Packet Interdeparture Time and Packet Delay in Multichannel Communication Systems with Different Capture Level Groups

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Abstract

In this paper, the probability distributions of the packet interdeparture time, number of packets successfully transmitted in parallel, and packet delay time in the multichannel packet radio communication system with different capture level groups are analyzed. The channel access protocols considered include slotted ALOHA IFT (Immediate-First-Transmission) and DFT (Delayed-First-Transmission). Through the analysis, the distribution, average, and coefficient of variation of these performance measures are explicitly derived and are numerically compared with those of the system without capture.

1. Introduction

The design and development of the systems such as multichannel multi-hop systems and interconnected network systems require not only such average performance measures but also higher moments of the packet interdeparture time and number of channels having successful transmission in parallel.

In this paper, we give expressions for the moment generating functions of the packet interdeparture time, packet delay and number of packets successfully transmitted in parallel. We can explicitly calculate not only the averages of these performance measures but also their higher moments by numerical differentiation.

2. System Model

The system consists of a finite population of N users that are divided in to L different power level groups with indexes 1 to L, accessing a set of parallel M channels to transmit their packets. Such that if $i < j$ a group i packet always dominates a group j packet at the same time slot and over the same channel. The users in the same group have the same power, if two or more packets from the same group are simultaneously transmitted on the same channel at the same slot, collision of the packets occurs. The l^{th} $(l = 1, 2, ..., L)$ group has N_l users, where $\sum_{l=1}^{L} N_l = N$. The time axis is slotted into equal length segments of duration corresponding to the transmission time of a packet. All users are assumed to be synchronized and to start their transmissions only at the beginning of a slot. Every user has his own buffer which can store at most one packet at any time. Each user can be in either of two states: the thinking state if it does not have a packet in its own buffer to transmit; the backlogged state if it has a packet awaiting or undergoing transmission. In the IFT protocol case, a packet appertaining to group l is generated only at the beginning of a slot with probability λ_l, and upon a new packet arrives, the user transmits the packet with probability one, immediately. In the DFT protocol case, a thinking station in group l generates a new packet with probability λ_l at the end of a slot. Upon new packet arrival, the station joins the backlogged state at once, then schedules its transmission after a geometrically distributed backoff. A backlogged station in group l retransmits a packet after backoff slots with mean $1/\mu_l$.

3. Joint Probability Distribution

In this section, we analyze the joint probability distribution of packet interdeparture time and the number of packets successfully transmitted. Here, we define the packet interdeparture time as the time interval between two successive successful transmissions in which at least one packet is successfully transmitted. Let W represent a random variable of such packet interdeparture time and X $(X = 1, 2, ..., M)$ represent the number of channels having successful transmission in parallel when the departure satisfying the above definition occurs. We firstly denote, by $P_{\mathbf{IJ}}^{(s)*}(\xi)$, a set of the joint moment generating function of the system state transition in one slot when the system state changes from $x(t) = \mathbf{I}$ to $x(t+1) = \mathbf{J}$ and the number of successful channels is X. Then we have

$$P_{\mathbf{IJ}}^{(s)*}(\xi_1, \xi_2, ..., \xi_L) = \sum_{\mathbf{n}} \sum_{\mathbf{s}} P(\mathbf{J} - \mathbf{I} + \mathbf{s}, \mathbf{n}, \mathbf{s} \mid \mathbf{I}) \cdot \xi_1^{s_1} \xi_1^{s_2} \cdots \xi_1^{s_L}, \quad (1)$$

where for the IFT protocol, $0 \le i_l \le N_l$, $0 \le j_l \le N_l$, $1 \le n_l \le N_l$, $1 \le l \le L$, $1 \le s_l \le min(M, n_l)$; for the DFT protocol, $1 \le i_l \le N_l$, $0 \le j_l \le min(M, n_l)$. We also define the conditional transition probability

$$P_{\mathbf{IJ}}^{(s)} = \text{prob}\{x(t+1) = \mathbf{J}, \text{ at least one packet is successfully transmitted in slot } t+1 \mid x(t) = \mathbf{I}\} = P_{\mathbf{IJ}}^{(s)*}(1).$$

