Skip System for Time-Shared Computer Scheduling

AKIHIRO HASHIMOTO* AND TADAO WAKAYAMA*

Abstract

A Round-Robin (RR) system is well-known as a descipline for assigning processor time to many requests in a time-shared computer system. But this system lacks in flexibility. In this paper, a more flexible one named Skip system, which is a slight extension of a RR system and is easily implemented, is proposed and analyzed. In a RR system a request is served in its every cycle, but in this skip system a request may or may not be served in its cycle according to its past history.

1. *Introduction*

The simplest scheduling algorihm in a time-shared computer system is a Roud-Robin (RR) system, which has been analyzed several times, and it is known that the response time of a request is linealy dependent on its processing time. A system such as Multi-Level priority Queue (MLPQ) system in the CTSS [1] is more practical in order to assign a higher priority to a request whose processing time is short and in this case the average wait time is known to become shorter [2].

Genelalizing a RR system, a system in which the i-th quantum q_i of a request can be varied according to the amount of processing time previously received has been also proposed. Let it be called Variable Time Quantum (VTQ) system. This VTQ system is more flexible than a RR system, in which the request whose processing time is short can be given a higher priority. But the VTQ system is too general and it is not so easy to know its characteristics from the results of its analysis [3].

The Skip system proposed and analyzed here is a restricted VTQ system such that q_i equals zero or constant. Nevertheless the characteristics of the system can be varied over a wide range and the rule of management of its queue is almost the same as that in a RR system and becomes very simpler than that in MLPQ system.

2. *Analysis of Skip system*

We define a cycle as a process that a request joining the end of the queue

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* The Electrical Communication Laboratory of Nippon Telegraph and Telephone Public Corporation, Tokyo.

receive a certain time (including zero) of service and leaves the system or returns again to the end of the queue.

Let a VTQ system in which the quantum q_i of the i-th cycle is given by

$$q_i=\begin{cases} q, & (i\in S) \\ 0, & (i\notin S) \end{cases}$$

be called a Skip system, where S is a set of a possibly infihite number of predetermined positive integers as follows :

$$S=\{s_1,\ s_2,\ \cdots\cdots\cdots\},\qquad s_i:\text{ positive integer}$$
$$1\leq s_1\leq s_2\leq\cdots\cdots\cdots$$

When $i\in S$, let its cycle be a processing cycle, otherwise a null cycle.

The following analysis of the Skip system is an extension of the analysis of a RR system by Shemer [3] and some of the value estimation given below are somewhat approximate.

It is reasonable to assume that the arrival of requests is a Poison process with arrival rate λ, since there are an infinite number of independent users, and moreover we assume that the processing time of the requests has a negative exponential distribution with parameter μ. Consider the variable λ and μ to be statistically independent, and μ to be independent of the servicing descipline, and also maximum queue size to be unlimited. Furthermore, we neglect an overhead time and swapping time.

Utilizing these assumptions, the probability that a service is completed (namely, the number of requests in the system reduces from n to $n-1$) on one of the requests in a small time increment Δt is $\mu\Delta t$, and similarly the probability of an arrival (namely, from n to $n+1$) in Δt is $\lambda\Delta t$. Therefore, the steady-state probaility P_n that n requests are concurrently in the system is

$$P_n=\rho^n(1-\rho),\qquad n=0,1,2\cdots\cdots\cdots\ ;\ \rho=\lambda/\mu<1, \tag{1}$$

and the expected number of requests $E(n)$ is

$$E(n)=\sum_{n=0}^{\infty}nP_n=\rho(1-\rho)\frac{d}{d\rho}\sum_{j=1}^{\infty}\rho^j=\frac{\rho}{1-\rho}. \tag{2}$$

The number N of processing cycles required to finish the service of a request whose processing time is t is given by

$$(N-1)\ q<t\leq Nq,\quad N:\text{ positive interger.} \tag{3}$$

Letting s_N be the N-th element of S, the total number M of cycles is

$$M=s_N. \tag{4}$$

The expected value $E(R_M)$ of the response time R_M is determined by the expected wait time W_M required to receive M cycles and processing time t as follows :

$$E(R_M)=W_M+t. \tag{5}$$

Let τ_i be the expected wait time in the i-th cycle, then W_M is given by

$$W_M=\sum_{i=1}^{M}\tau_i. \tag{6}$$

Let η be the rate of the number of processing cycles to the total number of cycles. Then we have

$$\eta=\sum_{i=1}^{\infty}i\int_{(i-1)q}^{iq}\mu e^{-\mu t}dt\Big/\sum_{i=1}^{\infty}s_i\int_{(i-1)q}^{iq}\mu e^{-\mu t}dt. \tag{7}$$

