Vol.2012-AL-139 No.3
2012/3/14

均衡型 (C_5, C_{10})-Foil デザインと関連デザイン

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グラフ理論において、グラフの分解問題は主要な研究テーマである。C_5 を５点を通るサイクル、C_{10} を１０点を通るサイクルとする。１点を共有する辺素な t 個の C_5 と t 個の C_{10} からなるグラフを (C_5, C_{10})-$2t$-foil という。本研究では、完全グラフ K_n を 均衡的に (C_5, C_{10})-$2t$-foil 部分グラフに分解する均衡型 (C_5, C_{10})-foil デザインについて述べる。さらに、均衡型 C_{15}-foil デザイン、均衡型 C_{30}-foil デザイン、均衡型 C_{45}-foil デザイン、均衡型 C_{60}-foil デザイン、均衡型 C_{75}-foil デザイン、均衡型 C_{90}-foil デザイン、均衡型 C_{105}-foil デザイン、均衡型 C_{120}-foil デザイン、均衡型 C_{135}-foil デザイン、均衡型 C_{150}-foil デザインについて述べる。

Balanced (C_5, C_{10})-Foil Designs and Related Designs

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In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory. This paper gives balanced (C_5, C_{10})-foil designs, balanced C_{15}-foil designs, and balanced C_{30}-foil designs, and balanced C_{45}-foil designs, and balanced C_{60}-foil designs, and balanced C_{75}-foil designs, and balanced C_{90}-foil designs, and balanced C_{105}-foil designs, and balanced C_{120}-foil designs, and balanced C_{135}-foil designs, and balanced C_{150}-foil designs.

1. Balanced (C_5, C_{10})-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_{10} be *the* 5-*cycle* and *the* 10-*cycle*, respectively. *The* (C_5, C_{10})-$2t$-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_{10}'s with a common vertex and the common vertex is called *the center of the* (C_5, C_{10})-$2t$-*foil*. When K_n is decomposed into edge-disjoint sum of (C_5, C_{10})-$2t$-foils and every vertex of K_n appears in the same number of (C_5, C_{10})-$2t$-foils, we say that K_n *has a balanced* (C_5, C_{10})-$2t$-*foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced* (C_5, C_{10})-*foil design*.

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Theorem 1. K_n has a balanced (C_5, C_{10})-$2t$-foil design if and only if $n \equiv 1 \pmod{30t}$.

Proof. **(Necessity)** Suppose that K_n has a balanced (C_5, C_{10})-$2t$-foil decomposition. Let b be the number of (C_5, C_{10})-$2t$-foils and r be the replication number. Then $b = n(n-1)/30t$ and $r = (13t+1)(n-1)/30t$. Among r (C_5, C_{10})-$2t$-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of (C_5, C_{10})-$2t$-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $4tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/30t$ and $r_2 = 13(n-1)/30$. Therefore, $n \equiv 1 \pmod{30t}$ is necessary.
(Sufficiency) Put $n = 30st + 1$ and $T = st$. Then $n = 30T + 1$. Construct a (C_5, C_{10})-$2T$-foil as follows:
$\{(30T+1, T, 18T, 28T+1, 12T+1), (30T+1, 8T+1, 10T+2, 14T+2, 20T+3, 9T+2, 18T+3, 13T+2, 5T+2, T+1)\} \cup$
$\{(30T+1, T-1, 18T-2, 28T, 12T+2), (30T+1, 8T+2, 10T+4, 14T+3, 20T+5, 9T+3, 18T+5, 13T+3, 5T+4, T+2)\} \cup$
$\{(30T+1, T-2, 18T-4, 28T-1, 12T+3), (30T+1, 8T+3, 10T+6, 14T+4, 20T+7, 9T+4, 18T+7, 13T+4, 5T+6, T+3)\} \cup$
... $\cup$
$\{(30T+1, 1, 16T+2, 27T+2, 13T), (30T+1, 9T, 12T, 15T+1, 22T+1, 10T+1, 20T+1, 14T+1, 7T, 2T)\}$.
Decompose the (C_5, C_{10})-$2T$-foil into s (C_5, C_{10})-$2t$-foils. Then these starters comprise a balanced (C_5, C_{10})-$2t$-foil decomposition of K_n.

