0$ and hence $x_{k(p-1)} = x_{kp}$.

(2) If $EOT_k(\mathcal{V}_p, \mathcal{V}_{p+1}) > \lambda$ for some p, then $\mu_{kp} < \mu_{k(p+1)}$, which implies $\mu_{k(p+1)} > 0$ and hence $x_{k(p)} = x_{k(p+1)}$.

□

Theorem1 *Let $(\boldsymbol{x}, \boldsymbol{\mathcal{V}})$ be an optimum solution of ML-DVFS, and let λ and $\boldsymbol{\mu}$ be Lagrangian multipliers which satisfies KKT conditions. When EOT function is monotonic, and there exists an integer p^* such that*

$$EOT_k(\mathcal{V}_{p^*-1}, \mathcal{V}_{p^*}) < \lambda < EOT_k(\mathcal{V}_{p^*}, \mathcal{V}_{p^*+1})$$

then,

$$0 = x_{k0} = x_{k1} = x_{k2} = \cdots = x_{k(p^*-1)}$$
$$x_{kp^*} = x_{k(p^*+1)} = \cdots = x_{k(L-1)} = x_{kL} = 1$$

Considering the definition of variable $x_{i\ell}$, such solution indicates that a task t_k is driven by the sole supply voltage $\mathcal{V}_{p^*}$.

□

Theorem2 *Let $(\boldsymbol{x}, \boldsymbol{\mathcal{V}})$ be an optimum solution of ML-DVFS, and let λ and $\boldsymbol{\mu}$ be Lagrangian multipliers which satisfies KKT conditions. When EOT function is monotonic, and there exists an integer p^* such that*

$$EOT_k(\mathcal{V}_{p^*}, \mathcal{V}_{p^*+1}) = \lambda$$

then,

$$0 = x_{k0} = x_{k1} = x_{k2} = \cdots = x_{k(p^*-1)}$$
$$0 \le x_{kp^*} \le 1$$
$$x_{k(p^*+1)} = x_{k(p^*+2)} = \cdots = x_{k(L-1)} = x_{kL} = 1$$

Considering the definition of variable $x_{i\ell}$, such solution indicates that a task t_k is split into two part at a ratio of $x_{kp^*} - x_{k(p^*-1)} = x_{kp^*}$ to $x_{k(p^*+1)} - x_{kp^*} = 1 - x_{kp^*}$, and the former subtask is driven by the supply voltage $\mathcal{V}_{p^*}$ and the latter by another supply voltage $\mathcal{V}_{p^*+1}$.

□

Lemma2 *Under the task ID numbering rule, when*

$$EOT_k(\mathcal{V}_{p^*-1}, \mathcal{V}_{p^*}) < \lambda \le EOT_k(\mathcal{V}_{p^*}, \mathcal{V}_{p^*+1})$$

holds for a task t_k*, then, for a task* t_{k+1}*,*

$$EOT_{k+1}(\mathcal{V}_{p-1}, \mathcal{V}_p) < \lambda \leq EOT_{k+1}(\mathcal{V}_p, \mathcal{V}_{p+1})$$

holds with $p \geq p^*$*.*

□

Corollary1 *In an optimum solution of ML-DVFS, task* t_k*,* $1 \leq k \leq n-1$*, is always driven by a supply voltage no larger than the supply voltage which drives task* t_{k+1}*.*

□

3.2 $\nabla_{\mathcal{V}}$-relevant conditions

With respect to the second KKT condition (2), each individual condition associated with $\partial/\partial\mathcal{V}_q$, $q \in \{1, \cdots, L\}$, in $\nabla_{\mathcal{V}}$, is written as;

$$\frac{\partial E_{total}}{\partial \mathcal{V}_q} + \lambda \cdot \frac{\partial (T_{total} - T_{max})}{\partial \mathcal{V}_q} = 0$$

which can be embodied as follows.

$$\sum_{i=1}^{n} \left(\frac{\partial \left(e_i^{(D)}(\mathcal{V}_q) + \frac{P^{(S)}(\mathcal{V}_q)}{f(\mathcal{V}_q)} \right)}{\partial \mathcal{V}_q} \cdot m_i \cdot (x_{iq} - x_{i(q-1)}) \right) + \lambda \cdot \sum_{i=1}^{n} \left(\frac{\partial \left(\frac{1}{f(\mathcal{V}_q)} \right)}{\partial \mathcal{V}_q} \cdot m_i \cdot (x_{iq} - x_{i(q-1)}) \right) = 0$$

