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Enhancing PET images by means of Bates' blind deconvolution

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1. Introduction

The PET is one of the medical treatment inspection methods by the detection of two coincident gamma-rays from radioactive sources. However, it is desirable that the resolution of the PET images be improved for the purpose. Naturally effort must be spent for the improvement of the devices. The purpose of this work is to examine whether or not the improvement of the resolution of the PET images can be achieved by image-restoration. For the purpose, we explored Bates' blind deconvolution [1] that is regarded as an extended method of the one-dimensional signal conditioning with zero-values of signals. In this note, we present some new technical tools that are useful for the Bates' blind deconvolution. As an example, we present a PET image that we successfully enhanced by means of the Bates' blind deconvolution.

2. Technical tools for the Bates' Zero-sheet method

2.1 Classification method of zero-values by a differential equation

An essential problem in the Bates' blind deconvolution is how we classify zero-sheets of given images. In actual images such as PET images, the image sizes are large. Therefore, we need special techniques for classifying the zero-sheets of the images. We developed a zero-sheet classification method that uses a differential equation for the zero-values of given images.

The z-transform of a given image $g(x,y)$ of the size $M'\times N'$ is represented as

$$G(u,v)=\frac{1}{M'N'}\sum_{x=0}^{M'-1}\sum_{y=0}^{N'-1} g(x,y)\,u^x v^y. \qquad (1)$$

The differential equation for zero-values $\beta_k(\rho_u,\phi_u)$ $(k=1,2,\cdots,N'';N''\le N'-1)$ of complex variable v is written as

$$\left.\frac{d\beta_k(\rho_u=\rho_0,\,\phi_u)}{d\phi_u}\right|_{\rho_u=\rho_0} = -\left.\frac{\left(\dfrac{\partial G}{\partial \phi_u}\right)}{\left(\dfrac{\partial G}{\partial \beta_k}\right)}\right|_{\rho_u=\rho_0}, \qquad (2)$$

where

$$\frac{\partial G}{\partial \phi_u}=\frac{1}{M'N'}\sum_{x=0}^{M'-1}\sum_{y=0}^{N'-1} g(x,y)ix\left(\rho_u e^{i\phi_u}\right)^x \beta_k{}^y \qquad (3)$$

and

$$\frac{\partial G}{\partial \beta_k}=\frac{1}{M'N'}\sum_{x=0}^{M'-1}\sum_{y=0}^{N'-1} g(x,y)\left(\rho_u e^{i\phi_u}\right)^x y\beta_k{}^{y-1}. \qquad (4)$$

The differential equation is actually simultaneous equations of the real and imaginary parts of β_k's. The differential equation for zero-values $\gamma_k(\rho_v,\phi_v)$ $(k=1,2,\cdots,M'';M''\le M'-1)$ of complex variable u is obtained by $\beta_k\to\gamma_k$, $\rho_u\to\rho_v$, $\phi_u\to\phi_v$ and $x\leftrightarrow y$ in Eqs. (2)-(4), but $g(x,y)$ must be left as it is. In our method, we beforehand obtain, in a desired accuracy, zero values for some discrete points of ϕ_u using *Mathematica*. Then we connect the zero values obtained at the discrete points of ϕ_u by means of the differential equation (2). This method enables us to classify, in a practical processing time, the zero values for an actual image such as a PET image.

2.2 Conditional expression for judging a blur element of a small size

We found a discriminant expression by which one can detect a blur element without classifying the zero-values. The conditional expression is for blur-elements of the size 2×2. It is expressed as

$$\left(\frac{d\beta_k(\rho_u,\,\phi_u)}{d\rho_u}\right)\left(\frac{d^3\beta_k(\rho_u,\,\phi_u)}{d\rho_u{}^3}\right)=\frac{3}{2}\left(\frac{d^2\beta_k(\rho_u,\,\phi_u)}{d\rho_u{}^2}\right)^2. \qquad (5)$$

The conditional expression (5) is a relation among the first, second and third derivatives of β_k's with respect to ρ_u. They are all given analytically in terms of the derivatives of $G(\rho_u,\phi_u,\beta_k)$ with respect to ρ_u and β_k. For details see ref. [2]. The expression is very elegant in the sense that everything needed in evaluating the expression is given analytically. In order to judge whether or not a zero-value β_k is of a blur-element of the size 2×2, we only substitute the β_k evaluated at a point (ρ_u,ϕ_u) into the conditional expression (5). We can evaluate the expression (5) for any ρ_u and ϕ_u. The conditional expression for γ_k's is obtained by $\beta_k\to\gamma_k$, $\rho_u\to\rho_v$, $\phi_u\to\phi_v$ in the conditional expression (5).