Example 1.1. Balanced (C_5, C_{10})-2-foil design of K_{31}.
$\{(31, 1, 18, 29, 13), (31, 9, 12, 16, 23, 11, 21, 15, 7, 1)\}$.

This starter comprises a balanced (C_5, C_{10})-2-foil decomposition of K_{31}.

Example 1.2. Balanced (C_5, C_{10})-4-foil design of K_{61}.
$\{(61, 2, 36, 57, 25), (61, 17, 22, 30, 43, 20, 39, 28, 12, 3)\} \cup$
$\{(61, 1, 34, 56, 26), (61, 18, 24, 31, 45, 21, 41, 29, 14, 4)\}$.
This starter comprises a balanced (C_5, C_{10})-4-foil decomposition of K_{61}.

Example 1.3. Balanced (C_5, C_{10})-6-foil design of K_{91}.
$\{(91, 3, 54, 85, 37), (91, 25, 32, 44, 63, 29, 57, 41, 17, 4)\} \cup$
$\{(91, 2, 52, 84, 38), (91, 26, 34, 45, 65, 30, 59, 42, 19, 5)\} \cup$
$\{(91, 1, 50, 83, 39), (91, 27, 36, 46, 67, 31, 61, 43, 21, 6)\}$.
This starter comprises a balanced (C_5, C_{10})-6-foil decomposition of K_{91}.

Example 1.4. Balanced (C_5, C_{10})-8-foil design of K_{121}.
$\{(121, 4, 72, 113, 49), (121, 33, 42, 58, 83, 38, 75, 54, 22, 5)\} \cup$
$\{(121, 3, 70, 112, 50), (121, 34, 44, 59, 85, 39, 77, 55, 24, 6)\} \cup$
$\{(121, 2, 68, 111, 51), (121, 35, 46, 60, 87, 40, 79, 56, 26, 7)\} \cup$
$\{(121, 1, 66, 110, 52), (121, 36, 48, 61, 89, 41, 81, 57, 28, 8)\}$.
This starter comprises a balanced (C_5, C_{10})-8-foil decomposition of K_{121}.

Example 1.5. Balanced (C_5, C_{10})-10-foil design of K_{151}.
$\{(151, 5, 90, 141, 61), (151, 41, 52, 72, 103, 47, 93, 67, 27, 6)\} \cup$
$\{(151, 4, 88, 140, 62), (151, 42, 54, 73, 105, 48, 95, 68, 29, 7)\} \cup$
$\{(151, 3, 86, 139, 63), (151, 43, 56, 74, 107, 49, 97, 69, 31, 8)\} \cup$
$\{(151, 2, 84, 138, 64), (151, 44, 58, 75, 109, 50, 99, 70, 33, 9)\} \cup$
$\{(151, 1, 82, 137, 65), (151, 45, 60, 76, 110, 51, 101, 71, 35, 10)\}$.
This starter comprises a balanced (C_5, C_{10})-10-foil decomposition of K_{151}.

Example 1.6. Balanced (C_5, C_{10})-12-foil design of K_{181}.
$\{(181, 6, 108, 169, 73), (181, 49, 62, 86, 123, 56, 111, 80, 32, 7)\} \cup$
$\{(181, 5, 106, 168, 74), (181, 50, 64, 87, 125, 57, 113, 81, 34, 8)\} \cup$
$\{(181, 4, 104, 167, 75), (181, 51, 66, 88, 127, 58, 115, 82, 36, 9)\} \cup$
$\{(181, 3, 102, 166, 76), (181, 52, 68, 89, 129, 59, 117, 83, 38, 10)\} \cup$
$\{(181, 2, 100, 165, 77), (181, 53, 70, 90, 131, 60, 119, 84, 40, 11)\} \cup$
$\{(181, 1, 98, 164, 78), (181, 54, 72, 91, 133, 61, 121, 85, 42, 12)\}$.
This starter comprises a balanced (C_5, C_{10})-12-foil decomposition of K_{181}.