It can be further rewritten as;

$$EVT(\mathcal{V}_q, \boldsymbol{x}_q, \boldsymbol{x}_{q-1}) = \lambda \qquad (6)$$

where $EVT(\mathcal{V}_q, \boldsymbol{x}_q, \boldsymbol{x}_{(q-1)})$ (energy-vs-time efficiency, or EVT-efficiency) is defined as follows;

$$EVT(\mathcal{V}_q, \boldsymbol{x}_q, \boldsymbol{x}_{q-1}) = -\frac{\sum_{i=1}^{n} \left(\frac{\partial \left(e_i^{(D)}(\mathcal{V}_q) + \frac{P^{(S)}(\mathcal{V}_q)}{f(\mathcal{V}_q)} \right)}{\partial \mathcal{V}_q} \cdot m_i \cdot (x_{iq} - x_{i(q-1)}) \right)}{\sum_{i=1}^{n} \left(\frac{\partial \left(\frac{1}{f(\mathcal{V}_q)} \right)}{\partial \mathcal{V}_q} \cdot m_i \cdot (x_{iq} - x_{i(q-1)}) \right)}$$

Here we will make the following assumption about EVT functions.

Assumption3 $EVT(\mathcal{V}, \boldsymbol{x}_q, \boldsymbol{x}_{(q-1)})$ *is assumed to be monotonically increasing with respect to* $\mathcal{V}$*.*

4. Solutions of ML-DVFS

4.1 Optimum Voltage Assignment for Fixed Supply Voltages

Before showing a solution algorithm for our ML-DVFS problem, we briefly discuss a solution for DVFS with fixed supply voltages. Discussions done in section III.A (∇_x-relevant conditions) are important bases for our solution algorithm.

Since $\boldsymbol{\mathcal{V}}$ is given and fixed, KKT conditions (1), (3) and (4) with fixed $\boldsymbol{\mathcal{V}}$ are our concern, and our algorithm is based on Theorems 1 and 2. The outline of the algorithm is as follows.

Algorithm for Fixed-DVFS:

Preliminary step: We will confirm that $T_{total} \leq T_{max}$ is achieved by assigning the highest supply voltage $\mathcal{V}_L$ to all tasks. If it is not achieved, we will quit the routine without any solution.

We will also confirm that $T_{total} > T_{max}$ is achieved by assigning the lowest supply voltage $\mathcal{V}_1$ to all tasks. If it is not, we will quit the routing with such a solution as $V_i = \mathcal{V}_1$ for every task t_i.

Step 1: We will compute $EOT_k(\mathcal{V}_\ell, \mathcal{V}_{\ell+1})$ for all k, $1 \leq k \leq n$, and for all ℓ, $1 \leq \ell \leq L-1$, and sort them in ascending order. Let $LIST$ be the sorted list of EOTs, and $LIST[i]$ denotes the ith element in $LIST$. Note that $LIST[1]$ must be $EOT_n(\mathcal{V}_1, \mathcal{V}_2)$, and $LIST[n \times (L-1)]$ must be $EOT_1(\mathcal{V}_{L-1}, \mathcal{V}_L)$.

Step 2: Let λ_t and J be a trial (value) of Lagrangian multiplier λ and an integer for controlling loop iteration, respectively. Initially we will set $J = n \times (L-1)$ and $\lambda_t = LIST[n \times (L-1)]$.

Step 3: Assuming $\lambda = \lambda_t$ and using Theorem 1 and 2, we will find $\boldsymbol{x}$, and check T_{total} achieved by this tentative solution. For $LIST[J] = EOT_k(\mathcal{V}_p, \mathcal{V}_{p+1})$, x_{kp} is given as large as possible while keeping $T_{total} \leq T_{max}$.

Step 4: If $T_{total} = T_{max}$ is achieved, quit with the current tentative solution as the final solution. Otherwise, i.e, $x_{kp} = 1$ is achieved with $T_{total} < T_{max}$, then update $J \leftarrow J - 1$ and $\lambda_t = LIST[J]$, and go to Step 3.

□

4.2 Solution for ML-DVFS

In this section, we will consider ML-DVFS solutions satisfying Theorem 1 for all tasks. From Theorem 1 and Corollary 1, under our task ID numbering rule, tasks which are driven by a same supply voltage have consecutive ID numbers. As for simplicity of notation, let a pair of integers with square brackets $[a, b]$ denote a set of tasks having consecutive ID numbers starting at a and ending at b, i.e., $\{t_a, t_{a+1}, \cdots, t_b\}$.