The expected value $\bar{q}$ of the time length q' really used when a quantum q is assigned to a request is given by

$$\bar{q}=\frac{\text{the average processing time}}{\text{the average number of cycles}}$$

$$=\frac{\sum_{i=1}^{\infty}\int_{(i-1)q}^{iq}\mu te^{-\mu t}dt}{\sum_{i=1}^{\infty}s_i\int_{(i-1)q}^{iq}\mu e^{-\mu t}dt}=\frac{\eta}{\mu}(1-e^{-\mu q}). \tag{8}$$

The probability that when a request J arrived at the system the left time of quantun q of the request J_0 which was being processed is r is given by

$$\int_{q-r}^{\infty}\mu e^{-\mu t}dt\Big/\int_{0}^{q}ds\int_{s}^{\infty}\mu e^{-\mu t}dt=\frac{\mu e^{-\mu(q-r)}}{1-e^{-\mu q}}.$$

Therefore, the expected value $\bar{q}_o$ of the time length q_o in which the servive of J_o in its processing cycle is completed is given by

$$\bar{q}_o=\int_{0}^{q}\frac{\mu e^{-\mu(q-r)}}{1-e^{-\mu q}}\left[\int_{0}^{r}t\mu e^{-\mu t}dt+\int_{r}^{\infty}r\mu e^{-\mu t}dt\right]dr=\frac{1}{\mu}-\frac{qe^{-\mu q}}{1-e^{-\mu q}}. \tag{9}$$

Utilizing the eq. (7), the average number of requests waiting in the queue of length n which will receive services in their cycle is

$$\left.\frac{dG_n(x)}{dx}\right|_{x=1}\equiv\left.\frac{d}{dx}(1-\eta+x\eta)^n\right|_{x=1}. \tag{10}$$

Let τ_i be the conditional expected wait time of a request between finishing the $(i-1)$-*st* cycle and initiating the i-th cycle, given that the request has undergone $(i-1)$ cycle allocations. Here, τ_1 means the expected value of the wait time t_1 of a newly arriving reqeust J in its first cycle. Then

$$\tau_1=E(t_1)=E\left(\sum_{n=1}^{\infty}P_nq_o+\left.\frac{d}{dx}\sum_{n=2}^{\infty}P_nq'G_n(x)\right|_{x=1}\right)=\rho\bar{q}_o+\frac{\rho^2\bar{q}}{1-p}. \tag{11}$$

Next, τ_2 of the request J in its scond cycle can be given as follows utilizing the number of requests in the system. Namely, the time required for the first cycle of J is (t_1+q_1) and the average number of new requests which will arrive during this time is $(t_1+q_1)\cdot\lambda$. The probability that the service of a request J_o which was being serviced when a request J arrived is not completed and J_o is returned to the end of the queue is given by

$$\int_{0}^{q}\frac{\mu e^{-\mu(q-r)}}{1-e^{-\mu q}}-\left[\int_{r}^{\infty}\mu e^{-\mu t}dt\right]dr=\frac{\mu qe^{-\mu q}}{1-e^{-\mu q}}. \tag{12}$$

The average number of requests which were in the queue of length n and are returned to the queue again is

$$\left.\frac{dH_n(x)}{dx}\right|_{x=1}\equiv\left.\frac{d}{dx}\left\{\eta(1-e^{-\mu q})+x(1-\eta+\eta e^{-\mu q})\right\}^n\right|_{x=1}=n(1-\eta+\eta e^{-\mu q}). \tag{13}$$

Thus we have

$$\tau_2=E\left\{q'\left((t_1+q_1)\lambda+\sum_{n=1}^{\infty}P_n\frac{\mu qe^{-\mu q}}{1-e^{-\mu q}}+\frac{d}{dx}\sum_{n=2}^{\infty}P_nH_{n-1}(x)\Big|_{x=1}\right)\right\}$$

$$=\bar{q}\left[(\tau_1+q_1)\lambda+\frac{\rho\mu qe^{-\mu q}}{1-e^{-\mu q}}+\frac{\rho^2}{1-\rho}(1-\eta+\eta e^{-\mu q})\right]=\tau_1\gamma+\delta_1+\varepsilon, \tag{14}$$

where

$$\gamma=\lambda\bar{q}+(1-\eta)+\eta e^{-\mu q}$$
$$\delta_1=\lambda q_1\bar{q}$$
$$\varepsilon=\eta\rho e^{-\mu q}(q-\bar{q}_o)-(1-\eta).$$