Consider now the conditional moment generating function $T_{\mathbf{I}}^*(z, \xi)(i_l = 0, 1, ..., N_l;\ \text{for } l = 1, 2, ..., L)$ of the joint probability distribution of the random variables W and X given $x(t) = \mathbf{I}$. Let $\mathbf{T}^*(z, \xi) = \{T_a^*(z, \xi); 0 \le a \le A-1\}^T$ be a A dimensional column vector. Then we get

$$\mathbf{T}^*(z, \xi) = z\left\{\mathbf{I} - z\left[\mathbf{P} - \mathbf{P}^{(s)}\right]\right\}^{-1} \mathbf{P}^{(s)*}(\xi)\mathbf{H}, \quad (2)$$

where $\mathbf{P}^{(s)*}(\xi)$ and $\mathbf{P}^{(s)}$ are $A \times A$ matrices, whose elements are $P_{\mathbf{IJ}}^{(s)*}(\xi)$ and $P_{\mathbf{IJ}}^{(s)}(i_l = 0, 1, ..., N_l, j_l = 0, 1, ..., N_l;\ \text{for } l = 1, 2, ..., L)$, respectively. $P_{0\mathbf{J}}^{(s)} = 0$ and $P_{0\mathbf{J}}^{(s)*}(\xi) = 0$ $(j_l = 0, 1, ..., N_l;\ \text{for } l = 1, 2, ..., L)$. $\mathbf{H}$ is the column vector with A elements all of which equal one, $\mathbf{P}$ is the matrix of the transition probabilities of the system.

Let $\mathbf{\Phi} = \{\phi_0, \phi_1, ..., \phi_{A-1}\}$ be an A dimensional row vector representing the stationary state probability distribution. It is given by solving the following equations (note that $\mathbf{P}^{(s)} = \mathbf{P}^{(s)*}(1)$):

$$\mathbf{\Phi} = \mathbf{\Phi}\left\{\mathbf{I} - \left[\mathbf{P} - \mathbf{P}^{(s)}\right]\right\}^{-1} \mathbf{P}^{(s)}, \quad \sum_{a=0}^{A-1} \phi_a = 1. \quad (3)$$

Based on $\mathbf{T}^*(z, \xi)$ and $\mathbf{\Phi}$, we can finally get $P^*(z, \xi)$ as

$$P^*(z, \xi) = \mathbf{\Phi}\mathbf{T}^*(z, \xi). \quad (4)$$

4. Packet Delay Process

We now give an expression for the moment generating function $D^{(l)*}(z)$ of the packet delay for group l. The packet delay of group l $(l = 1, 2, ..., L)$ involves two parts: 1) the delay from a packet arrival epoch of group l to the end of the first slot where the any packets are successfully transmitted; 2) the delay from the end of the slot to the completion of the packet transmission of group l. We denote, by $D_{\mathbf{I}}^{(l)*}(z)$, $(i_l = 0, 1, ..., N_l; l = 1, 2, ..., L)$, the conditional moment generating function of the second delay that a user finding himself among i_l backlogged users at the end of slot t, given $x(t) = \mathbf{I}$. In order to obtain $D_{\mathbf{I}}^{(l)*}(z)$, we consider the conditional transition probabilities as follows:

$$V_{\mathbf{IJ}}^{(l)} = prob\{x(t+1) = \mathbf{J}, \text{ a backlogged user of group } l \text{ finding himself in state } \mathbf{I} \text{ at the end of slot } t \text{ transmits his packet successfully in slot } t+1 \mid x(t) = \mathbf{I}\}.$$

Since the behavior of stations are homogeneous, the probability that i_l backlogged users contained by s_l successfully transmitted is s_l/i_l for $l = 1, 2, ..., L$. The conditional transition probabilities for the IFT protocol can be obtained by the equations,

$$V_{\mathbf{IJ}}^{(l)} = \sum_{\mathbf{n}} \sum_{\mathbf{s}} \sum_{\mathbf{k}} F\{k_l \mid n_l, s_l, n_l - j_l + i_l - s_l\} P\{\mathbf{J} - \mathbf{I} + \mathbf{s}, \mathbf{n}, \mathbf{s} \mid \mathbf{I}\} \frac{k_l}{i_l}$$

$$\left(\begin{array}{c} 1 \le i_l \le N_l, 0 \le j_l \le N_l \\ 1 \le n_l \le N_l, 1 \le l \le L \\ 1 \le s_l \le min(M, n_l) \\ max(0, i_l - j_l) \le k_l \le min(s_l, n_l - j_l + i_l - s_l) \end{array}\right), \quad (5)$$

$F\{k_l \mid n_l, s_l, b_l\}$ is the probability that the backlogged users of group l successfully transmit k_l packets under the condition that n_l packet transmissions are started, among which b_l packets are old and s_l packets are successfully transmitted for $l = 1, 2, ..., L$. For the DFT protocol, $V_{\mathbf{IJ}}^{(l)}$ can be obtained by the equations,

$$V_{\mathbf{IJ}}^{(l)} = \sum_{\mathbf{n}} \sum_{\mathbf{s}} P\{\mathbf{J} - \mathbf{I} + \mathbf{s}, \mathbf{n}, \mathbf{s} \mid \mathbf{I}\} \frac{s_l}{i_l}$$
$$\left(\begin{array}{c} 1 \le i_l \le N_l, \ 0 \le j_l \le N_l \\ 1 \le n_l \le i_l, \ 1 \le s_l \le min(M, n_l) \end{array}\right). \quad (6)$$

