An Application of Game Refinement Theory

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Abstract. This paper presents an application of the game refinement theory to a class of round-match games. It describes the basic idea of game refinement theory while showing the process of game design using two-person round-match RoShamBo. It then deals with *Saicyugen*, a three-person card game. Since a round-match is the first-level meta game, it can be refined by the same way as for the board games. Simulation experiments performed show the effectiveness of the idea.

1 Introduction

We have studied the property of the decision space in game playing[5][6][8][9]. The decision space is the minimal search space without forecasting. It provides the common measures for almost all board games. The dynamics of decision options in the decision space has been investigated and we observed that this dynamics was a key factor for game entertainment. Then we proposed the measure of the refinement in games[6].

Interesting games are always uncertain until the last end of games. Thus the variation in available options stays constant all over games. Here the games are a kind of seesaw game between possible results. In contrast, one player quickly dominates over the other in uninteresting games. Here options are likely to be diminishing quickly. Therefore, the refined games are more likely to be in a seesaw game.

From the property of seesaw games, we proposed a measure of refinement in game playing. This measure should reflect some aspects of the attractiveness of games. We then compared a class of board games by means of this measure. We especially compare various chess variants by means of this measure and other characteristics[6]. The measure was $\frac{\sqrt{B}}{D}$ where B stands for the average number of possible moves and D stands for the average game length.

We should notice that the rules and details of a game should have changed over the long history of games in most games and the current games are the evolutionary outcomes of a long history from the original games. For example, many variants of chess-like board games are known in history and the modern chess is the descendant of these variants. Here consider the historical changes of refinement measure in chess variants. Suppose that a new variant is born from the old version of a game. If the old variant is less refined than a new variant, such a variant should be replaced by the new one, because it is less attractive. Therefore, the game itself should be refined in a long history of a game, and the current game is the evolutionary outcome of the long history of games. The refinement measure should be expected to increase in a long history of a game. Consequently the history of games can be viewed as an evolutionary optimization of game-refinement factors as well as complexity.

In this paper we focus on an application of the measure to a class of round-match games. Since a round-match is a game, the proposed measure would be also a key factor for the attractiveness of the round-match games. The point is how to estimate the number of options and game length for such games. Section 1 presents a two-person round-match game so-called RoShamBo, with experiments and discussion. An appopriate rounds will be discussed if we focus on the proposed measure. We then consider another type of round-match games *Saicyugen* (Most Mediocre), played by three players. It is a meta game with level 0 and 1. Finally, the concluding remarks are given.

2 Two-Person Round-Match RoShamBo

RoShamBo (also known as Rock-Paper-Scissors) is a simple game, but the best strategy can be quite complicated when playing against fallible opponents. The game is played between two players. On each turn, the players simultaneously choose one of "rock", "paper", or "scissors". If they choose the same item, the result is a tie; otherwise rock crushes scissors, paper covers rock, or scissors cuts paper. A match consists of a series of turns (rounds) between the two players.

The game is trivial from a game-theoretic point of view. The optimal mixed strategy is to

Table 1. Experimental results under the basic rules.

Winning point	2	3	4	5	6	7	8	9	10	11
Round to end	3.74	6.19	8.72	11.31	13.95	16.61	19.28	21.97	24.70	27.18
$\frac{\sqrt{B}}{D}$	0.461	0.280	0.200	0.153	0.124	0.104	0.090	0.080	0.070	0.064
Winning point	12	13	14	15	16	17	18	19	20	
Round to end	30.21	32.95	35.72	38.56	41.27	44.09	46.85	49.67	52.48	
$\frac{\sqrt{B}}{D}$	0.057	0.053	0.048	0.045	0.042	0.039	0.037	0.035	0.033	

choose an action uniformly at random (one-third probability of each). This will ensure a break-even result in the long run, regardless of how strong (or how weak) the opponent is. However, against predictable opponents, a player can attempt to detect patterns in the opponent's play, and exploit those tendencies with an appropriate counter-strategy[1].

The two-person round-match RoShamBo provides at each round with three options: win, loss or tie. The length of the game is the number of the total rounds played. Below the basic rules for the experiments performed in this paper are shown.

- The winner at each round gets one point while no point for the loser. Otherwise (tie) also no point.
- If one's total winning points reaches a given regulation point, then the player wins the game.

To change the characteristics of the game, several additional rules will be given based on the following idea.

- If one wins some consecutive rounds, the bonus points are given. The consecutive wins must not contain a draw round.

The additional rules will make the simple game more sophisticated while increasing the potential of changing the outcome.

2.1 Experimental design

We set the computer experiments as follows.

- The winner obtains one point.
- Various regulation points (say n) are to be examined.
- Rounds go on until the winner is determined.
- When one player meets the additional rule, he/she obtains n point.
- Each player selects an action at random.
- The first experiment takes 100,000 trials with the regulation point ranging from 2 to 20, where the additonal rules are not used.
- In the second experiment, 100,000 trials are performed with $n = 10$ for each additional rule.

Results and discussion (1) We show, in Table 1, the results of the first experiment. The game length (i.e., average number of rounds to end, denoted as D) and the measure $\frac{\sqrt{B}}{D}$ are given. The degree of freedom (denoted as B) for each player in the two-person round-match RoShamBo is three since it has only three possibilities at each round: win, loss or tie. As $\frac{\sqrt{B}}{D}$ becomes larger, the influence by the chance to the outcome of the game increases. Similarly, as $\frac{\sqrt{B}}{D}$ becomes smaller, the outcome of the game highly depends on the skill[7].

As mentioned in the introduction section, the modern chess is the descendant of many chess old variants and must be well-refined. The measure value (roughly 0.07[6]) for chess may reflect the well balance between skill and chance. We notice that the value of $\frac{\sqrt{B}}{D}$ is 0.07 in Table 1 when the winning point is 10. We then suspect that the two-person round-match RoShamBo with winning point 10 may be well-refined at some level.

