On Generation, Storage and Processing of Graphics

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This paper is concerned with on-line man-machine communication. It is impossible now to expect the computers to have the creative power, while it is possible to enhance man's creativity with the aid of high speed and accurate data-processing power of computers. When computers matche to human capabilities in their communication mode and response time, an optimum man-machine system will be able to be built. With these considerations, we have made some experiments on graphic communication, manipulation and storage. Some of their results and devices used are as follows;

[1] The digital hybrid computer (incremental-whole number combined general purpose computer) and its input and output devices.

[2] The simulation of human process in drawing graphics with rulers and compasses.

[3] The display of three dimensional objects and its manipulation, such as the rotation and the perspective projection.

[4] The expression of arbitrary curves and their storage.

This paper is the first step of our research on man-machine interaction to enhance qualities and to have flexibility in automated systems in the future.

1. *Devices*

The machine used for this study is the digital hybrid computer which was

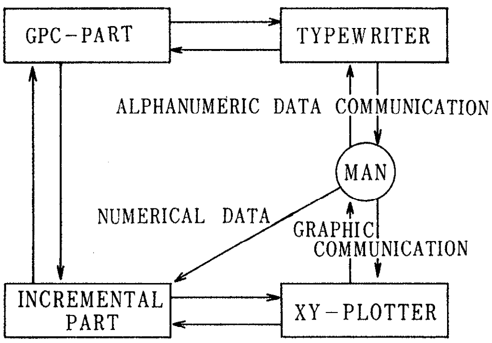

Fig. 1. The digital hybrid computer system.

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designed by one of the authors. It consists of stored program general purpose computer (GPC) combined with incremental computing part and the incremental input-output devices. Schematic diagram of the system is shown in *Fig. 1*. The incremental computing part has more extended functions than the usual digital differential analyzer (DDA), especially in the sequential control abilities. It is made of mainly magneto-strictive delay lines with one mega-cycle clock rate and contains 28 digital integrators which are also used as extended decision and non-linear-elements. It also contains 28 digital potentiometers. The outputs of each elements can be distributed to other elements, XY plotters and GPC part as control signals. Simultaneously each element can receive output pulses from the graphic input devices. The GPC part consists of the magnetic core memory and the electro-magnetic delay line registers. Each part can operate independently or work together, emitting interrupting signals and transferring data and instructions.

The functions of the GPC part are to supervise general main flow and to perform complex calculation and conversion, which the incremental part is not suitable for. The incremental part generates mainly continuous quantities and control signals concerning those continuous data.

An incremental XY plotter is used for drawing output data in the automatic mode, while, during the operation of the manual mode, error detecting on-off switch attached to the pen-holder of this XY plotter senses the hand motion and the signal of which is fed to the incremental part, driving the XY plotter to the position where error signal eliminates. Thus the XY plotter works as a graphic I/O device and the pen position is always registered dynamically in the computer registers. The motor driving signals show the incremental path of the pen, and the pen-up and pen-down signals and other interrupt signals are also fed to the computer.

The reasons to adopt such instruments as man-machine interface devices are as follows;

[1] We have neither CRT display with a light pen nor a suitable computer as well as budget to afford them. We had to make the I/O devices and computer almost in our laboratory.

[2] Positioning accuracy of the incremental XY plotter-pen is higher than in the CRT system with a light pen.

[3] The display on a sheet of paper requires no regeneration as in CRT, so storage capacity is kept minimum.

[4] There is neither DA nor AD conversion. Every quantity is discrete and is easy to handle by digital computer.

[5] Because of the apparent parallel operation of the incremental computer and the driving capability of plural XY plotters, the related figures and movement are obtained simultaneously. This greatly serves human understandings of three dimensional problems.

The disadvantages of the using of these mechanical devices instead of the CRT-light pen combination is in their slow speed and non-erasability.

The special capabilities of the incremental part and its application is written in the other papers.[1], [2], [3]

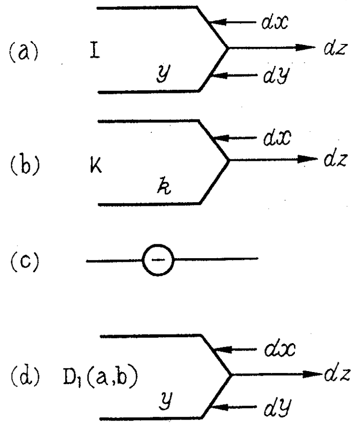


Fig. 2. The computing elements.

