3Q−5

確率微分方程式の数値解法の誤差解析[1]

齊藤 善弘[2], 三井 斌友[3]
聖徳学園女子短期大学[4], 名古屋大学[5]

1. Introduction

We consider stochastic initial value problem (SIVP) for scalar autonomous Ito stochastic differential equation (SDE) given by

$$\begin{cases} dX(t) = f(X)dt + g(X)dW(t), & t \in [0,T], \\ X(0) = x, \end{cases} \tag{1}$$

where $W(t)$ represents the standard Wiener process and initial value x is a fixed value. In [1,4-11] and other papers numerical schemes for SDE (1) were proposed, which recursively compute sample paths (trajectories) of solution $X(t)$ at step-points. Numerical experiments for these schemes can be seen in some papers([5-8]). We proposed error analysis separating error term into two parts that is, stochastic and deterministic parts ([10]). However we investigated only in deterministic part. We will show results of error behaviour of stochastic part.

2. Stochastic and deterministic parts in the global error

We will describe our error analysis proposed in [10]. For obtaining the mean-square error of the approximation, it is important that the exact value of the realizations of the solution $X(T)$ can be determined. As mentioned in Introducton, the error analysis has not been carried out successfully. In our view, it could be solved by separating mean-square error into two parts (stochastic and deterministic), namely

$$\|X(T) - \bar{X}_N\| \le \|X(T) - \hat{X}_N\| + \|\hat{X}_N - \bar{X}_N\|, \tag{2}$$

where $\|\cdot\|$ denotes

$$\|X\| = \{\mathrm{E}(|X|^2)\}^{\frac{1}{2}},$$

and $\hat{X}_N$ the discretized exact solution realized by using pseudo-random numbers generated in digital computer. By this reason, we will call $\hat{X}_N$ the realized exact solution, and the first term in the right-hand side as stochastic part, the second term as deterministic part. We anticipate that the effect of order of numerical schemes appears in deterministic part. A similar notion in weak sense is seen in [11]. Stochastic part will have to be investigated by some stochastic tests.

For example, if the solution $X(t)$ of SDE (1) can be expressed

$$X(t) = F(t; W(t), \int_0^t W(s)ds, \int_0^t s dW(s), \cdots), \tag{3}$$

[1] Error analysis of numerical schemes for stochastic differential equations
[2] Yoshihiro SAITO
[3] Taketomo MITSUI
[4] Shotoku Gakuen Women's Junior College
[5] Nagoya University

for $F \in C^\infty$, then the realized exact solution is

$$\hat{X}_n = F(t; \sum_{i=0}^{n-1} \Delta W_i, \sum_{i=0}^{n-1} \Delta Z_i, \sum_{i=0}^{n-1} \Delta \bar{Z}_i, \cdots). \tag{4}$$

Therefore we can easily calculate the deterministic part, whereas stochastic part will be calculated with the distances between $\sum_{i=0}^{n-1} \Delta W_i$ and $W(t)$, $\sum_{i=0}^{n-1} \Delta Z_i$ and $\int_0^t W(s)ds$, and so on at each step points ($t = ih$; $i = 0, \cdots, n-1$) in distribution sense.

Note that our error analysis is slightly different from one carried out by Liske and Platen [5, 6] or Newton [7]. They estimated directly the error $\|X(T) - \bar{X}_N\|^2$. Here our realized exact solution corresponds to the truncated one for the exact solution they thought of.

参考文献

[1] T.C. GARD, *Introduction to Stochastic Differential Equations*, Marcel Dekker, New York, 1988.

[2] R. JANSSEN, *Discretization of the Wiener-process in difference-methods for stochastic differential equations*, Stochatic Process. Appl., **18**(1984), 361–369.

[3] S. KANAGAWA, *The rate of convergence for approximate solutions of stochastic differential equations*, Tokyo J. Math., **12**(1989), 33–48.

[4] P.E. KLOEDEN AND E. PLATEN, *A survey of numerical methods for stochastic differential equations*, J.Stoch.Hydrol.Hydraulics, **3**(1989), 155–178.

[5] P.E. KLOEDEN AND E. PLATEN, *The Numerical Solution of Stochastic Differential Equations*, Springer, Berlin, 1992.

[6] H. LISKE AND E. PLATEN, *Simulation studies on time discrete diffusion approximations*, Mathematics and Computers in Simulation, **29**(1987), 253–260.

[7] N.J. NEWTON, *Asymptotically efficient Runge-Kutta methods for a class of Ito and Stratonovich equations*, SIAM J. Appl. Math., **51** (1991), 542–567.

[8] E. PARDOUX AND D.TALAY, *Discretization and simulation of stochastic differential equations*, Acta Appl.Math, **3**(1985), 23–47.

[9] Y. SAITO AND T. MITSUI, *Discrete approximations for stochastic differential equations*, Trans. Japan SIAM, **2**(1992), 1–16 (in Japanes).

[10] Y. SAITO AND T. MITSUI, *Simulation of stochastic differential equations*, to appear in Ann. Inst. Statist. Math..

[11] D.TALAY, *Efficient numerical schemes for the approximation of expectations of functionals of the solution of a SDE and applications*, Lectures Notes in Control and Information Sciences, **61**, Springer-Verlag, Berlin, 1984.