3. Enhancement of a PET image

Figure 1 shows a PET image of human brain that we downloaded from the web site http://www.cc.nih.gov/pet/images.html [3]. The extracted image shown

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in Fig. 1 seems to be blurred to some extent. Our aim is to clarify whether or not any blur component is convolved in the extracted PET image. The size of the extracted image is 85×85, which is rather small.

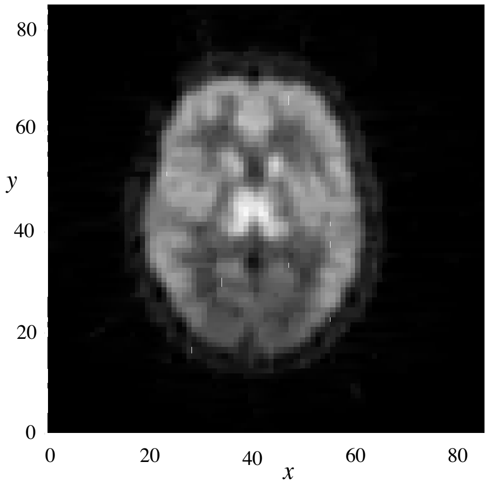

Fig. 1 A PET image of human brain that was downloaded from an image list on the web site http://www.cc.nih.gov/pet/images.html [3]. The size of the image is 85×85.

We analyzed the zero-sheets of variables u and v, using the two mathematical tools presented in the previous section. First, we searched optimal values of ρ_u and ρ_v. Eventually we found $\rho_u = \rho_v = 0.8$ as an optimal value. Although we applied the conditional expression (5) to the zero-values calculated with $\rho_u = \rho_v = 0.8$, we could not find any blur-element of the size 2×2.

Figure 2a is the image that we restored by removing the zero-sheet of a blur from the whole zero-sheet of the observed image of Fig. 1. In the restored image the light and shade of the image looks enhanced compared to those of the observed image. In Fig. 2b, the brightness of the 44th row is compared in the enhanced image and in the original image. In some parts the dynamic range seems to be enlarged in the enhanced image. We believe that a blur was to some extent removed in the restored image.

To the end of this note, we should mention the blur image that we removed from the observed image. We could find a blur-element in u-space. The blur-element consists of two zero-values. This means that the size of the blur-element in x-space is three. Although we do not have a space in this note to present the zero-value of the blur, it has apparently ϕ_v dependence. This implies that the size of the blur-element in y-space is two or larger than two. If the size of the blur-element in y-space is 1, then the zero-values of u do not have any ϕ_v dependence. Hence, we can guess that the blur-element is of $3 \times n$ $(n \geq 2)$. However, we did not find any blur-element in v-space. This is a puzzle. At present, we do not have any clear answer to this. We are still analyzing this problem.

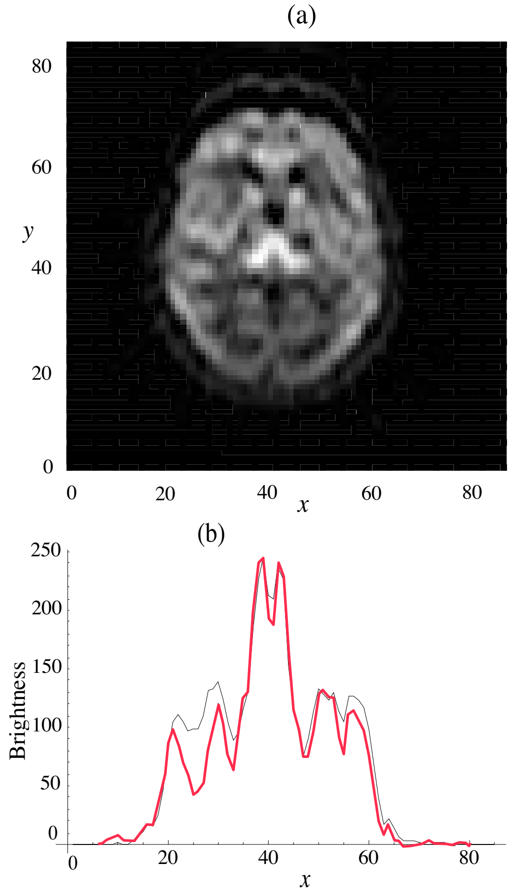

Fig. 2 (a) The image restored from the observed image of Fig. 1. (b) The red and black lines respectively indicate the brightness of the restored image and extracted image.

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[3] A PET image downloaded from F-18 cyclofoxy (http://www.cc.nih.gov/pet/images.html), in private communication with B. Bogley at PET Department, NIH Clinical Center.