5K-8

Representation of Rotational Sweeps using Quadratic Surfaces and Toruses

2次曲面体とトーラスを用いた回転スイープの表現

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1. Introduction

Flexible modeling of arbitrary solid objects is one of the most important problems of computer graphics. This paper presents a new efficient method of modeling rotational sweeps which are curved objects defined by rotating a given curve around an arbitrary axis.

Glasses or bottles are common examples of rotational sweeps. Since the rotational sweep is one of the most natural object forms produced by using a lathe, its efficient modeling is especially important for industrial applications.

It is popular to approximate the 3D surface with many small planes (patches). However, optical phenomena like reflection and refraction are not correctly computed by using patches even if smooth shading is employed. Consequently, it becomes important to approximate the objects with lower order curved surfaces for fast calculation, making use of the fact that they are rotational sweeps.

Kajiya[1] proposed a method for this purpose based on spline approximation and strip tree algorithm. However, his algorithm is recursive, and computational cost is very high.

Wijk[2] reduced the problem to solve the equation of 6th order in non-recursive form. Though his method is more efficient than Kajiya's method, it takes still very long time for numerical computation.

Horiuchi et.al.[3] approximated the given free curve with a series of straight lines, and applied smooth shading to a series of local cones which are formed by rotating the straight lines around the axis. This method is based on solving the quadratic equation to detect the ray-object intersection. However, in their method, unnatural phenomena occur at the jointing portions of cones, which is essentially the same problem as approximating the surface with many patches.

No reports have been published on truly ideal methods which satisfy both of computational cost and synthesized image quality conditions.

This paper presents a new efficient method of modeling objects which assures the both conditions stated above. In the proposed method, the object surface is approximated by many quadratic surfaces with preserving the tangent continuity at their jointing portions.

2. Conic spline interpolation of curves

Quadratic curve approximation of a given arbitrary curve is called conic spline approximation[4][5]. It must be noted that only the curves that make quadratic surfaces by rotational sweeping are defined by eq.1 as,

$$\bar{w}^T \quad A \quad \bar{w} = 0, \qquad (eq.1)$$

where $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & b_{20} \\ 0 & b_{20} & a_{00} \end{bmatrix}$ and $\bar{w}^T = [\, x \quad y \quad 1\,]$.

A given curve is approximated by controlled quadratic curves defined by eq.1 fundamentally. Circular arcs are used to interpolate the outline around singular points. A singular point is a point which satisfies one of the following conditions.

(1) A point is not on the rotational axis, and the tangent of the curve at that point is perpendicular to the axis.

(2) A point is on the rotational axis, and the tangent of the curve at that point is parallel to the axis.

By rotating the approximated curve around the axis, the resultant rotational sweep is described by a CSG model of quadratic surfaces and toruses.

3. Generation of rotational sweep

The quadratic surface formed by rotating the quadratic curve around the axis is defined by eq.2 as,

$$\bar{x}^T \quad B \quad \bar{x} = 0, \qquad (eq.2)$$

where $B = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & b_{20} \\ 0 & 0 & a_{11} & 0 \\ 0 & b_{20} & 0 & a_{00} \end{bmatrix}$

and $\bar{x}^T = [\, x \quad y \quad z \quad 1\,]$.

The neighborhood of singular points are approximated by circular arcs, and, toruses are formed at the corresponding portions as shown in Figure 1.

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If the radius of circular arc is r and the distance between the central axis and the center of the circular arc is d, the torus is expressed by eq. 3 as,

$$\chi^T \quad C \quad \chi = 0, \qquad (eq.3)$$

where $C = \begin{bmatrix} 1 & 1 & 1 & -d^2 - r^2 \\ 1 & 1 & 1 & d^2 - r^2 \\ 1 & 1 & 1 & -d^2 - r^2 \\ -d^2 - r^2 & d^2 - r^2 & -d^2 - r^2 & (d^2 - r^2)^2 \end{bmatrix}$

and $\chi^T = [x^2 \quad y^2 \quad z^2 \quad 1]$.

Therefore the problem to calculate the ray-torus intersection is reduced to algebraically solving the equation of forth order.

4. Experimental results

Figure 2 shows the images synthesized by the rotational sweep data constructed by the method proposed here. Ray-casting was used for rendering the images. A wine glass shown in Figure 2 was generated from fourteen control points to define the quadratic curves of cross section. One circular arc is used to approximate the neighbor of a singular point. Consequently, 3D model of the wine glass is defined as a CSG of one torus and twenty five quadratic surfaces.

The proposed method is apparently much faster than Wijk's method[2]. Wijk's method is based on solving the equation of 6th order, while the proposed method needs only solving quadratic equations for most parts, and equations of 4th order for singular parts.

computational cost would be comparable with Horiuchi et.al.'s method[3], if same number of primitives are used, while image quality must be very much improved. In Horiuchi et.al.'s methods in order to avoid unfavorable phenomena which easily occur, more primitives (cones) should be used, and, the computational cost becomes much higher.

Since Horiuchi et.al.'s[3] reported that their method is about 20 times faster than Wijk's method[2], the proposed method seems to be at least 20 times faster than Wijk's method[2]. Wijk used cubic spline interpolation to approximate the given curve to assure the continuity up to the second derivative (curvature). While, in the proposed method, conic spline is used to approximate the given curve, and only tangent continuity at the jointing portions of primitives is assured. However, for various characteristics in human vision, there is no big difference between them, and computational cost is drastically reduced.

5. Conclusion

This paper proposed a new efficient method of modeling rotated sweeps using quadratic surfaces and toruses which guarantee tangent continuity all over the surfaces.

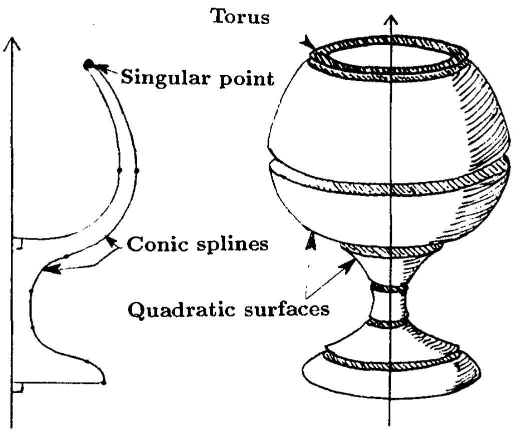

Figure 1

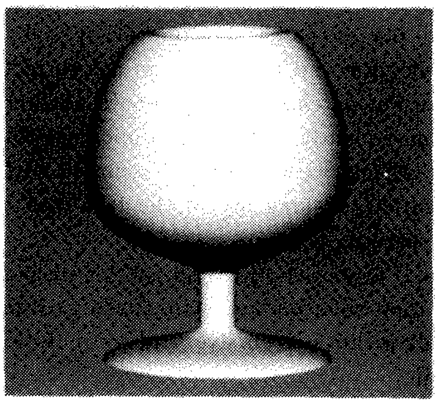

Figure 2 Wine glass

In comparison with previously reported methods such as Kajiya[1], Wijk[2], and Horiuchi et.al.[3], the method proposed here reduces computational cost drastically without losing the high picture quality, because ray-object intersection of quadratic surfaces and toruses are done algebraically, not by numerical computation.

References

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