Let $\mathbf{D}^{(l)*}(z)$ denote the A dimensional column vector whose elements are $\{D_{\mathbf{I}}^{(l)*}(z); \ 0 \le i_l \le N_l, l = 1, 2, ..., L\}$. Using the above conditional transition probabilities, the conditional moment generating function $D_{\mathbf{I}}^{(l)*}(z)$ of packet delay of group l for the second part is thus given by

$$\mathbf{D}^{(l)*}(z) = z\left\{\mathbf{I} - z\left[\mathbf{P} - \mathbf{V}^{(l)}\right]\right\}^{-1} \mathbf{V}^{(l)}\mathbf{H}, \quad (7)$$

where for the IFT protocol, $0 \le i_l \le N_l, \ 0 \le j_l \le N_l$; for the DFT protocol, $1 \le i_l \le N_l, \ 1 \le j_l \le N_l$. Next we consider the first delay part. We let $q_0(l)$ represent the probability of a packet of group l being successfully transmitted at the first slot upon its arrival, and let $q_{\mathbf{J}}^{(l)}$ represent the probability that a packet of group l finds itself among j_l backlogged packets at the end of slot $t+1$ upon its arrival, given $x(t) = \mathbf{I}, \ \ l = 1, 2, ..., L$. For the IFT protocol, these probabilities can be obtained by the following expressions,

$$q_0^{(l)} = K \sum_{\mathbf{J}} \sum_{\mathbf{I}} \sum_{\mathbf{n}} \sum_{\mathbf{s}} \sum_{\mathbf{k}} \pi_{\mathbf{I}} H\{k_l \mid n_l, s_l, j_l - i_l + s_l\} k_l \cdot P\{\mathbf{J} - \mathbf{I} + \mathbf{s}, \mathbf{s}, \mathbf{n} \mid \mathbf{I}\}$$
$$\left(\begin{array}{c} 1 \le n_l \le N_l, \ 1 \le l \le L, \ 1 \le s_l \le min(M, N_l) \\ 0 \le i_l \le N_l - 1, \ 0 \le j_l \le N_l - 1 \\ max(0, 2s_l + j_l - i_l - n_l) \le k_l \\ k_l \le min(s_l, j_l - i_l + s_l) \end{array}\right), \quad (8)$$

where $\pi_{\mathbf{I}}$ is the stationary state probability distribution.

$$q_{\mathbf{J}}^{(l)} = K \sum_{\mathbf{n}} \sum_{\mathbf{s}} \sum_{\mathbf{I}} \pi_{\mathbf{I}} \sum_{\mathbf{k}} H\{k_l \mid n_l, s_l, j_l - i_l + s_l\} \cdot P\{\mathbf{J} - \mathbf{I} + \mathbf{s}, \mathbf{n}, \mathbf{s} \mid \mathbf{I}\}(j_l - i_l + s_l - k_l)$$
$$\left(\begin{array}{c} 0 \le n_l \le N_l, \ 1 \le l \le L, \ 0 \le s_l \le min(M, N_l) \\ 0 \le i_l \le min(N_l - 1, j_l + s_l - 1) \\ max(0, 2s_l + j_l - i_l - n_l) \le k_l \\ k_l \le min(s_l, j_l - i_l + s_l) \end{array}\right), \quad (9)$$

$H\{k_l \mid n_l, s_l, a_l\}$ is the probability that k_l new packets are successfully transmitted with the condition that n_l packet transmissions are started, among which a_l packets are new, and s_l packets are successfully transmitted for group l. K is a normalizing constant such that $\sum_{\mathbf{J}} q_{\mathbf{J}}^{(l)} = 1$. The probability $q_{\mathbf{J}}^{(l)}$ for the DFT protocol can be obtained by

$$q_{\mathbf{J}}^{(l)} = K \sum_{\mathbf{s}} \sum_{\mathbf{I}} \pi_{\mathbf{I}} \sum_{\mathbf{n}} P(\mathbf{J} - \mathbf{I} + \mathbf{s}, \mathbf{n}, \mathbf{s} \mid \mathbf{I})(j_l - i_l + s_l)$$
$$\left(\begin{array}{c} 1 \le j_l \le N_l \\ 0 \le s_1 + s_2 + ... + s_L \le M \\ 0 \le i_l \le min(N_l, j_l + s_l) \\ s_l \le n_l \le i_l \end{array}\right). \quad (10)$$