Results and discussion (2) We expect to refine the two-person round-match RoShamBo with winning point 10 using the additional rules. Thirteen additional rules are used for the next experiment. These additional rules may change sensitively the game property while increasing or decreasing the game length. In this experiment, 100,000 trials are performed for each additional rule. We show, in Table 2, the results of the experiment.

We first notice that the addition of special rules is more delicate than the change of the regulation point when we compare the two experiments. From Table 2, we assume that five variants #4 to #8 are somehow interesting. The values of $\frac{\sqrt{B}}{D}$ for these variants are in the range of [0.071 – 0.080]. It is hard to determine the

Table 2. Experimental results under the addinional rules.

#	Additional rules	Round	$\frac{\sqrt{B}}{D}$
1	winning two-in-row once	6.02	0.288
2	winning two-in-row twice	14.66	0.118
3	winning three-in-row once	15.48	0.112
4	winning two-in-row three times	21.67	0.080
5	winning four-in-row three once	21.78	0.079
6	winning three-in-row twice	23.83	0.073
7	winning five-in-row three once	23.94	0.072
8	winning six-in-row three once	24.52	0.071
9	winning seven-in-row three once	24.69	0.070
10	winning four-in-row three twice	24.69	0.070
11	winning eight-in-row once	24.69	0.070
12	winning three-in-row three time	24.71	0.070
13	winning nine-in-row once	24.73	0.070

best variant since the entertaining impact depends highly on the skill of players. If players strongly prefer the chance factor, then they will choose #4 or #5 variant.

We have shown a process of game design. Starting with the primitive version of the two-person round-match RoShamBo, the first stage of the game design determined the regualtion point that the game may have a reasonable value of $\frac{\sqrt{B}}{D}$. The second stage was followed by the addition of the special rules that the game property may have changed more sensitively. We thus see that $\frac{\sqrt{B}}{D}$ can be a reasonable indication for game entertainment for the two-person round-match game.

We hereby give a remark on the property of a game which has the three outcomes: win, loss and draw.

Remark 1. Two-person round-match is a game where the degree of the freedom is three.

Note that Checkers is a game for which the average number of possible moves is nearly three.

2.2 Round-match RoShamBo and Chess compared

The round-match is a level-0 meta game which has some rounds at which a game is played. The game to play at each round is the two-person RoShamBo in the previous experiments. However, we recognize that the measure $\frac{\sqrt{B}}{D}$ will be applied to any kind of games if the degree of freedom (possiblities for players) and playing time (game length) can be defined.

Consider the round-match for chess championship between the world champion and a challenger. The question arises: "how many rounds are sufficiently to decide the new champion?" We observe that the two-person round-match RoShamBo provides at each round with three options: win, loss or tie. To enhance the game, it is good to play the 10-point match, namely roughly 24 round-match game. In a chess match, there are also three possibilities at each round: win, loss or draw.

The indication (well balance between skill and chance) suggests that about 24 round-match would be reasonable. Indeed, 24 round-match system has been used in chess[3]. We claim that this is not by chance, but the evolutionary selection of the long history of round-match games.

3 Most Mediocre

Most Mediocre (Saicyugen in Japanese) is a deceptively simple game modified from Mediocre invented by Douglas Hofstadter[2]. In Mediocre, each player selects a number. The player who selects the middlemost number is the winner and gets a score. This is known as level 0 mediocrity. A level 1 game of mediocrity involves playing N games of level 0 mediocrity. Then, the winner is the person with the middlemost number of level 0 wins.

However, in Most Mediocre each player selects a card (i.e., a number) from his/her hand, while playing N games of level 0 and M games of level 1. Below we show the standard rules of Most Mediocre.

- 52 cards used without joker.
- There are three players. 17 (say X) cards are dealt to each.

Table 3. *Saicyugen* variants.

#	X	M	N	Options	Rounds	$\frac{\sqrt{B}}{D}$
1	17	3	5	10	45	0.07
2	17	3	3	13	27	0.13
3	9	3	3	5	27	0.08
4	10	3	3	6	27	0.09

- At each turn, one selects a card (i.e., number) from his/her hand. The player who selects the middlemost number is the winner and gets a score that is the number on the card.
- There are superiority based on suit of card. Therefore, we have only the winner at each turn.
- The winner's card will be faced up, otherwise faced down.
- A level 1 game involves playing 5 (say M) rounds of level 0 mediocrity. The winner is the person with the middlemost number of level 0 wins.
- A level 2 game involves playing 3 (say N) rounds of level 1 mediocrity. Then, the final winner is the person with the middlemost number of level 1 wins.

There are four *Saicyugen* variants with different value X, M and N. We show, in Table 3, the variants and our calculation on the options B and game length D for each variant. In the standard *Saicyugen*, at the first round each player has seventeen options, sixteen options at the second round, and so on. Therefore, the average number of actions for each player is 10. The game length is given by $3 \times 5 \times 3 = 45$.

According to the GPCC e-mail discussions, #4 variant is known to be most interesting while #2 variant is boring[4]. The values of $\frac{\sqrt{B}}{D}$ for three variants (#1, #3 and #4) are in the range of $[0.07 - 0.09]$. The indication suggests that #1 variant will be more interesting than #4 variant as the skill of players increases.

4 Concluding Remarks

The future works will consider the general case where multiple players (more than 3) participate the competition. In the tournament, a game at each round is played by two players or two teams with limited number of rounds. Usually, such a tournament is organized while using Swiss System or its modified pairing system. We then have to estimate the potential options at each round, and will know sufficient rounds in the sense of the attractiveness of multi-player round-match games.

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