マッチングを用いたパターン形成アルゴリズム

藤永　直[†1]　小野廣隆[†2]
来嶋秀治[†1]　山下雅史[†1]

本論文では，パターン形成問題と呼ばれる，非同期的に動くロボット群を幾何的な配置に並ばせる問題を考察する．ここでは，各ロボットは各自固有の座標系に従って，他のロボットの配置と目的の配置を観測するできるものとする．n 点の座標からなる集合の対 A, B の間のマッチングを求めるアルゴリズム "clockwise matching" を構成し，"clockwise matching" によって任意のパターン形成問題を解くことができることを示す．

Pattern Formation Through Optimum Matching

Nao Fujinaga,[†1] Ono Hirotaka,[†2] Kijima Shuji[†1]
and Masafumi Yamashita [†1]

A geometric pattern formation problem by autonomous mobile robots is investigated in this paper. In the "embedded pattern formation problem" disscussed in this paper, the target pattern is assumed to be visible from the robots via their own (local) coordinate system. We show that our matching algorithm "clockwise matching" which calculates the matching between a pair of patterns A and B both comprising of n coordinates, solves any embedded pattern formation problem.

1. Introduction

This paper considers a system of anonymous mobile robots on Euclidean plane. Each robot, given an algorithm, repeats a "Look–Compute–Move" cycle, to observe the other robots' positions (in Look phase), to compute the next position by using the algorithm (in Compute phase), and to move toward the next position (in Move phase). The robots are anonymous in the sense that they do not have identifiers (and are not identified just by their looks neither), and are controlled by the same algorithms. A basic and crucial assumption on the system is that they are not aware of the global coordinate system, and all the actions by robots are via their local coordinate systems, which may be inconsistent each other.

The problem of forming a given formation by a set of such robots is called the (geometric) pattern formation problem and has been studied extensively[4)5)3)]. The difficulty of solving formation problem lies on the asynchrony of the robots and at present three synchronous models are considered according to degree of asynchrony; In fully synchronous model (FSYNCH), all robots simultaneously execute the Look-Compute-Move cycles. In asynchrounous model (ASYNCH), on the other hand, we makes no assumption about synchronization. Finally, in semi-synchronous model (SSYNCH) execution of each cycle is assumed to be instanteneous, hence Look and Move phase of two distinct robots never overlap.

Although formable pattern is characterized by[6)] in FSYNCH and SSYNCH model, in ASYNCH model, much is unknown due to difficulity of systematically descriving formation step of every asynchrounous execution of algorithm. Thus in this paper we tacle on the asynchronous robot model (with different visibility assumption).

In these literatures target formation is assumed to be invisible by robots and thus pattern formation problem is formulated as a problem of forming the "shape" of given pattern. In contrust to conventional assumption, in this paper, we assume target pattern is visible like landmarks, via their own coordinate system, hence the target pattern is specified as a set of points on a global coordinate system. We call a formation problem of this setting *embedded pattern formation problem*. Embedded pattern formation problem can be solved by matching between robots and target points if that can be uniquely calculated among robots in spite of their inconsistent local coordinate systems and asynchronous execution of formation algorithm.

In this paper, we construct an algorithm, which we call "clockwise matching" and

†1 九州大学大学院システム情報科学研究院
Graduate School of Information Science and Electrical Engineering, Kyushu University
†2 九州大学大学院経済学研究院
Department of Electrical Engineering and Computer Science, Kyushu University

show that it presents a desired canonical matching between robots and target points during formation step.

Organization.

The rest of paper is organized as follows: In section 2.1, we introduce our model of robots. In section 2.3 we define terminology that we use in this paper. In section 2.2 we define our algorithm "clockwise matching" and present the main theorems. In section 3 we argure that "clockwise matching" is well defined. The proof of the theorems is omitted in this paper.

2. Preliminary

2.1 Model

In this section, we introduce our model of robots. We consider a set of robots A and a set of target points B of equal cardinality. A_p denotes the positions of the robots, and B_p denotes the positions of the target points. (Formally, we call an injection $p : A \cup B \to \mathbb{R}^2$, an embedding. X_p denotes the set $\{p(a) : a \in X\}$ for a set $X \subseteq A \cup B$ and an embedding p.) In the model, each robot asynchronously repeats the following Look-Compute-Move cycle;

Look observes positions of other robots A_p and target points B_p via its own (local) coordinate system T (i.e., T is some mapping consisting of rotation, translation and uniform scaling),

Compute and from input $(A_{T\circ p}, B_{T\circ p})$, computes a perfect bipartite matchings $\psi(A_{T\circ p}, B_{T\circ p})$ between A and B minimizing total Euclidean distance and decides its own target point of B_p by an *algorithm* ψ,

Move and moves directly toward the point with ϵ (or larger) length if there is no other robot(s) along the way.