In the case of τ_i $(i\geq 3)$, considering the average length of the queue in the $(i-1)$ -*st* cycle to be $\tau_{i-1}/\bar{q}$, we have

$$\tau_i=\bar{q}\left[(\tau_{i-1}+q_{i-1})\lambda+\frac{\tau_{i-1}}{\bar{q}}(1-\eta+\eta e^{-\mu q})\right]=\gamma\tau_{i-1}+\delta_{i-1},\ (i\geq 3), \tag{15}$$

where

$$\delta_i=\lambda q_i\bar{q}=\begin{cases}\lambda q\bar{q}, & (i\in S)\\ 0, & (i\notin S)\end{cases}$$

Rewriting the eq. (15), we have

$$\tau_i=\tau_1\gamma^{i-1}+\varepsilon\gamma^{i-2}+\delta_1\gamma^{i-2}+\delta_2\gamma^{i-3}+\cdots+\delta_{i-1},\ (i\geq 2). \tag{16}$$

From these results, we have

$$W_M=\sum_{i=1}^{M}\tau_i=\frac{\tau_1(1-\gamma^M)}{1-\gamma}-\frac{\varepsilon(1-\gamma^{M-1})}{1-\gamma}+\frac{\delta_1(1-\gamma^{M-1})}{1-\gamma}+\frac{\delta_2(1-\gamma^{M-2})}{1-\gamma}+\cdots+\delta_{M-1}. \tag{17}$$

3. *An example of Skip system*

When $S=\{s_1=1, s_2, \cdots\}$ is given, let a_n be given by

$$a_n=s_{n+1}-s_n,\ (n=1, 2,\cdots\cdots)$$

and let such a system be called a_n-Skip system. A 1-Skip system is a RR system.

Now, rewriting the above mentioned results, we have

$$E(R_M)=W_M+t=\sum_{i=1}^{M}\tau_i+t=\frac{\tau_1(1-\gamma^M)}{1-\gamma}+\frac{\varepsilon(1-\gamma^{M-1})}{1-\gamma}$$
$$+\frac{\delta}{1-\gamma}(N-1-\sum_{i=1}^{N-1}\gamma^{M-s_i})+t, \tag{19}$$

where

$$M=s_N,\qquad \eta=(1-e^{-\mu q})^{-1}\cdot(1+\sum_{i=1}^{\infty}a_ie^{-i\mu q})^{-1},\qquad \rho=\frac{\lambda}{\mu}$$

$$(N-1)q<t\leq Nq,\quad \gamma=\lambda\bar{q}+(1-\eta)+\eta e^{-\mu q},\qquad \bar{q}_o=\frac{1}{\mu}\left(1-\frac{1}{\mu q}(1-e^{-\mu q})\right)$$
$$\delta=\lambda q\bar{q},$$

$$\tau_1=\rho\bar{q}_o+\frac{\rho^2}{1-\rho}\cdot\bar{q},\quad \varepsilon=\eta\rho e^{-\mu q}(q-\bar{q}_o)-(1-\eta),\qquad \bar{q}=\frac{\eta}{\mu}(1-e^{-\mu q}).$$

By selecting S or $\{a_n\}$ properly, we can get various Skip systems. In the following with reference to the Fig. 1 through Fig. 3 we show some examples of the results of such Skip systems :

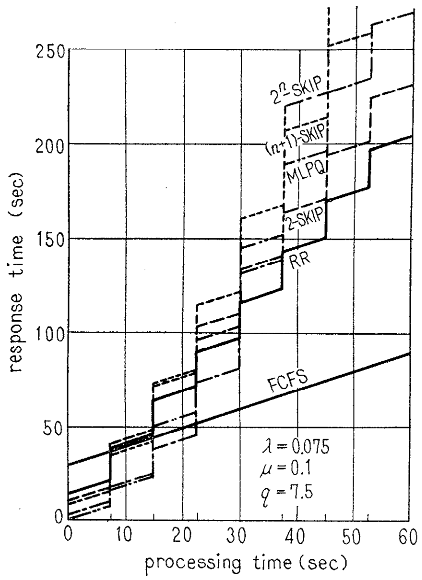

Fig. 1. Comparison of Expected Response Time.