2. Balanced C_{15}-Foil Designs

Let C_{15} be *the cycle* on 15 vertices. *The C_{15}-t-foil* is a graph of t edge-disjoint C_{15}'s with a common vertex and the common vertex is called *the center of the C_{15}-t-foil*. When K_n is decomposed into edge-disjoint sum of C_{15}-t-foils and every vertex of K_n appears in the same number of C_{15}-t-foils, it is called that K_n *has a balanced C_{15}-t-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced C_{15}-foil design*.

Theorem 2. K_n has a balanced C_{15}-t-foil design if and only if $n \equiv 1 \pmod{30t}$.

Proof. (Necessity) Suppose that K_n has a balanced C_{15}-t-foil decomposition. Let b be the number of C_{15}-t-foils and r be the replication number. Then $b = n(n-1)/30t$ and $r = (14t+1)(n-1)/30t$. Among r C_{15}-t-foils having a vertex v of K_n, let r_1 and r_2 be the numbers of C_{15}-t-foils in which v is the center and v is not the center, respectively. Then $r_1 + r_2 = r$. Counting the number of vertices adjacent to v, $2tr_1 + 2r_2 = n - 1$. From these relations, $r_1 = (n-1)/30t$ and $r_2 = 14(n-1)/30$. Therefore, $n \equiv 1 \pmod{30t}$ is necessary.
(Sufficiency) Put $n = 30st + 1, T = st$. Then $n = 30T + 1$. Construct a C_{15}-T-foil as follows:
$\{\ (30T+1, T, 18T, 28T+1, 12T+1, 20T+2, 8T+1, 10T+2, 14T+2, 20T+3, 9T+2, 18T+3, 13T+2, 5T+2, T+1),$
$(30T+1, T-1, 18T-2, 28T, 12T+2, 20T+4, 8T+2, 10T+4, 14T+3, 20T+5, 9T+3, 18T+5, 13T+3, 5T+4, T+2),$
$(30T+1, T-2, 18T-4, 28T-1, 12T+3, 20T+6, 8T+3, 10T+6, 14T+4, 20T+7, 9T+$

$4, 18T+7, 13T+4, 5T+6, T+3)$,
...,
$(30T+1, 1, 16T+2, 27T+2, 13T, 22T, 9T, 12T, 15T+1, 22T+1, 10T+1, 20T+1, 14T+1, 7T, 2T)\ \}$.
Decompose this C_{15}-T-foil into s C_{15}-t-foils. Then these starters comprise a balanced C_{15}-t-foil decomposition of K_n.

Example 2.1. Balanced C_{15} design of K_{31}.
$\{(31, 1, 18, 29, 13, 22, 9, 12, 16, 23, 11, 21, 15, 7, 2)\}$.
This stater comprises a balanced C_{15}-decomposition of K_{31}.

Example 2.2. Balanced C_{15}-2-foil design of K_{61}.
$\{(61, 2, 36, 57, 25, 42, 17, 22, 30, 43, 20, 39, 28, 12, 3)$,
$(61, 1, 34, 56, 26, 44, 18, 24, 31, 45, 21, 41, 29, 14, 4)\}$.
This stater comprises a balanced C_{15}-2-foil decomposition of K_{61}.

Example 2.3. Balanced C_{15}-3-foil design of K_{91}.
$\{(91, 3, 54, 85, 37, 62, 25, 32, 44, 63, 29, 57, 41, 17, 4)$,
$(91, 2, 52, 84, 38, 64, 26, 34, 45, 65, 30, 59, 42, 19, 5)$,
$(91, 1, 50, 83, 39, 66, 27, 36, 46, 67, 31, 61, 43, 21, 6)\}$.
This stater comprises a balanced C_{15}-3-foil decomposition of K_{91}.

Example 2.4. Balanced C_{15}-4-foil design of K_{121}.
$\{(121, 4, 72, 113, 49, 82, 33, 42, 58, 83, 38, 75, 54, 22, 5)$,
$(121, 3, 70, 112, 50, 84, 34, 44, 59, 85, 39, 77, 55, 24, 6)$,
$(121, 2, 68, 111, 51, 86, 35, 46, 60, 87, 40, 79, 56, 26, 7)$,
$(121, 1, 66, 110, 52, 88, 36, 48, 61, 89, 41, 81, 57, 28, 8)\}$.
This stater comprises a balanced C_{15}-4-foil decomposition of K_{121}.