Note that robots cannot communicate each other, and use distinct local coordinate systems where each robot only knows its own system, hence does not know others' ones. In this paper, each local coordinate system is consist of rotation, translation and uniform scaling, this means that the robots do not have common knowledge about the north. (directions of y-axis of local coordinate systems may not agree.) However, since we do not allow mirror transformation, the robots have common handedness, i.e., bases of their local coordinate system have same orientation. Thus, in order to solve the formation task, they must compute the matching in a way that is independent of local coordinate system as well as asynchrony of execution. However, if there uniquely exists the minimum perfect matching $M = \{a_i b_i \in A \times B : i = 1, \ldots, n\}$ between A and B, minimizing $\sum_{a_i b_i \in M} |p(a_i) - p(b_i)|$ (where $p(a)$ indicates the position of a and $|x|$ is the euclidean norm of x), a formation is easily done. In this paper, we are concerned with symmetrical configurations of A_p and B_p, for which there are more than one minimum perfect matchings between A and B. See e.g. Fig. 1.

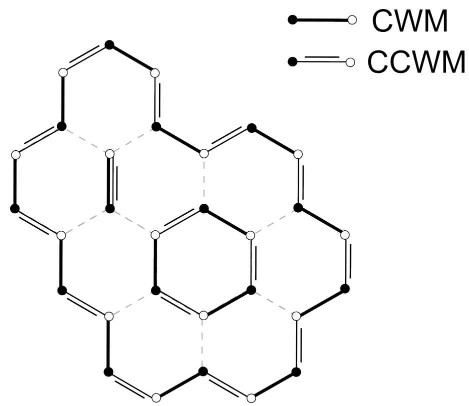

図 1 An example of CWM.

2.2 Terminology

To describe our altorithm "clockwise matching" we define our terminology (c.f.,[1),2)]).

The *frontier* of a set $X \subseteq \mathbb{R}^2$ is the set Y of all points $y \in \mathbb{R}^2$ such that every neighbourhood of y intersect with both X and $\mathbb{R}^2 \setminus X$. Let $O \subset \mathbb{R}^2$ be an open set. The relation of two points being connected by some continuous map $[0,1] \to O$ defines an equivalence relation on O. The corresponding equivalence classes are the regions of O. For any bounded set $X \subset \mathbb{R}^2$, $\mathbb{R}^2 \setminus X$ has unique unbounded region. We call the frontier of the region, the *periphery* of X and denotes it by $P(X)$.

Throughout this paper, we consider a pair of disjoint finite sets A and B of equal cardinality, and an embedding p (i.e., injection $A \cup B \to \mathbb{R}^2$), as well as graph $G = (V, E)$

with a vertex set $V \subseteq A \cup B$, a edge set $E \subseteq A \times B$. We draw vertices of A with black and vertices of B with white in Figures 1-7. The vertex set of a graph G is referred to as $V(G)$ and its edge set as $E(G)$.

K_2 denotes complete graph with 2 vertices. A graph G is *elementary* if the union of all perfect matchings of G is a connected subgraph of G. For a bipartite graph G and its perfect matchings M and M', the cycle C of G is an *alternating cycle of* M *and* M' if the edges of C appear alternately in M and M'.

For $e = ab \in E(G)$ and an embedding p, the *direction* $\overrightarrow{e}_p$ of e under p is the vector $p(b) - p(a)$. the drawing $\overline{e}_p$ of e under p, is the line segment $\{p(a) + t(p(b) - p(a)) : t \in [0, 1]\}$, and $\mathring{e}_p$, the *interior* of e under p, is the set $\overline{e} \setminus \{p(a), p(b)\}$. For a pair of vectors $x, y \in \mathbb{R}^2$, we say x and y to be *parallel* and denotes $x \parallel y$, if there exists $a > 0$ such that $ax = y$.

For a graph G and an embedding p, the *drawing* $\overline{G}_p$ of G under p, is the set $\bigcup_{e \in E(G)} \overline{e}_p$, and the *periphery* $P(G, p)$ of G under p is the subgraph G' of G with the edge set $E(G') = \{e \in E(G) : \overline{e}_p \subseteq P(\overline{G}_p)\}$ ($V(G')$ is all end vertices of the edges). For $Q \subseteq A \times B$, we define $A(Q)$ to be a set $\{a : ab \in Q\}$ and $B(Q)$ to be a set $\{b : ab \in Q\}$.