So details are omitted, and its computing elements and their symbols are shown in *Fig. 2.*

When $a=b=0$ and dx=maximum rate, $D_1(a, b)$ becomes as the servo amplifier. Other various uses of decision elements are shown in the ref. [2].

2. *Fundamental Graphic Manipulation*

Constructing geometric figures, at first man usually makes an image in his mind that he wants to draw. Then he draws circles, lines and curves as he conceives, controlling the devices such as compasses, rulers and a scale, and using his fundamental subroutines at will. He does not want to use computers not only in a simple problem but also in a complexed one. This is because the usual computers lack in graphic input-output and on line communication between man and computer and because they require the problem solving methods which are not accustomed to users.

In considering these situations, we decided for the first step to construct the system to give a man such aids as not so different from his usual habitude, and to give him clear understandings of the images which are difficult to make in his mind. So the computer is made to have characteristics stated as follows;

[1] The information of the graphics is supplied to the computer in graphic form directly, not in numerical value. When necessary, its descriptive characteristics and processing commands are given from the typewriter keys and other buttons. The responses are also displayed in graphic forms.

[2] The computer has basic processing routines. Their combination and the execution are directed on line and the sequences of the process are determined by a man during his use of the system.

[3] The form of the output is converted in several ways so as to ease human understandings. For example, the rotation, the translation, the expansion, the contraction and the perspective representation.

2.1. *The basic routines of rulers and compasses and the intersection of curves*

a. Rulers: Pressing a button to indicate to store the coordinates, two points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ are given on a sheet of paper with the pen attached to the XY plotter which is in manual mode. Then the XY plotter is changed to automatic mode and dt pulses are supplied. When accumulated dx and dy pulses becomes x_2-x_1 and y_2-y_1 respectively, the supply of dt pulses is stopped. The line drawing routine is shown in *Fig. 3. a.* The equations are;

$$dx=(x_2-x_1)dt$$
$$dy=(y_2-y_1)dt$$

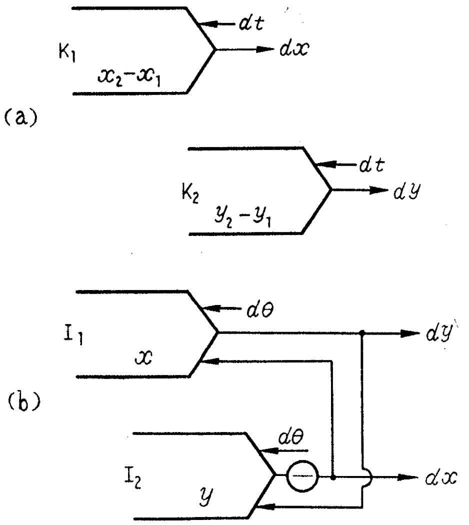


Fig. 3. Line and circle generator.

When one point and direction angle of a line are given, values of x_2-x_1, y_2-y_1 in the computing elements are substituted by $\cos\theta$ and $\sin\theta$ respectively, which can be calculated by the circle drawing routine.

b. Compasses: The center point and the radius or the point on the circumference are specified by the pen and buttons. The connection of the incremental computing elements is shown *Fig. 3. b.* The equation of the computing elements are;

$$dx=-r\sin\theta\cdot d\theta=-(y_2-y_1)d\theta$$
$$dy=r\cos\theta\cdot d\theta=(x_2-x_1)d\theta$$

where (x_1, y_1) is the center point, and (x_2, y_2) is the point on the circumference. The angle of the arc is detected by the decision element, comparing $\int d\theta$ with θ_0, where the angle θ_0 is given by the typewriter or by a specified angle with the horizontal line graphically.

The routine "compasses" is also used in the resolver or the coordinate rotation routine.

c. *The intersection:* The intersection of two curves is obtained in the incremental computer by minimizing the distance of the two points on each curve or by satisfying equations of one curve with running point on the other curve. These methods are described schematically in *Fig. 4.* Mathematically the former corresponds to the minimizing process of two-parameter function. The latter is that of one-parameter function. The reason of not using the GPC part is that the initial conditions of the incremental calculation are introduced by pointing the curves by the pen and that when the curves move, their intersection can be tracked with high speed.