Example 2.5. Balanced C_{15}-5-foil design of K_{151}.
$\{(151, 5, 90, 141, 61, 102, 41, 52, 72, 103, 47, 93, 67, 27, 6)$,
$(151, 4, 88, 140, 62, 104, 42, 54, 73, 105, 48, 95, 68, 29, 7)$,
$(151, 3, 86, 139, 63, 106, 43, 56, 74, 107, 49, 97, 69, 31, 8)$,
$(151, 2, 84, 138, 64, 108, 44, 58, 75, 109, 50, 99, 70, 33, 9)$,
$(151, 1, 82, 137, 65, 110, 45, 60, 76, 110, 51, 101, 71, 35, 10)\}$.
This stater comprises a balanced C_{15}-5-foil decomposition of K_{151}.

Example 2.6. Balanced C_{15}-6-foil design of K_{181}.
$\{(181, 6, 108, 169, 73, 122, 49, 62, 86, 123, 56, 111, 80, 32, 7)$,
$(181, 5, 106, 168, 74, 124, 50, 64, 87, 125, 57, 113, 81, 34, 8)$,
$(181, 4, 104, 167, 75, 126, 51, 66, 88, 127, 58, 115, 82, 36, 9)$,
$(181, 3, 102, 166, 76, 128, 52, 68, 89, 129, 59, 117, 83, 38, 10)$,
$(181, 2, 100, 165, 77, 130, 53, 70, 90, 131, 60, 119, 84, 40, 11)$,
$(181, 1, 98, 164, 78, 132, 54, 72, 91, 133, 61, 121, 85, 42, 12)\}$.
This stater comprises a balanced C_{15}-6-foil decomposition of K_{181}.

3. Balanced C_{15m}-Foil Designs

Let C_{15m} be *the cycle* on $15m$ vertices. *The C_{15m}-t-foil* is a graph of t edge-disjoint C_{15m}'s with a common vertex and the common vertex is called *the center of the C_{15m}-t-foil*. When K_n is decomposed into edge-disjoint sum of C_{15m}-t-foils and every vertex of K_n appears in the same number of C_{15m}-t-foils, it is called that *K_n has a balanced C_{15m}-t-foil decomposition* and this number is called *the replication number*. This decomposition is known as *a balanced C_{15m}-foil design*.

Theorem 3. K_n has a balanced C_{30}-t-foil design if and only if $n \equiv 1 \pmod{60t}$.

Example 3.1. Balanced C_{30} design of K_{61}.
$\{(61, 2, 36, 57, 25, 42, 17, 22, 30, 43, 20, 39, 28, 12, 3, 7, 4, 14, 29, 41, 21, 45, 31, 24, 18, 44, 26, 56, 34, 1)\}$.
This stater comprises a balanced C_{30}-decomposition of K_{61}.

Example 3.2. Balanced C_{30}-2-foil design of K_{121}.
$\{(121, 4, 72, 113, 49, 82, 33, 42, 58, 83, 38, 75, 54, 22, 5, 11, 6, 24, 55, 77, 39, 85, 59, 44, 34, 84, 50, 112, 70, 3),$
$(121, 2, 68, 111, 51, 86, 35, 46, 60, 87, 40, 79, 56, 26, 7, 15, 8, 28, 57, 81, 41, 89, 61, 48, 36, 88, 52, 110, 66, 1)\}.$
This stater comprises a balanced C_{30}-2-foil decomposition of K_{121}.

Example 3.3. Balanced C_{30}-3-foil designn of K_{181}.
$\{(181, 6, 108, 169, 73, 122, 49, 62, 86, 123, 56, 111, 80, 32, 7, 15, 8, 34, 81, 113, 57, 125, 87, 64, 50, 124, 74, 168, 106, 5),$
$(181, 4, 104, 167, 75, 126, 51, 66, 88, 127, 58, 115, 82, 36, 9, 19, 10, 38, 83, 117, 59, 129, 89, 68, 52, 128, 76, 166, 102, 3),$
$(181, 2, 100, 165, 77, 130, 53, 70, 90, 131, 60, 119, 84, 40, 11, 23, 12, 42, 85, 121, 61, 133, 91, 72, 54, 132, 78, 164, 98, 1)\}.$
This stater comprises a balanced C_{30}-3-foil decomposition of K_{181}.