Let $\mathcal{U}(A, B)$ denotes a set of all perfect matchings of A and B (bijection from A to B). We define cost $w_p(M)$ of the matching $M \in \mathcal{U}(A, B)$ under p by

$$w_p(M) = \sum_{ab \in M} |p(b) - p(a)| \ . \tag{1}$$

Let $\mathcal{M}(A, B, p)$ denotes a set of all matchings $M \in \mathcal{U}(A, B)$ which minimize $w_p(M)$ and for all $x, y \in M$, $\overline{x}_p \nsubseteq \overline{y}_p$. Note that as an element of $\mathcal{M}(A, B, p)$, we do not allow matching whose edge includes its another edge as in Fig. 2 (a), while allowing parallel edges as in Fig. 2 (b). See that $\mathcal{M}(A, B, p) \neq \emptyset$ (except $A = B = \emptyset$), but not necessarily $|\mathcal{M}(A, B, p)| = 1$. $G(A, B, p)$ denotes a bipartite graph with the vertex set $V = A \cup B$ and the edge set $E = \bigcup_{M \in \mathcal{M}(A,B,p)} M$. See Fig. **??** for an example of $G(A, B, p)$.

A tour of a graph G is a sequence of vertices $v_0 v_1 \dots v_k$, such that $v_i v_{i+1} \in E(G)$ for all $i = 0, 1, \dots, k-1$, and hamilton tour is a tour of G which visit all the vertices of G exactly once. For a cycle C and an embedding p, a clockwise tour of C under p is a hamilton tour $v_0 v_1 \dots v_{n-1}$, which minimize $\sum_{i=0}^{n-1} e_i \times e_{i+1 \bmod n}$ with ($\times$) being vector product operator, and $e_i = \overrightarrow{v_i v_{i+1}}_p$. For convinience, we define a clockwise tour

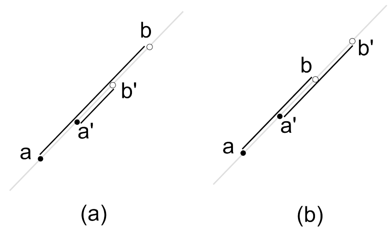

図 2

of a graph $C = K_2$ to be its hamilton tour.

2.3 Definition of CWM

Given a pair of disjoint finite sets A and B of equal cardinality, and an embedding $p : A \cup B \to \mathbb{R}^2$, let $G_1, G_2, \dots, G_N$ be connected components of $G(A, B, p)$, and let $C_i = P(G_i, p)$, $A_i = A(G_i \setminus C_i)$ and $B_i = B(G_i \setminus C_i)$. $\mathrm{CWM}(A, B, p)$ is a matching of $G(A, B, p)$, defined by

$$\mathrm{CWM}(A, B, p) = \begin{cases} \emptyset & \text{if } A = B = \emptyset \\ \bigcup_{i=1}^{N} \left\{ \mathrm{CWM}'(C_i, p) \cup \mathrm{CWM}(A_i, B_i, p) \right\} & \text{otherwise} \end{cases} . \tag{2}$$

where $\mathrm{CWM}'(C, p)$ is a set $\{v_0 v_1, v_1 v_2, \dots, v_{m-1} v_m\}$, with $v_0 v_1 \dots v_m v_0$ being any clockwise tour of C starting from $v_0 \in A$. (See Fig. 1 for illustration of CWM.)

Let A and B be finite sets of equal cardinality, p be an embedding, and T be a coordinate system. Then following theorems holds.

Theorem 1 (optimality)**.** $\mathrm{CWM}(A, B, p) \in \mathcal{M}(A, B, p)$.

Theorem 2 (coordinate system free)**.** $\mathrm{CWM}(A, B, p) = \mathrm{CWM}(A, B, T \circ p)$.

Theorem 3 (asynchrony free)**.** *If an embedding* q *satisfies*

(1) $\forall e \in \mathrm{CWM}(A, B, p), \overline{e}_q \subseteq \overline{e}_p$ *and* $\overrightarrow{e}_q \parallel \overrightarrow{e}_p$.

(2) $\forall x, y \in \mathrm{CWM}(A, B, p), \overline{x}_q \nsubseteq \overline{y}_q$.

then $\mathrm{CWM}(A, B, p) = \mathrm{CWM}(A, B, q)$.

Note that we expressed $\mathrm{CWM}(A, B, p)$ as a matching between A and B to state above theorems, however, it also can be seen as matchng between A_p and B_p, since we do not consider multiplicity, i.e., we only consider injective embeddings.

