# PARAdeg Computation and Scheduling of Self-Cleaning SWITCH-less Program Nets

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#### 1. Introduction

A program net (denoted as PN) is a graph representation of a data-flow program and its parallelism has been studied through computing PARAdeg = $\sum_{i} (\overline{f}(z_i)\tau_i)/\underline{T}$ , where  $\overline{f}(z_i)$  and  $\tau_i$  are maximum firing number and single firing time of node  $z_i$  respectively and  $\underline{T}$  is minimum firing time of PN [1]. For a SWITCH-less PN, the numerator of PARAdeg can be efficiently computed, however the denominator is usually costed with exponential computation time [1]. To improve this, we proposed an improved algorithm by a way of contracting nodes for self-cleaning SWITCH-less PNs with identical node firing time [2]. However the problem for the case of arbitrary node firing time has not been studied yet. And also it is important to schedule execution of PNs by using finite processors. In this paper, we first propose conditions that node contraction can be carried out for self-cleaning SWITCH-less PNs with arbitrary node firing time. Then we give scheduling methods by using PARAdeg processors to execute SWITCH-less PNs.

## 2. Preliminary

PN is constituted of AND-node ( $\bigcirc$ ), OR-node ( $\triangle$ ) and SWITCH-node ( $\nabla^{\circ}$ ) (with node firing time  $\tau_i$ ) representing operations, and directed edges representing transmission channel of tokens [1]. Node firing rules and the detaileds refer to reference [1].

Minimum firing time  $\underline{T}$  of a PN is the minimum time while infinite processors are allowed to fire each node  $z_i$   $\overline{f}(z_i)$  times. A SWITCH-less PN is of no SWITCH-node. A PN with 0 initial token distribution is self-cleaning if there is no token remaining on the edges after the firing of PN terminated [3]. In this paper PNs are assumed to be self-cleaning and SWITCH-less, and AND-nodes are allowed to possess multiple input and output edges.

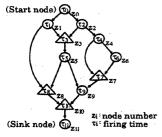
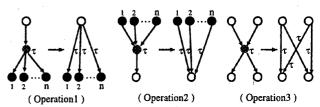


Fig. 1 : An example of self-cleaning SWITCH-less PN .

The improved algorithm is to compute  $\underline{T}$  for self-cleaning SWITCH-less PNs (as shown in Fig.1) with identical node firing time by applying AND-node contraction rules as shown in Fig.2 [2].



τ: firing time ♦: contraction node ●: AND or OR-node ○: AND-node

Fig.2: AND-node contraction rules.

#### 3. Conditions of Node Contraction

Suppose z be an AND-node with single input edge and firing time  $\tau$ . If two tokens enter the input edge in succession within time interval  $\tau$ , then z can not be contracted and our algorithm [2] fails to be used. The following proposition gives the condition for general AND-node contraction.

[Proposition 1] Let z be any AND-node with multiple input edges and  $\tau$  be its firing time. z can be contracted if the following condition holds:

 $\max(t_n^1, \dots, t_n^m) - \max(t_{n-1}^1, \dots, t_{n-1}^m) \ge \tau$ . Here,  $t_k^i$  shows the time when k-th token appears on the i-th input edge.

Further, following Proposition 2-4 can be derived from Proposition 1.

[Proposition 2] All the AND-nodes of a PN are possible to be contracted if the node firing times of all the nodes are identity  $(\tau_1 = \tau_2 = ... = \tau)$ . [Proposition 3] Let z be an AND-node with firing time  $\tau$  and  $z_i$  (i = 1,2,...,n) be an input node of z with firing time  $\tau_i$ . z can be contracted if the following condition holds:

 $\min(\tau_1, \tau_2, \cdots, \tau_n) \ge \tau$ .  $\square$  [Proposition 4] Let  $P_1, P_2, ..., P_n$  be n paths from start node to an AND-node z and  $\tau_j^i$  be the firing time of j-th node of  $P_i$  as shown in Fig.3. z can be contracted if the following conditions hold:

$$P_1: \max(\tau_1^{1}, \tau_2^{1}, \dots, \tau_m^{1}) \ge \tau, P_2: \max(\tau_1^{2}, \tau_2^{2}, \dots, \tau_n^{2}) \ge \tau,$$

$$P_n: \max(\tau_1^n, \tau_2^n, \dots, \tau_k^n) \ge \tau.$$

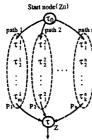


Fig.3: An example of paths from start node to an AND-node z.