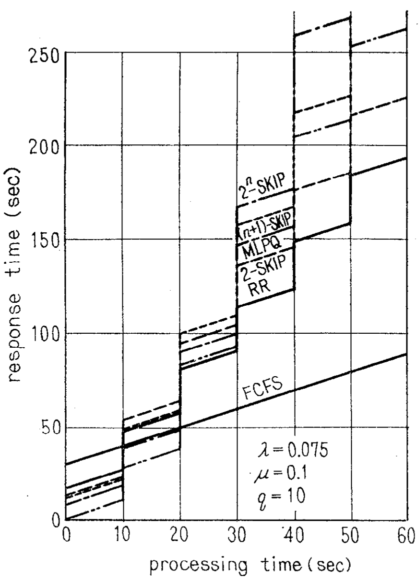

Fig. 2. Comparison of Expected Response Time.

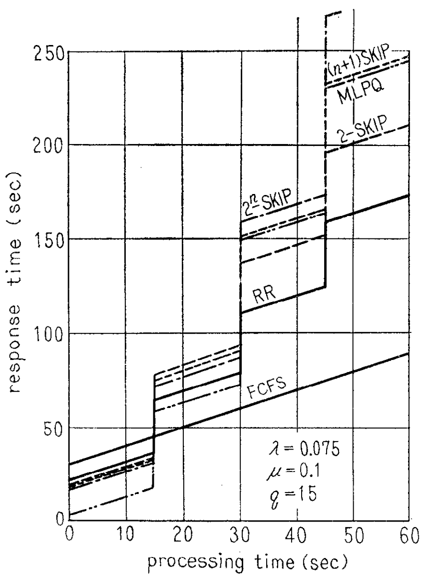

Fig. 3. Comparison of Expected Response Time.

(2-Skip system)

$$\gamma=1/(1+e^{-\mu q})$$

$$W_{2N-1}=\frac{\tau_1(1-\gamma^{2N-1})}{1-\gamma}+\frac{\varepsilon(1-\gamma^{2N-2})}{1-\gamma}+\frac{\delta}{1-\gamma}(N-(1+\gamma^2+\gamma^4+\cdots+\gamma^{2N-2}))$$

$((n+1)-$Skip system)

$$\gamma=(1-e^{-\mu q}),\quad M=N(N+1)/2$$

$$W_M=\frac{\tau_1(1-\gamma^M)}{1-\gamma}+\frac{\varepsilon(1-\gamma^{M-1})}{1-\gamma}+\frac{\delta}{1-\gamma}(N-(1+\gamma^N(1+\gamma^{N-1}(\cdots(1+\gamma^2(1))\cdots)$$

$(2^n-$Skip system)

$$\gamma=\frac{1-2e^{-\mu q}}{1-e^{-\mu q}}\text{ with } e^{-\mu q}<\frac{1}{2},\quad M=2^N-1$$

$$W_M=\frac{\tau_1(1-\gamma^M)}{1-\gamma}+\frac{\varepsilon(1-\gamma^{M-1})}{1-\gamma}+\frac{\delta}{1-\gamma}(N-(1-\gamma^{2N-1}(1+\gamma^{2N-2}(\cdots(1+\gamma(1)\cdots)$$

Fig. 1 through Fig. 3 show the results of these as well as the results of a first-come-first-served system, a RR system and a MLPQ system. We assumed that the MLPQ system shown there has the infinite number of levels and the quantum in every level is q, and all requests arrive only at the first level with Poison arrival of parameter λ and we also used the equation ([4], eq. (22a)) by Coffman.

4. *Conclusion*

In such a case as $\{a_n\}$ is given as the form of $a_{n+1}=f(a_n)$, it is easy to manage a queue and to vary the characteristics of response time. As one of such cases, we give the following example by setting a pair of positive integers I and J on a request, where the integer I represents a_n, and the integer J is used as a parameter in order to determine whether a service of a request in its cycle should be executed or skipped.

(1) $I=1$, $J=1$ for a newly arriving request (namely, $n=1$).

(2) If $I=J$ in a certain cycle of the request then $I=f(I)$ and $J=1$, and its service is executed.

(3) If $I\neq J$ in a certain cycle of the request then $J=J+1$ and its service is skipped.

In order to vary the characteristics of response time, we only change the form of f in the procedure of case (2). For example, if $I=1$ then this system becomes a RR system, if $I=I+1$ then $(n+1)$-Skip system, and if $I=I\times 2$ then 2^n-Skip system. Taking the merit of such a wide adaptability into consideration, this system would be effective if it was used in a time-shared computer system.

On the other hand, because the quantum in the Skip system is constant, programs must be swapped as many times as in a RR system. And in comparison with the swapping method in CTSS, the loss due to swapping time can be considered to be large.

The authors examined and compared of this schedulihg algoritnm with others by simulation. From the result of this simulation, it was shown that the standard derivation of response time by Skip system is nearly equal to that by RR system and smaller than that by MLPQ system.

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