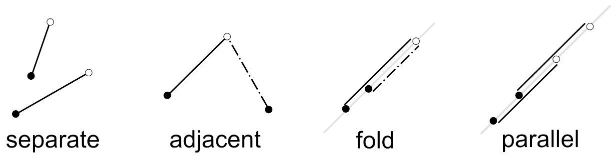

図 3 Possible relation of two edges of $G(A, B)$.

3. Clockwise Matching

In this section, we argue that the "clockwise-matching" $\mathrm{CWM}(A, B, p)$ of A, B and its embedding p, is well defined. To begin with, we analyze a graph $G = G(A, B, p)$.

3.1 Graph $G(A, B, p)$

Though we want to consider G to be a plane graph by replacing each edge e with a line $\overline{e}$ [⋆1], G is not a plane graph in general unfortunately. First, we look at the relation between two edges of G. The following holds even if G is not connected graph.

Lemma 4. *There are four cases between two edges $x = ab$ and $y = a'b'$ of G. See Fig. 3 for illustration.*

- *(separate) $\overline{x} \cap \overline{y} = \emptyset$.*
- *(fold) x and y have exactly one common end vertex and $\overrightarrow{x} \parallel -\overrightarrow{y}$.*
- *(adjacent) x and y have exactly one common end vertex and not (fold).*
- *(parallel) $\overrightarrow{x} \parallel \overrightarrow{y}$ and $\overline{x} \not\subseteq \overline{y}$.*

Proof. We classify the other cases which do not satisfy the above relation as follow. For each case, we derive contradiction. See Fig. 4. Let $x = ab$ and $y = a'b'$ be two edges that are not adjacent and let $M, M' \in \mathcal{M}(A, B)$ be matchings such that $x \in M$, $y \in M'$. Since x and y are not adjacent, $a \neq a'$ and $b \neq b'$.

- Case 1. $M = M'$.
  - (cross) Two lines $\overline{x}$ and $\overline{y}$ cross at a point.
  - (opposite-direction) $\overrightarrow{x} \parallel -\overrightarrow{y}$ and $\overline{x} \cap \overline{y} \neq \emptyset$.
  - (include) $\overrightarrow{x} \parallel \overrightarrow{y}$ and $\overline{x} \subseteq \overline{y}$.
- Case 2. $M \neq M'$.

⋆1 We omit the reference embedding if it is clear.

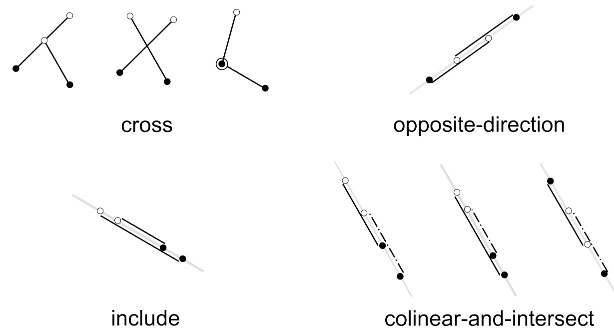

図 4 Impossible relation of two edges of $G(A, B)$.

  - (cross) Two lines $\mathring{x}$ and $\mathring{y}$ cross at a point.
  - (colinear-and-intersect) a, b, a', b' are colinear and $\overline{y} \cap \overline{x} \neq \emptyset$.

Let $\tilde{x} = ab', \tilde{y} = a'b$.

- Case 1. $M = M'$. Let $W = M \setminus \{x, y\} \cup \{\tilde{x}, \tilde{y}\}$. Obviously, $W \in \mathcal{U}(A, B)$.
  - (cross) Let s be the crossing point. By the triangle inequality, $|a-b'|+|a'-b| < |a-s| + |s-b| + |a'-s| + |s-b'| = |a-b| + |a'-b'|$. Thus $w(W) = w(M \setminus \{x, y\} \cup \{\tilde{x}, \tilde{y}\}) = w(M) - \big\{|a-b|+|a'-b'|\big\} + \big\{|a-b'|+|a'-b|\big\} < w(M)$. This contradict with $M \in \mathcal{M}(A, B)$.
  - (opposite-direction) Same as the case (cross).
  - (include) This contradict with $M \in \mathcal{M}(A, B)$.
- Case 2. $M \neq M'$.
  - (cross) Let $C = (M \oplus M') \setminus \{x, y\} \cup \{\tilde{x}, \tilde{y}\}$. Then, each connected component of C forms an alternating cycle and $w(C) < w(M \oplus M')$ by the same argument as the case (cross) of Case 1. Thus, there exists $W, W' \in \mathcal{U}(A, B)$ such that $w(W \oplus W') < w(M \oplus M')$ and $M \cap M = W \cap W'$. This is because, as edges of W and W', you can take edges of the alternating cycle alternately for each connected component of C and for the rest of the W and W', you can take edges both from $M \cap M'$. Thus, $w(W) + w(W') = w(W \oplus W') + 2w(W \cap W') < w(M \oplus M') + 2w(M \cap M') = w(M) + w(M')$. Furthermore, since M and M' are optimum matchings, $w(M) = w(M')$. Therefore, $w(W) < w(M)$ or