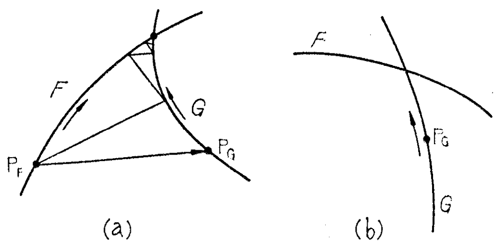


Fig. 4. Intersection of two curves.

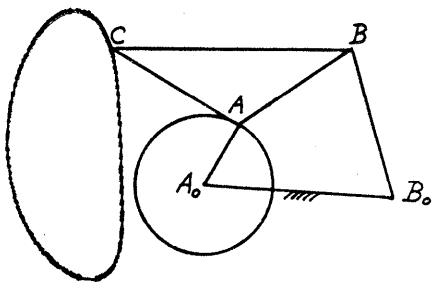


Fig. 5. A four-bar linkage.

2. 2. *The examples of the applications*

The motion of four-bar linkages are obtained using the routines described above. Usually, the design process of the linkages is tedious, because of many parameters involved, such as the length of the four bars and the position of the coupler point. In *Fig. 5,* the point A is on the circumference of the circle centered at A_0. The point B is fixed as the intersection of the two circles, the one defined by the center A and the radius $\overline{AB}$, and the other defined by the center B_0 and the radius $\overline{BB_0}$. The coupler point C is also determined by the fixed triangle ABC. The problem is to construct such a four-bar linkage that the point C has the specified motion. With the routines and their combinations, the parameters are easily changed, the points being specified by the pen, and the locus

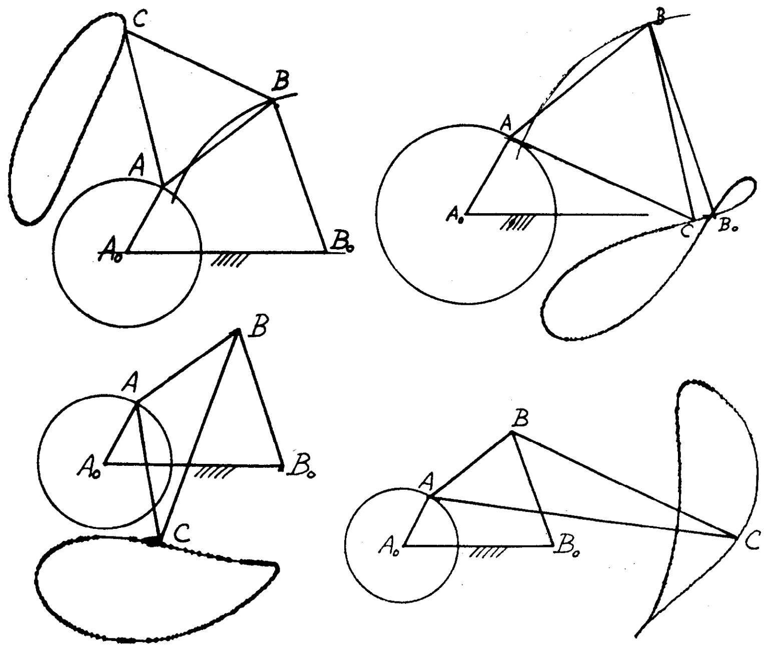


Fig. 6. The coupler curves.

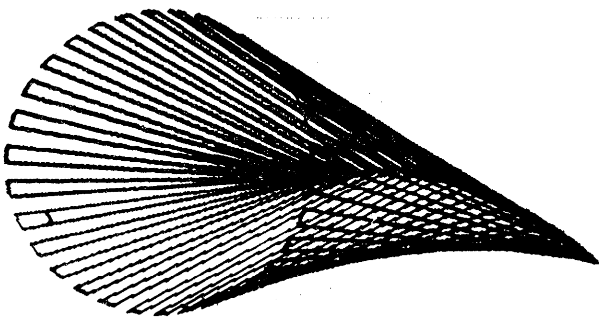


Fig. 7. The motion of the connecting rod of a four-bar linkage.

of the motion is drawn on the XY plotter. Some of them are shown in *Figs. 6* and *7*. Not only the motion of the points but also the motion of the rods can be obtained. Moreover, the display of the solution is real time; the movement of the pen which simulates the motion of the point gives real feeling to the user.