Our algorithm for solving ML-DVFS is based on the enumeration of partitions of tasks into L consecutive tasks and λ-centric voltage computations. Now we let $[s_1, e_1]$, $[s_2, e_2]$, $\cdots$, $[s_L, e_L]$ be an instance of L consecutive task partition, where $s_1 = 1$, $s_i = e_{i-1} + 1$ for $2 \le i \le L$ and $e_L = n$. Note that there are ${}_{n-1}C_{L-1}$ different partitions, which is exponential order of L (the number of voltage levels). However, L is not so large number in practice, and enumerating partitions might be tractable.

KKT conditions for ML-DVFS request that (6) is fulfilled for every part of partition with the same λ. Here we will recall (6), but it is arranged by the syntax of task partitioning.

$$EVT(\mathcal{V}_q, s_q, e_q) = \lambda, \quad q \in \{1, 2, \cdots, L\} \tag{7}$$

where

$$EVT(\mathcal{V}_q, s_q, e_q) = -\frac{\displaystyle\sum_{i=s_q}^{e_q} \left(\frac{\partial \left(e_i^{(D)}(\mathcal{V}_q) + \frac{P^{(S)}(\mathcal{V}_q)}{f(\mathcal{V}_q)} \right)}{\partial \mathcal{V}_q} \cdot m_i \right)}{\displaystyle\sum_{i=s_q}^{e_q} \left(\frac{\partial \left(\frac{1}{f(\mathcal{V}_q)} \right)}{\partial \mathcal{V}_q} \cdot m_i \right)}$$

When EVT function is monotonic with respect to $\mathcal{V}_q$, we can solve (7) under the constraint

$$T_{total}(\boldsymbol{x}, \boldsymbol{\mathcal{V}}) - T_{max} = 0.$$

One possible algorithm is to use bisection search of λ [6].

After finding optimum supply voltages for each of possible task partitions, one best solution among them is selected.

5. Conclusions

In this paper, we have discussed the optimum design of DVFS with limited voltage levels (ML-DVFS). Based on the KKT conditions for the optimum solution of a nonlinear optimization problem, several properties of the optimum solution of ML-DVFS problem have been revealed. Through discussions, we found that one Lagrangian multiplier (it is named λ in this paper) appears in all constraints originated from ∇_x and from $\nabla_{\mathcal{V}}$, and plays an important role for characterizing optimum solutions and for solving the optimization problem. However, the author feels that the meaning of λ in DVFS problem is not sufficiently comprehended yet. Proposed ML-DVFS algorithm is naive enough to rely on the enumeration of task partitioning. Deeper understanding about λ is expected to help in improving the algorithm.

参考文献

[1] M. Horowitz, T. Indernaur, R. Gonzalez, “Low-Power Digital Design”, Proc. IEEE Symp. Low-Power Electron., pp.8-11, 1994.

[2] Tohru Ishihara, Hiroto Yasuura, “Voltage Scheduling Problem for Dynamically Variable Voltage Processors”, Proc. International Symposium on Low Power Electronics and Design, pp.197-202, 1998.

[3] Dongkun Shin, Jihong Lim, Seongsoo Lee, “Low-Energy Intra-Task Voltage Scheduling Using Static Timing Analysis”, Proc. Design Automation Conference, pp. 438-443, 2001.

[4] Kihwan Choi, Ramakrishna Soma, Massoud Pedram, “Fine-Grained Dynamic Voltage and Frequency Scaling for Precise Energy and Performance Tradeoff Based on the Ratio of Off-Chip Access to On-Chip Computation Times”, IEEE Trans. CAD ofIntegrated Circuits and Systems, Vol. 24, No.1, pp.18-28, 2005.

[5] Sangyoung Park, Jaehyun Park, Donghwa Shin, Yanzhi Wang, Qing Xie, Massoud Pedram, and Naehyuck Chang, “Accurate Modeling of the Delay and Energy Overhead of Dynamic Voltage and Frequency Scanling in Modern Microprocessors”, IEEE Trans. CAD of Integrated Circuits and Systems, Vol.32, No.5, pp.695-708, 2013.

[6] Mineo Kaneko, “A Study on DVFS for Heterogeneous Task Set”, IEICE Technical report on VLD, VLD2015-47, pp.63-68, 2015.