In the study [2], we treated only the PNs with identical node firing time. Hence we can extend our algorithm [2] to suiting for the PNs with arbitrary node firing time by applying Proposition 4. The way to do this is to limit node contraction only to those AND-nodes satisfying Proposition 4.

## 4. Scheduling Algorithm

Scheduling problem is one of NP-complete problems even we limit nets to AND-node only ones [4]. Therefore here we propose three heuristic scheduling methods to execute SWITCH-less PNs by using PARAdeg processors and then evaluate the performance of the methods.

The following algorithm is to label each node before scheduling.

## ( Labeling Node )

Step1: Regard  $\overline{f}(z_i)$  and  $\tau_i$  as node weights of  $z_i$  respectively to calculate longest distances  $D_f(z_i)$  and  $D_{\tau}(z_i)$  from  $z_i$  to the sink node t by using BFS (Breadth First Search).

Step 2: Label each node  $z_i$  with  $D(z_i) = D_f(z_i) \times D_\tau(z_i)$  and sort nodes in a list L in accordance with descending order of  $D(z_i)$ .

Step3: Stop.

 $D(z_i)$  indicates the maximum firing time from node  $z_i$  to the sink node.

Now we give our scheduling algorithms for PNs, which are respectively based on the following three priorities among firable nodes:

Method 1: Use L as priority list.

Method 2: Take priority of OR-node.

Method 3: Take priority of the larger  $\tau_i$ .

Here in case of nodes with same priority when using Methods 2 and 3, Method 1 is to be applied. The following algorithm works for all of Methods 1, 2 and 3 using *PARAdeg* processors.

#### $\langle$ Scheduling of $PN \rangle$

Step 1: Carry out  $\langle$  Labeling Node  $\rangle$  and set t = 0.

Step2: If there are unoccupied processors, then choose firing nodes from firable ones according to  $Method\ i\ (i=1\ {\rm or}\ 2\ {\rm or}\ 3)$  and execute their firings.

Step3: Update t = t+1 and release processors that have finished executing firing at current time

Step4: If there is no occupied processor and no firable node, stop. Otherwise goto Step2.

We have applied  $\langle$  Scheduling of  $PN \rangle$  for executing 10 arbitrary PNs. Table 1 shows the results by Methods 1, 2 and 3 in  $\langle$  Scheduling of  $PN \rangle$  as well as the result by using sufficient |V| processors. Each data indicates processors' ratio of working measured as (total working time of processors)/ $(PARAdeg \times (executing time of PN))$ . From Table 1, all three methods have certainly better performance than using |V| processors. Among the three methods, Method 2 is best in average. That is because firing of an ORnode may probably create more firable nodes than firing other nodes.

Table 1: Experimental results.

|         | method 1 | method 2 | method 3 | V  processors |
|---------|----------|----------|----------|---------------|
| PN-1    | 59.5 %   | 67.4 %   | 67.4 %   | 16.9 %        |
| PN-2    | 68.9 %   | 68.9 %   | 70.4 %   | 21.1 %        |
| PN-3    | 75.4 %   | 77.2 %   | 77.2 %   | 15.1 %        |
| PN-4    | 65.9 %   | 65.9 %   | 67.5 %   | 18.6 %        |
| PN-5    | 72.2 %   | 78.8 %   | 72.2 %   | 21.7 %        |
| PN-6    | 75.9 %   | 75.9 %   | 73.3 %   | 8.4 %         |
| PN-7    | 81.8 %   | 81.8 %   | 81.8 %   | 9.1 %         |
| PN-8    | 70.9 %   | 72.6 %   | 67.8 %   | 20.6 %        |
| PN-9    | 81.2 %   | 77.8 %   | 84.8 %   | 18.2 %        |
| PN-10   | 77.8 %   | 81.7 %   | 77.8 %   | 18.7 %        |
| Average | 73.0 %   | 74.8 %   | 74.0 %   | 16.8 %        |

## 5. Concluding remarks

We have firstly proposed conditions, that node contraction can be carried out for self-cleaning SWITCH-less PNs with arbitrary node firing time, in order to keep using of our previous algorithm[2] and then given scheduling methods to execute SWITCH-less PNs by PARAdeg processors. The performance evaluation for scheduling has shown that Method 2 is best among the three. It still remains to be solved to evaluate performance of (1) the previous algorithm[2] for PNs with arbitrary node firing time and (2) our scheduling methods for much more PNs.

#### Reference

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