Example 3.4. Balanced C_{30}-4-foil design of K_{241}.
$\{(241, 8, 144, 225, 97, 162, 65, 82, 114, 163, 74, 147, 106, 42, 9, 19, 10, 44, 107, 149, 75, 165, 115, 84, 66, 164, 98, 224, 142, 7),$
$(241, 6, 140, 223, 99, 166, 67, 86, 116, 167, 76, 151, 108, 46, 11, 23, 12, 48, 109, 153, 77, 169, 117, 88, 68, 168, 100, 222, 138, 5),$
$(241, 4, 136, 221, 101, 170, 69, 90, 118, 171, 78, 155, 110, 50, 13, 27, 14, 52, 111, 157, 79, 173, 119, 92, 70, 172, 102, 220, 134, 3),$
$(241, 2, 132, 219, 103, 174, 71, 94, 120, 175, 80, 159, 112, 54, 15, 31, 16, 56, 113, 161, 81, 177, 121, 96, 72, 176, 104, 218, 130, 1)\}.$
This stater comprises a balanced C_{30}-4-foil decomposition of K_{241}.

Example 3.5. Balanced C_{30}-5-foil design of K_{301}.
$\{(301, 10, 180, 281, 121, 202, 81, 102, 142, 203, 92, 183, 132, 52, 11, 23, 12, 54, 133, 185, 93, 205, 143, 104, 82, 204, 122, 280, 178, 9),$
$(301, 8, 176, 279, 123, 206, 83, 106, 144, 207, 94, 187, 134, 56, 13, 27, 14, 58, 135, 189, 95, 209, 145, 108, 84, 208, 124, 278, 174, 7),$
$(301, 6, 172, 277, 125, 210, 85, 110, 146, 211, 96, 191, 136, 60, 15, 31, 16, 62, 137, 193, 97, 213, 147, 112, 86, 212, 126, 276, 170, 5),$
$(301, 4, 168, 275, 127, 214, 87, 114, 148, 215, 98, 195, 138, 64, 17, 35, 18, 66, 139, 197, 99, 217, 149, 116, 88, 216, 128, 274, 166, 3),$
$(301, 2, 164, 273, 129, 218, 89, 118, 150, 219, 100, 199, 140, 68, 19, 39, 20, 70, 141, 201, 101, 221, 151, 120, 90, 220, 130, 272, 162, 1)\}.$
This stater comprises a balanced C_{30}-5-foil decomposition of K_{301}.

Theorem 4. K_n has a balanced C_{45}-t-foil design if and only if $n \equiv 1 \pmod{90t}$.

Example 4.1. Balanced C_{45} design of K_{91}.
$\{(91, 3, 54, 85, 37, 62, 25, 32, 44, 63, 29, 57, 41, 17, 4, 9, 5, 19, 42, 59, 30, 65, 45, 34, 26, 64, 38, 84, 52, 2, 51, 49, 50, 83, 39, 66, 27, 36, 46, 67, 31, 61, 43, 21, 6)\}.$
This stater comprises a balanced C_{45}-decomposition of K_{91}.

Example 4.2. Balanced C_{45}-2-foil design of K_{181}.
$\{(181, 6, 108, 169, 73, 122, 49, 62, 86, 123, 56, 111, 80, 32, 7, 15, 8, 34, 81, 113, 57, 125, 87, 64, 50, 124, 74, 168, 106, 101, 105, 4, 104, 167, 75, 126, 51, 66, 88, 127, 58, 115, 82, 36, 9),$
$(181, 3, 102, 166, 76, 128, 52, 68, 89, 129, 59, 117, 83, 38, 10, 21, 11, 40, 84, 119, 60, 131, 90, 70, 53, 130, 77, 165, 100, 2, 99, 97, 98, 164, 78, 132, 54, 72, 91, 133, 61, 121, 85, 42, 12)\}.$
This stater comprises a balanced C_{45}-2-foil decomposition of K_{181}.