3. *The Display and the Conversion of Three Dimensional Objects*

The rotation and several kinds of projections, such as the orthogonal, the isometric and the perspective projections, are necessary for us to handle the three dimensional objects.

3. 1. *The rotation*

The process, rotating a object about an axis fixed to it, can be interpreted as follows (*Fig. 8*). First, the space fixed coordinante system S_0 is rotated about its z axis by the angle ψ, this system denoted by S_1, which is again rotated about its y axis by the angle θ. Then the original x axis is coincident with the specified axis of the rotation. This system is S_2. Now considering an object in the fixed co-ordinate system S_3, which is at first coincident with the system S_2. Then the object is rotated around the axis by the angle ϕ. This is equivalent to the rotation of S_2 by the angle $-\phi$, if seen from S_3. Then S_2 is reversed to original system S_0, through S_1. Writing the transformation matrix from S_i to S_j by $[T_{ji}]$, the rotation matrix about the fixed axis is given by;

$$[R]=[T_{01}]\cdot[T_{12}]\cdot[T_{32}]\cdot[T_{21}]\cdot[T_{10}]=[T_{10}]^{-1}\cdot[T_{21}]^{-1}\cdot[T_{32}]\cdot[T_{21}]\cdot[T_{10}]$$

$[T_{j+1,j}]$ can be calculated easily within the modified circle routine. *Fig. 9* is the process or the program in the incremental computer. The unit vector component $(1, 0, 0)$ in S_0 is converted to other coordinate system by successive rotation about one coordinate axis. With this routine, one can obtain first column of the general rotation matrix, the other column can be obtained by putting initial condition $(0, 1, 0)$ and $(0, 0, 1)$. This rotation matrix is denoted by $[R]$. Then the original vector P_0 is transformed to P_1.

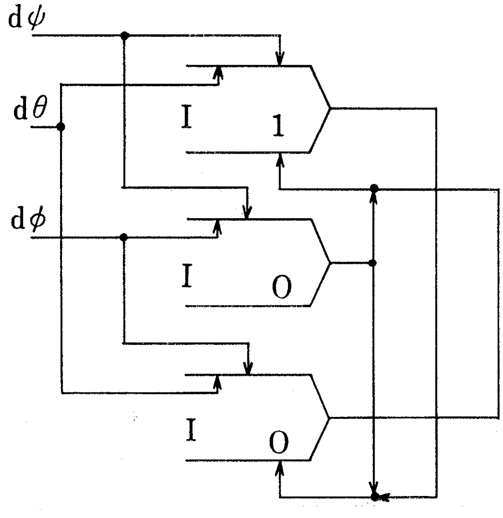


Fig. 8. The rotation around an axis L.

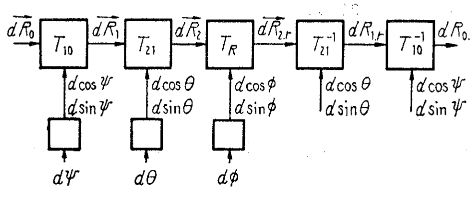
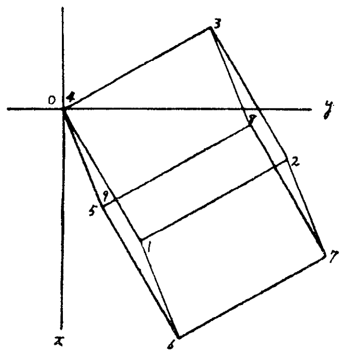


Fig. 9. The process of rotation.



$\theta=4=\phi=30°$

Fig. 10. The rotation of a cube.

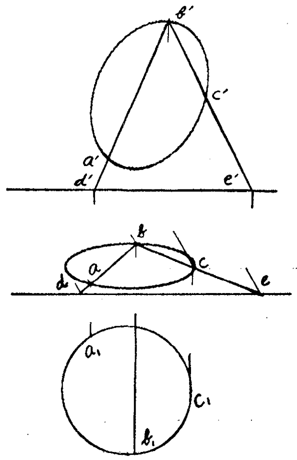


Fig. 11. The projection of a circle, passing through three points in the space.

$$P_1=[R]\cdot P_0$$

a. Example 1

Fig. 10 shows the rotating of a cube.

b. Example 2

Fig. 11 shows a circle which passes through the three points A, B and C, given in the space. The procedure to draw the figures resembles that in the descriptive geometry, except we use the routines of compasses and rulers instead of "true" compasses and rulers.

