Balanced (C_4, C_{14})-$2t$-Foil Decomposition Algorithm of Complete Graphs

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1. Introduction

Let K_n denote *the complete graph* of n vertices. Let C_4, C_{14} be *the 4-cycle* and *the 14-cycle*, respectively. *The* (C_4, C_{14})*-2t-foil* is a graph of t edge-disjoint C_4's and t edge-disjoint C_{14}'s with a common vertex and the common vertex is called *the center of the* (C_4, C_{14})*-2t-foil*. When K_n is decomposed into edge-disjoint sum of (C_4, C_{14})-$2t$-foils, it is called that K_n *has a* (C_4, C_{14})*-2t-foil decomposition*. Moreover, when every vertex of K_n appears in the same number of (C_4, C_{14})-$2t$-foils, it is called that K_n *has a balanced* (C_4, C_{14})*-2t-foil decomposition* and this number is called *the replication number*.

2. Balanced (C_4, C_{14})-$2t$-foil decomposition of K_n

Theorem. K_n has a balanced (C_4, C_{14})-$2t$-foil decomposition if and only if $n \equiv 1 \pmod{36t}$.

Proof. **(Necessity)** Suppose that K_n has a balanced (C_4, C_{14})-$2t$-foil decomposition. Let b be the number of (C_4, C_{14})-$2t$-foils and r be the replication number. Then $b = n(n-1)/36t$ and $r = (16t+1)(n-1)/36t$. Among r (C_4, C_{14})-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_4, C_{14})-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/36t$ and $r_2 = 16(n-1)/36$. Therefore, $n \equiv 1 \pmod{36t}$ is necessary.

(Sufficiency) Put $n = 36st + 1$ and $T = st$. Then $n = 36T + 1$.

Case 1. n = 37. Construct a balanced (C_4, C_{14})-2-foil decomposition of K_{37}:

$B_i = \{(i, i+31, i+2, i+32), (i, i+1, i+12, i+15, i+25, i+8, i+22, i+4, i+19, i+10, i+6, i+30, i+18, i+16)\}$
$(i = 1, 2, ..., 37)$.

Case 2. n $\geq$ 73. Construct n (C_4, C_{14})-$2T$-foils as follows:

$B_i = \{(i, i+30T+1, i+T+1, i+31T+1), (i, i+1, i+10T+2, i+13T+2, i+22T+3, i+6T+2, i+19T+3, i+34T+3, i+16T+3, i+8T+2, i+4T+2, i+28T+2, i+16T+2, i+15T+1)\}$
$\cup \{(i, i+30T+2, i+T+3, i+31T+2), (i, i+2, i+10T+4, i+13T+3, i+22T+5, i+6T+3, i+19T+5, i+34T+4, i+16T+5, i+8T+3, i+4T+4, i+28T+3, i+16T+4, i+15T+2)\}$
$\cup \{(i, i+30T+3, i+T+5, i+31T+3), (i, i+3, i+10T+6, i+13T+4, i+22T+7, i+6T+4, i+19T+7, i+34T+5, i+16T+7, i+8T+4, i+4T+6, i+28T+4, i+16T+6, i+15T+3)\}$
$\cup \ldots$
$\cup \{(i, i+31T, i+3T-1, i+32T), (i, i+T, i+12T, i+14T+1, i+24T+1, i+7T+1, i+21T+1, i+35T+2, i+18T+1, i+9T+1, i+6T, i+29T+1, i+18T, i+16T)\}$
$(i = 1, 2, ..., n)$.

Decompose each (C_4, C_{14})-$2T$-foil into s (C_4, C_{14})-$2t$-foils. Then they comprise a balanced (C_4, C_{14})-$2t$-foil decomposition of K_n. ■

Example 1. A balanced (C_4, C_{14})-2-foil decomposition of K_{37}.

$B_i = \{(i, i+31, i+2, i+32), (i, i+1, i+12, i+15, i+25, i+8, i+22, i+4, i+19, i+10, i+6, i+30, i+18, i+16)\}$
$(i = 1, 2, ..., 37)$.

Example 2. A balanced (C_4, C_{14})-4-foil decomposition of K_{73}.

$B_i = \{(i, i+61, i+3, i+63), (i, i+1, i+22, i+28, i+47, i+14, i+41, i+71, i+35, i+18, i+10, i+58, i+34, i+31)\}$

$\cup \{(i, i+62, i+5, i+64), (i, i+2, i+24, i+29, i+49, i+15, i+43, i+72, i+37, i+19, i+12, i+59, i+36, i+32)\}$
$(i = 1, 2, ..., 73)$.

Example 3. A balanced (C_4, C_{14})-6-foil decomposition of K_{109}.
$B_i = \{(i, i+91, i+4, i+94), (i, i+1, i+32, i+41, i+69, i+20, i+60, i+105, i+51, i+26, i+14, i+86, i+50, i+46)\}$
$\cup \{(i, i+92, i+6, i+95), (i, i+2, i+34, i+42, i+71, i+21, i+62, i+106, i+53, i+27, i+16, i+87, i+52, i+47)\}$
$\cup \{(i, i+93, i+8, i+96), (i, i+3, i+36, i+43, i+73, i+22, i+64, i+107, i+55, i+28, i+18, i+88, i+54, i+48)\}$
$(i = 1, 2, ..., 109)$.

Example 4. A balanced (C_4, C_{14})-8-foil decomposition of K_{145}.
$B_i = \{(i, i+121, i+5, i+125), (i, i+1, i+42, i+54, i+91, i+26, i+79, i+139, i+67, i+34, i+18, i+114, i+66, i+61)\}$
$\cup \{(i, i+122, i+7, i+126), (i, i+2, i+44, i+55, i+93, i+27, i+81, i+140, i+69, i+35, i+20, i+115, i+68, i+62)\}$
$\cup \{(i, i+123, i+9, i+127), (i, i+3, i+46, i+56, i+95, i+28, i+83, i+141, i+71, i+36, i+22, i+116, i+70, i+63)\}$
$\cup \{(i, i+124, i+11, i+128), (i, i+4, i+48, i+57, i+97, i+29, i+85, i+142, i+73, i+37, i+24, i+117, i+72, i+64)\}$
$(i = 1, 2, ..., 145)$.

Example 5. A balanced (C_4, C_{14})-10-foil decomposition of K_{181}.
$B_i = \{(i, i+151, i+6, i+156), (i, i+1, i+52, i+67, i+113, i+32, i+98, i+173, i+83, i+42, i+22, i+142, i+82, i+76)\}$
$\cup \{(i, i+152, i+8, i+157), (i, i+2, i+54, i+68, i+115, i+33, i+100, i+174, i+85, i+43, i+24, i+143, i+84, i+77)\}$
$\cup \{(i, i+153, i+10, i+158), (i, i+3, i+56, i+69, i+117, i+34, i+102, i+175, i+87, i+44, i+26, i+144, i+86, i+78)\}$
$\cup \{(i, i+154, i+12, i+159), (i, i+4, i+58, i+70, i+119, i+35, i+104, i+176, i+89, i+45, i+28, i+145, i+88, i+79)\}$
$\cup \{(i, i+155, i+14, i+160), (i, i+5, i+60, i+71, i+121, i+36, i+106, i+177, i+91, i+46, i+30, i+146, i+90, i+80)\}$
$(i = 1, 2, ..., 181)$.

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