Example 4.3. Balanced C_{45}-3-foil design of K_{271}.
$\{(271, 9, 162, 253, 109, 182, 73, 92, 128, 183, 83, 165, 119, 47, 10, 21, 11, 49, 120, 167, 84, 185, 129, 94, 74, 184, 110, 252, 160, 8, 159, 151, 158, 251, 111, 186, 75, 96, 130, 187, 85, 169, 121, 51, 12),$
$(271, 6, 156, 250, 112, 188, 76, 98, 131, 189, 86, 171, 122, 53, 13, 27, 14, 55, 123, 173, 87, 191, 132, 100, 77, 190, 113, 249, 154, 149, 153, 4, 152, 248, 114, 192, 78, 102, 133, 193, 88, 175, 124, 57, 15),$
$(271, 3, 150, 247, 115, 194, 79, 104, 134, 195, 89, 177, 125, 59, 16, 33, 17, 61, 126, 179, 90, 197,$

135, 106, 80, 196, 116, 246, 148, 2, 147, 145, 146, 245, 117, 198, 81, 108, 136, 199, 91, 181, 127, 63, 18)}.
This stater comprises a balanced C_{45}-3-foil decomposition of K_{271}.

Theorem 5. K_n has a balanced C_{60}-t-foil design if and only if $n \equiv 1 \pmod{120t}$.

Example 5.1. Balanced C_{60} design of K_{121}.
{(121, 4, 72, 113, 49, 82, 33, 42, 58, 83, 38, 75, 54, 22, 5, 11, 6, 24, 55, 77, 39, 85, 59, 44, 34, 84, 50, 112, 70, 67, 69, 2, 68, 111, 51, 86, 35, 46, 60, 87, 40, 79, 56, 26, 7, 15, 8, 28, 57, 81, 41, 89, 61, 48, 36, 88, 52, 110, 66, 1)}.
This stater comprises a balanced C_{60}-decomposition of K_{121}.

Example 5.2. Balanced C_{60}-2-foil design of K_{241}.
{(241, 8, 144, 225, 97, 162, 65, 82, 114, 163, 74, 147, 106, 42, 9, 19, 10, 44, 107, 149, 75, 165, 115, 84, 66, 164, 98, 224, 142, 135, 141, 6, 140, 223, 99, 166, 67, 86, 116, 167, 76, 151, 108, 46, 11, 23, 12, 48, 109, 153, 77, 169, 117, 88, 68, 168, 100, 222, 138, 5),
(241, 4, 136, 221, 101, 170, 69, 90, 118, 171, 78, 155, 110, 50, 13, 27, 14, 52, 111, 157, 79, 173, 119, 92, 70, 172, 102, 220, 134, 131, 133, 2, 132, 219, 103, 174, 71, 94, 120, 175, 80, 159, 112, 54, 15, 31, 16, 56, 113, 161, 81, 177, 121, 96, 72, 176, 104, 218, 130, 1)}.
This stater comprises a balanced C_{602}-2-foil decomposition of K_{241}.

Theorem 6. K_n has a balanced C_{75}-t-foil design if and only if $n \equiv 1 \pmod{150t}$.

Example 6.1. Balanced C_{75} design of K_{151}.
{(151, 5, 90, 141, 61, 102, 41, 52, 72, 103, 47, 93, 67, 27, 6, 13, 7, 29, 68, 95, 48, 105, 73, 54, 42, 104, 62, 140, 88, 4, 87, 83, 86, 139, 63, 106, 43, 56, 74, 107, 49, 97, 69, 31, 8, 17, 9, 33, 70, 99, 50, 109, 75, 58, 44, 108, 64, 138, 84, 2, 3, 1, 82, 137, 65, 110, 45, 60, 76, 110, 51, 101, 71, 35, 10)}.
This stater comprises a balanced C_{75}-decomposition of K_{151}.