3.2. *The perspective views*

The perspective views of an object can be obtained as the intersection of a vertical plane and a line of sight. The vertical plane is defined as yz plane with horizontal y axis and vertical z axis. The x axis is perpendicular to this plane.

Let the point on the body, (x_b, y_b, z_b) and the location of the eye, (x_e, y_e, z_e). The point on the perspective, (x, y, z) is determined from the next equations.

$$(x_e - x_b)dy + y_e dx_b - x_e dy_b = 0$$
$$(x_e - x_b)dz + z_e dx_b - x_e dz_b = 0$$

This process is shown in *Fig. 12,* and some example are in *Figs. 13, 14* and *15.*

4. *The Storage and the Projection of Arbitrary Curves*

In the manual mode, the pen attached to the XY plotter is used to trace or draw arbitrary curves. The given curves are approximated by some equations, to reduce the amount of data to be stored. In this paper, a part of the curves is substituted by a circular arc or a quadratic polynomial. More sophisticated approximation will be given in the near future, when the approximation methods of space curves and surfaces are introduced.

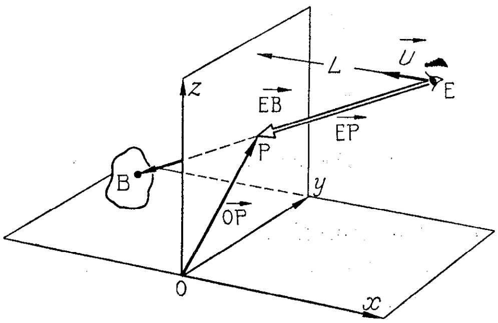


Fig. 12. The process of the perspective view.

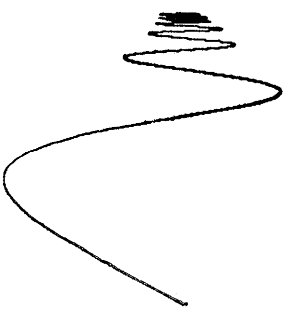


Fig. 13. The example of the perspective (1).

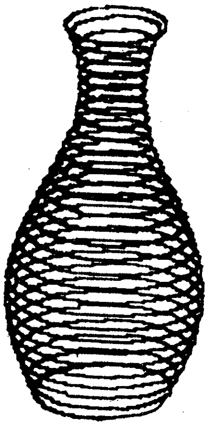


Fig. 14. The example of the perspective (2).

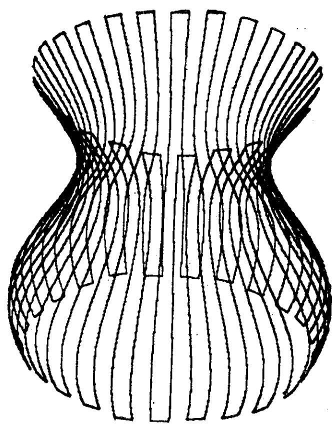


Fig. 15. The example of the perspective (3).

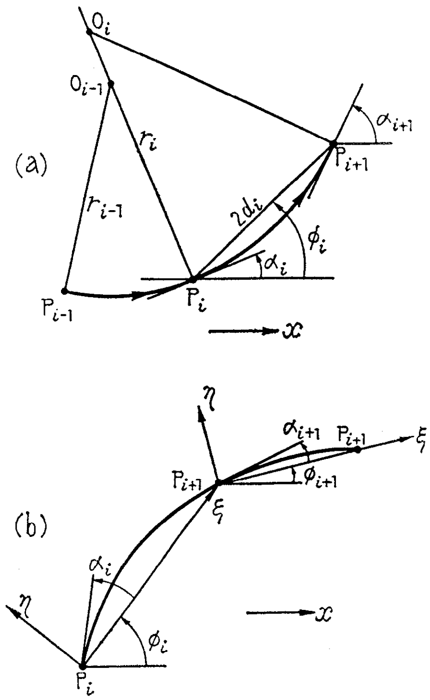


Fig. 16. The approximation of a curve.