Finally, we obtain the unconditional moment generating functions $D^{(l)*}(z)$ of the packet delay for $l = 1, 2, ..., L$ as follows:
IFT protocol case

$$D^{(l)*}(z) = z\, q_0^{(l)} + z\, \mathbf{Q}^{(l)}\, \mathbf{DL}^{(l)*}(z), \quad (11)$$

DFT protocol case

$$D^{(l)*}(z) = \mathbf{Q}^{(l)}\, \mathbf{DL}^{(l)*}(z), \quad (12)$$

where $\mathbf{Q}^{(l)} = \{q_1^{(l)}, q_2^{(l)}, ..., q_{A-1}^{(l)}\}$ is an $(A-1)$ dimensional row vector and $\mathbf{DL}^{(l)*}(z) = \left\{D_1^{(l)*}(z), D_2^{(l)*}(z), ..., D_{A-1}^{(l)*}(z)\right\}$ are $(A-1)$ dimensional column vectors excluding element $D_0^{(l)*}(z)$.

5. Numerical Results

We denote, by $V_{ar}[G]$, the variance of the random variable G, $V_{ar}[G] = E\left[G^2\right] - \{E[G]\}^2$, and by C_G, the coefficient of variation of the random variable G, $C_G = \sqrt{V_{ar}[G]}/E[G]$. We numerically compare the averages and coefficients of variation of the packet interdeparture time and packet delay of the multichannel systems with capture and without capture under slotted ALOHA IFT protocol. For all the numerical results, we consider that the number of groups is 3, the number of channels is 2, the number of stations is 12. The curves of coefficients of variation of interdeparture time W and packet delay D are showed in Fig.1 and Fig.2, respectively. Fig.1 shows that with capture the value of coefficient of variation decreases to low levels when the arrival rate of new packets is too high. However, for the single channel case, the coefficient of variation is too large even when λ is large. Fig.2 shows that for all cases coefficients of variation of the packet delay D are very stable in the system with capture.

6. Conclusions

In this paper, the moment generating functions of the interdeparture time, packet delay and number of packets successfully transmitted in parallel for the multichannel slotted ALOHA system with capture were given. The coefficients of variation of the packet interdeparture time and packet delay were numerically compared with those of the system without capture. These results are useful for analyzing multi-hop networks employing these random access protocols. Numerical results showed that the performance measures of the system can be improved by employing capture.

References

[1] R. Nelson and L. Kleinrock, IEEE Trans. Commun., pp. 684-694, 1984. [2] W. Yue, Proc. of ISCIE'90, pp. 371-372, 1990.

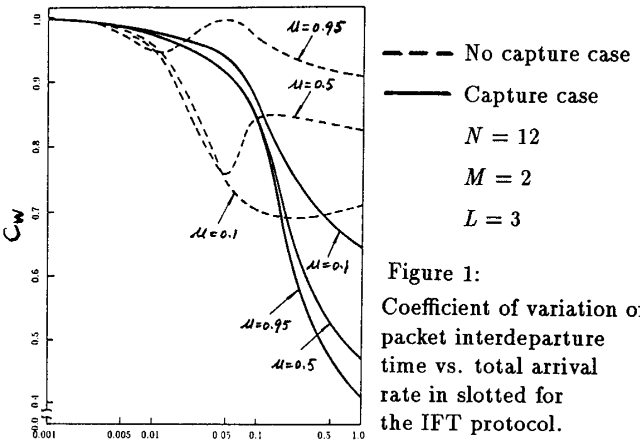

Figure 1:
Coefficient of variation of packet interdeparture time vs. total arrival rate in slotted for the IFT protocol.

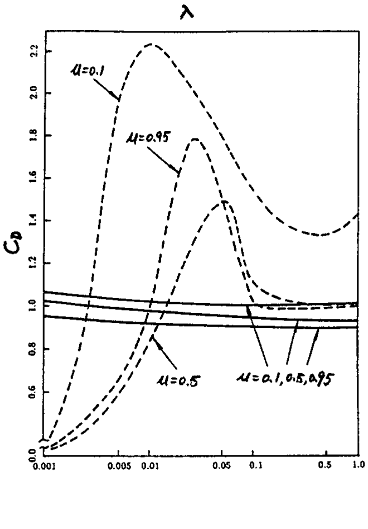

Figure 2:
Coefficient of variation of packet delay vs. total arrival rate in slotted for the IFT protocol.