Example 6.2. Balanced C_{75}-2-foil design of K_{301}.
{(301, 10, 180, 281, 121, 202, 81, 102, 142, 203, 92, 183, 132, 52, 11, 23, 12, 54, 133, 185, 93, 205, 143, 104, 82, 204, 122, 280, 178, 169, 177, 8, 176, 279, 123, 206, 83, 106, 144, 207, 94, 187, 134, 56, 13, 27, 14, 58, 135, 189, 95, 209, 145, 108, 84, 208, 124, 278, 174, 167, 173, 6, 172, 277, 125, 210, 85, 110, 146, 211, 96, 191, 136, 60, 15),
(301, 5, 170, 276, 126, 212, 86, 112, 147, 213, 97, 193, 137, 62, 16, 33, 17, 64, 138, 195, 98, 215, 148, 114, 87, 214, 127, 275, 168, 4, 7, 3, 166, 274, 128, 216, 88, 116, 149, 217, 99, 197, 139, 66, 18, 37, 19, 68, 140, 199, 100, 219, 150, 118, 89, 218, 129, 273, 164, 2, 163, 161, 162, 272, 130, 220, 90, 120, 151, 221, 101, 201, 141, 70, 20)}.
This stater comprises a balanced C_{75}-2-foil decomposition of K_{301}.

Theorem 7. K_n has a balanced C_{90}-t-foil design if and only if $n \equiv 1 \pmod{180t}$.

Example 7.1. Balanced C_{90} design of K_{181}.
{(181, 6, 108, 169, 73, 122, 49, 62, 86, 123, 56, 111, 80, 32, 7, 15, 8, 34, 81, 113, 57, 125, 87, 64, 50, 124, 74, 168, 106, 101, 105, 4, 104, 167, 75, 126, 51, 66, 88, 127, 58, 115, 82, 36, 9, 19, 10, 38, 83, 117, 59, 129, 89, 68, 52, 128, 76, 166, 102, 3, 5, 2, 100, 165, 77, 130, 53, 70, 90, 131, 60, 119, 84, 40, 11, 23, 12, 42, 85, 121, 61, 133, 91, 72, 54, 132, 78, 164, 98, 1)}.
This stater comprises a balanced C_{90}-decomposition of K_{181}.

Theorem 8. K_n has a balanced C_{105}-t-foil design if and only if $n \equiv 1 \pmod{210t}$.

Example 8.1. Balanced C_{105} design of K_{211}.
{(211, 7, 126, 197, 85, 142, 57, 72, 100, 143, 65, 129, 93, 37, 8, 17, 9, 39, 94, 131, 66, 145, 101, 74, 58, 144, 86, 196, 124, 6, 123, 117, 122, 195, 87, 146, 59, 76, 102, 147, 67, 133, 95, 41, 10, 21, 11, 43, 96, 135, 68, 149, 103, 78, 60, 148, 88, 194, 120, 4, 119, 115, 118, 193, 89, 150, 61, 80, 104, 151, 69, 137, 97, 45, 12, 25, 13, 47, 98, 139, 70, 153, 105, 82, 62, 152, 90, 192, 116, 2, 3, 1, 114, 191, 91, 154, 63, 84, 106, 155, 71, 141, 99, 49, 14)}.
This stater comprises a balanced C_{105}-decomposition of K_{211}.

Theorem 9. K_n has a balanced C_{120}-t-foil design if and only if $n \equiv 1 \pmod{240t}$.

Example 9.1. Balanced C_{120} design of K_{241}.

$\{(241, 8, 144, 225, 97, 162, 65, 82, 114, 163, 74, 147, 106, 42, 9, 19, 10, 44, 107, 149, 75, 165, 115,$
$84, 66, 164, 98, 224, 142, 135, 141, 6, 140, 223, 99, 166, 67, 86, 116, 167, 76, 151, 108, 46, 11, 23,$
$12, 48, 109, 153, 77, 169, 117, 88, 68, 168, 100, 222, 138, 133, 137, 4, 136, 221, 101, 170, 69, 90,$
$118, 171, 78, 155, 110, 50, 13, 27, 14, 52, 111, 157, 79, 173, 119, 92, 70, 172, 102, 220, 134, 3, 5, 2,$
$132, 219, 103, 174, 71, 94, 120, 175, 80, 159, 112, 54, 15, 31, 16, 56, 113, 161, 81, 177, 121, 96, 72,$
$176, 104, 218, 130, 1)\}.$
This stater comprises a balanced C_{120}-decomposition of K_{241}.