$w(W') < d(M)$. This contradict with $M \in \mathcal{M}(A, B)$.

– (colinear-and-intersect) In this case, though each connected component of $M \oplus M'$ must form alternating cycle, this is impossible because two edges in the same matching cannot have relation (cross) or (opposite-direction) as we proved in Case 1.

□

We consider the connected component of G. When G is connected, let's see, what kind of graph we can draw on the plane. To begin with edge e_1, by Lemma 4, as the next edge e_2, we can draw edge which have either (adjacent) or (fold) relation with e_1. If we choose (fold), the next edge e_3 must be the (parallel) relation with e_1. Then, for the next edge e_4, you can choose (adjacent) or (fold). However in case you choose (fold) you have to be careful not to draw the line too long and include e_2, and so on, and of course any of two edges could never cross each other.

With that observation, let us define the plane graph representation $D(G)$ of G as follow. We call an alternating path $a_1b_1 \ldots a_mb_m$ of G which satisfy $a_{i+1} \in \overline{a_ib_i}$ and $b_i \in \overline{a_{i+1}b_{i+1}}$ for all $i = 1 \ldots m-1$, a *folded-path*. Any edge is a folded-path with length 1. A *maximal folded-path* is a folded-path, which by extending the path with one more vertex, no longer holds above condition. $D(G)$ is a plane graph which is produced by replacing each maximal folded-path aPb of G with a line $\overline{ab}$. See that those two lines never intersect with each other except for end points and for any perfect matching of $D(G)$, there is a corresponding perfect matching of G since each aPb is an alternating path without branch from inner vertices. From the definition, Lemma 4 and the above arguments, we obtain the following Corollaries.

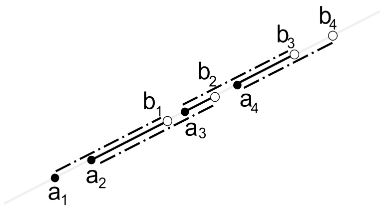

図 5 An example of folded-path.

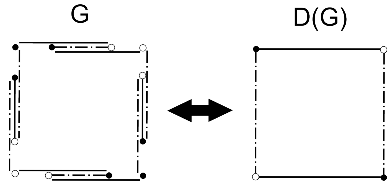

図 6 An example of G and $D(G)$.

Corollary 5. *G is bipartite elementary.*

Corollary 6. *$D(G)$ is plane bipartite elementary.*

We also remark the followings.

Theorem 7.[2] *Elementary bipartite graph is 2-connected.*

Theorem 8.[1] *Any face of 2-connected plane graph with more than 4 vertices is bounded by a cycle of the graph.*

The geometrical property defined on plane graph $D(G)$ is naturally lifted to the original graph G. i.e., for the periphery $x_0x_1 \ldots x_{m-1}x_0$ of $D(G)$, as there is unique maximal folded-path P_i of G that connect x_i and $x_{i+1 \bmod m}$ and $\overline{P_i} = \overline{x_ix_{i+1}}$, we can see that the periphery of G is the cycle $x_0P_0x_1P_1 \ldots x_{m-1}P_{m-1}x_0$ and is orientable. Thus the periphery $P(G)$ of G as well as clockwise tour of $P(G)$ is well defined. By the above theorems, we can say that the periphery of any connected component of G is either K_2 or an alternating cycle.

4. Conclusion

In this paper, we considered formation problem by asynchronous autonomous mobile robots and presented the algorithm "clockwise matching" which presents a kind of canonical matching in our formation step.

Our robot model assumed that each robot can observe positions of other robots and target points via its own local coordinate system whose direction of y axis may not agree while orientation agree with other robots' systems.

By "clockwise matching", robots always calculate matching between the robots and the target points which minimize the sum of distances. "clockwise matching" takes advantage of their common knowledge about orientation and precedes "clockwise" matching to "counter-clockwise" ones if there is more than one minimum weight matching.

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