4.1. *The circular arc approximation*

Given the chord length $2d_i$, the direction angle ϕ_i, and the angle of the tangent vector α_i, (*Fig. 16*) the radius of the circle is;

$$r_i=\left|\frac{d_i}{\sin(\alpha_i-\phi_i)}\right|$$

To draw a smooth curve, the angle α_i is;

$$\alpha_i=2\phi_{i-1}-\alpha_{i-1}$$

or $\alpha_i=\pi+2\phi_{i-1}-\alpha_{i-1}$ (if the point is a cusp)

The circle generator is called to draw the arc, in which the sign of $d\theta$ is determined as;

if $\alpha_i-\phi_i>0$ then clockwise
if $\alpha_i-\phi_i<0$ then counterclockwise
if $\alpha_i-\phi_i=0$ then a straight line is drawn.

4.2. *The quadratic polynomial approximation*

The arc $\widehat{P_iP_{i+1}}$ is approximated by a quadratic polynomial with respect to $\xi-\eta$ coordinate system, which is transformed to $x-y$ coordinate system.

$$\begin{pmatrix} x \\ y \end{pmatrix} = 2d_i \begin{pmatrix} \cos\phi_i & -\sin\phi_i \\ \sin\phi_i & \cos\phi_i \end{pmatrix} \begin{pmatrix} \xi \\ -1/2 \tan\alpha_i \cdot \xi(\xi-1)/2 \end{pmatrix}$$

where the angle of the tangent vector at P_i is given

$$\alpha_i = \phi_{i-1} - \phi_i - \alpha_{i-1}$$

or $\alpha_i = \pi + \phi_{i-1} - \phi_i - \alpha_{i-1}$ (if the point is a cusp)

The block diagram of simulated cuve generating process is shown in *Fig. 17.*

There are some other routines to handle the discontinuity of the curves and the modification of the generated curves. The example is shown in *Fig. 18,* which is a simulated hand written character, with fifteen points and the initial tangent vector given.

5. *Conclusion*

The on-line graphic communication and the processing methods are tried to en-

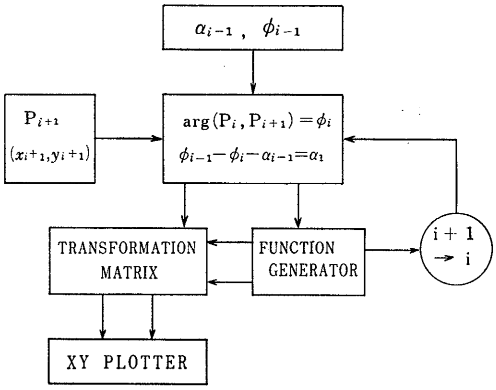


Fig. 17. The approximated curve generator.

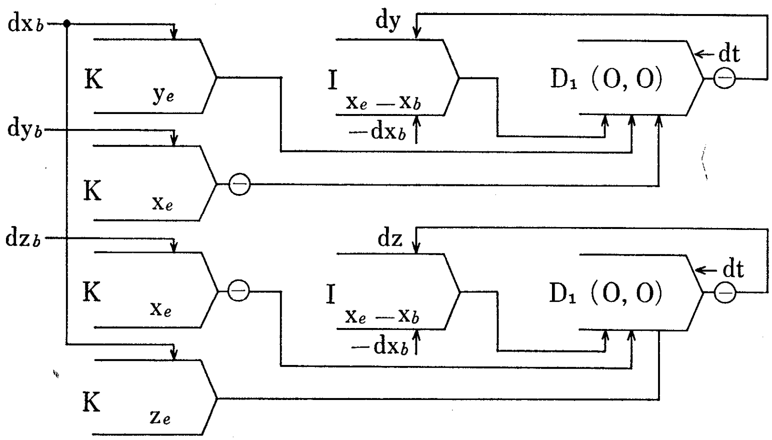


Fig. 18. The approximation of a hand written character.

hance man's activity, with a small scale computer and special I/O devices. The results obtained are;

[1] The method of using compasses and rulers is simulated by the computer, while the user decides the sequence of using those "tools". This method also includes the graphic input, the calculation of the intersection, the change of the scale and the rotation.

[2] Three dimensional display and processing such as the rotation, the projection and the perspective view are also performed by the incremental computer. They are very useful for man's perception of three dimensional objects.

[3] Kinematics motion is indicated dynamically. This gives real feeling aids, serving design engineer.

[4] Arbitrary curves are stored with small amount of the memory space. Those curves can be easily modified and regenerated.

[5] It is shown that the digital hybrid computer is very powerful to those kinds of works.

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