Theorem 10. K_n has a balanced C_{135}-t-foil design if and only if $n \equiv 1 \pmod{270t}$.

Example 10.1. Balanced C_{135} design of K_{271}.
$\{(271, 9, 162, 253, 109, 182, 73, 92, 128, 183, 83, 165, 119, 47, 10, 21, 11, 49, 120, 167, 84, 185,$
$129, 94, 74, 184, 110, 252, 160, 8, 159, 151, 158, 251, 111, 186, 75, 96, 130, 187, 85, 169, 121, 51,$
$12, 25, 13, 53, 122, 171, 86, 189, 131, 98, 76, 188, 112, 250, 156, 6, 155, 149, 154, 249, 113, 190,$
$77, 100, 132, 191, 87, 173, 123, 55, 14, 29, 15, 57, 124, 175, 88, 193, 133, 102, 78, 192, 114, 248,$
$152, 4, 7, 3, 150, 247, 115, 194, 79, 104, 134, 195, 89, 177, 125, 59, 16, 33, 17, 61, 126, 179, 90,$
$197, 135, 106, 80, 196, 116, 246, 148, 2, 147, 145, 146, 245, 117, 198, 81, 108, 136, 199, 91, 181,$
$127, 63, 18)\}.$
This stater comprises a balanced C_{135}-decomposition of K_{271}.

Theorem 11. K_n has a balanced C_{150}-t-foil design if and only if $n \equiv 1 \pmod{300t}$.

Example 11.1. Balanced C_{150} design of K_{301}.
$\{(301, 10, 180, 281, 121, 202, 81, 102, 142, 203, 92, 183, 132, 52, 11, 23, 12, 54, 133, 185, 93,$
$205, 143, 104, 82, 204, 122, 280, 178, 169, 177, 8, 176, 279, 123, 206, 83, 106, 144, 207, 94, 187,$
$134, 56, 13, 27, 14, 58, 135, 189, 95, 209, 145, 108, 84, 208, 124, 278, 174, 167, 173, 6, 172, 277,$
$125, 210, 85, 110, 146, 211, 96, 191, 136, 60, 15, 31, 16, 62, 137, 193, 97, 213, 147, 112, 86, 212,$
$126, 276, 170, 5, 9, 4, 168, 275, 127, 214, 87, 114, 148, 215, 98, 195, 138, 64, 17, 35, 18, 66, 139,$
$197, 99, 217, 149, 116, 88, 216, 128, 274, 166, 163, 165, 2, 164, 273, 129, 218, 89, 118, 150, 219,$
$100, 199, 140, 68, 19, 39, 20, 70, 141, 201, 101, 221, 151, 120, 90, 220, 130, 272, 162, 1)\}.$
This stater comprises a balanced C_{150}-decomposition of K_{301}.

参 考 文 献

1) Ushio, K. and Fujimoto, H.: Balanced bowtie and trefoil decomposition of complete tripartite multigraphs, *IEICE Trans. Fundamentals*, Vol. E84-A, No. 3, pp. 839–844 (2001).
2) Ushio, K. and Fujimoto, H.: Balanced foil decomposition of complete graphs, *IEICE Trans. Fundamentals*, Vol.E84-A, No.12, pp.3132–3137 (2001).
3) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E86-A, No.9, pp.2360–2365 (2003).
4) Ushio, K. and Fujimoto, H.: Balanced bowtie decomposition of symmetric complete multi-digraphs, *IEICE Trans. Fundamentals*, Vol.E87-A, No.10, pp.2769–2773 (2004).
5) Ushio, K. and Fujimoto, H.: Balanced quatrefoil decomposition of complete multigraphs, *IEICE Trans. Information and Systems*, Vol.E88-D, No.1, pp.19–22 (2005).
6) Ushio, K. and Fujimoto, H.: Balanced C_4-bowtie decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E88-A, No.5, pp.1148–1154 (2005).
7) Ushio, K. and Fujimoto, H.: Balanced C_4-trefoil decomposition of complete multigraphs, *IEICE Trans. Fundamentals*, Vol.E89-A, No.5, pp.1173–1180 (2006).