Octagonal Tracing Method for Drawing Formulated Pattern

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Abstract—A method is developed for drawing formulated pattern. A tracing octagon is set to lay its center on a curve and then a drawing vector is decided by evaluating the sign of the drawing function at vertexes of the octagon. Adopting, at the same time, a new decision rule for an evaluation sequence, called "course check", a tracing efficiency is improved and it becomes possible to draw curves to trace with difficulty by other method. Therefore, it can be applied to drawing integral curve of implicit function.

I INTRODUCTION

A method using the formulated pattern to represent the shapes for design and manufacturing has been proposed as the present development of Computer-Aided-Design and Computer-Aided-Manufacturing. An effective method is required for the purpose of drawing the formulated pattern. Although the staircase tracing method, the cross pattern tracing method and their variations have been studied for it, their disadvantages are due to poor traceability, insufficient tracing efficiency and approximation as discussed later. The method proposed in this paper aims at giving an effective algorithm for drawing not only explicit function but two-variable implicit function.

II FORMULATED PATTERN

The formulated pattern method for representing a shape was proposed by Okino. A complicated shape is split into elements of more simplified patterns and it is expressed as a set of them. Let Pij be a element of a shape, then the shape P is defined by $P=\bigcup_{j=1}^{m}(\bigcap_{i=1}^{n} P_{ij})$, where Pij is represented in form of an explicit function, an implicit function or a vector function. When a formulated pattern must be drawn, it is deduced to draw the integral curve of the equation f(x,y)=0, called the curve equation.

III THEORY AND ALGORITHM

3.1 Drawing Vector and Tracing Octagon

Drawing with digital XY plotter or NC-drafter, minimum drawing unit is eight vector of $+\delta x, -\delta x, +\delta y, -\delta y, +\delta x+\delta y, +\delta x-\delta y, -\delta x+\delta y, -\delta x-\delta y$. Those eight vectors are fundamental in general not only for a hard-copy device but also for a CRT-display device when precise curve are drawn, **therefore,**

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they are adopted as basic drawing unit. If we let A,E,B,F,C,G,D,H be terminal points of one drawing stroke, tetragon ABCD is a square as in Fig.1 and J,K,L,M,N,R,S,I denote middle points of them respectively and octagon JKLMNRSI is called "tracing octagon". Evaluation on vertexes of this tracing octagon minimizes a drawing error. The reason will be discussed in 3.2. In general, if we set a polygon with sides of the number corresponding to unit drawing vectors of the device and adopt the tracing square which has a side of twice length of a minimum drawing step, the following algorithm is available without arrangement. The length of the side is set larger when rough drawings are required or shorter when fine ones are requested respectively.

3.2 Fundamental Algorithm

Let the drawing curve equation GIJE(x,y)=0 be across at point Q on side IJ and let point O be the center of the octagon as illustrated in Fig.1. Observing microscopically, although the curve generally winds slightly, we let it be equivalent to the segment $\overline{OQ}$ at this portion because plotter have finite resolution capability, and so with point O being a little away from the curve. Therefore, the discussion will be focused the attention on the tracing center point and the cross point between the curve and the side of the octagon hereafter. Solution is to choose the vector closest to segment $\overline{OQ}$ among from drawing vectors, that is to say, it is resulted in choosing segment $\overline{OA}$ or segment $\overline{OH}$ replacing segment $\overline{OQ}$ as illustrated in Fig.2. Let the length of $\overline{AA'}$ or segment $\overline{HH'}$ be the drawing error, then the former is less than the latter. Line OA crossing the middle point of side IJ is, namely, closest to the drawing curve when the curve GIJE(x,y)=0 intersects side IJ. Let (xo,yo) denote the coordinates value of the point drawn just before and let us define as follows: [I]=sign(GIJE(xo+s,yo+s/2)), [J]=sign(GIJE(xo+s/2,yo+s)), [K]=sign(GIJE(xo-s/2,yo+s)), [L]=sign(GIJE(xo-s,yo+s/2)), [M]=sign(GIJE(xo-s,yo-s/2)), [N]=sign(GIJE(xo-s/2,yo-s)), [R]=sign(GIJE(xo+s/2,yo-s)), [S]=sign(GIJE(xo+s,yo-s/2)), where s is a half of a tracing square side. A drawing vector is decided by finding minus valued one among [I]·[J], [J]·[K], [I]·[S], [K]·[L], [S]·[R], [L]·[M], [R]·[N], [M]·[N]. It is the bisector of these two points and then the terminal point of the vector is to be the next tracing center point. Plotting on a CRT-display device, an output is the coordinates value of the tracing center point replacing the incremental vector.

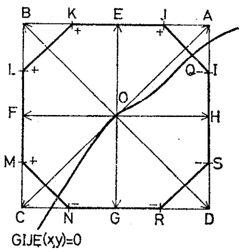


Fig.1 Tracing octagon and tracing square.

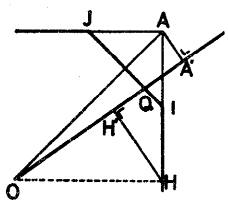


Fig.2 Minimum error when AA'=HH'.

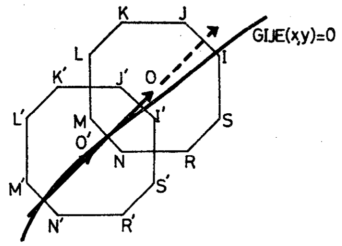


Fig.3 To trace first in the direction(dotted) of the previous vector $\overrightarrow{O'O}$ is to find the curve quickly.

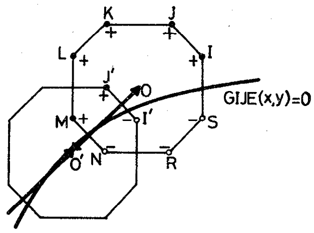


Fig.4 Going and returning between O' and O in the case of constantly clockwise (or counterclockwise) evaluating.

3.3 Influence of the Evaluation Sequence to Tracing Efficiency and Traceability

Curves are drawn by the process above mentioned, however, computing all values at eight vertexes is not efficient. Evaluating first in the prolonged direction of the latest drawing vector, curves can be traced quickly. Hence, let the tracing octagon just before be I'J'K'L'M'N'R'S' and let the drawing vector be $\overrightarrow{O'O}$, when the octagon to be traced is IJKLMNRS as shown in Fig.3, the first points to be evaluated are point I and point J. If they have the same sign, the next points to be evaluated are point K or point S. Tracing is tried in sequence of point L or R, point M or N until the occurrence of the sign inversion. The method (called "minimum method", here) of evaluating values at all eight vertexes and prolonging the drawing vector towards the less point, in the absolute value, among two points having the contrary sign each other is disadvantageous with respect to computing time. If the evaluation sequence is constantly counterclockwise starting from point I, impossibility of tracing will be resulted occasionally from going and returning between point O' and point O because of tracking the intersecting point (the portion drawn immediately before) on side MN before tracking the intersecting point on side SI as in Fig.4.

3.4 Comparative Discussion of Traceability

Let us discuss the traceability on occasion of tracing special curves. For any special curve, the problems as following never exist between the same segment of a pattern and the others. Some troubles arise at the portion where the interval of the two curves is narrower than a tracing square side, as follows: (1) Protruding portion of whose neck is narrower than a tracing square side is neglected; "Skipping over"; Fig.5. (2) A sharply pointed end narrower than a side of the tracing octagon is never drawn; Fig.6. (3) Drawing might be difficult due to tracking back at a curved pointed end; Fig.7. In the figures, the staircase tracing method is illustrated as chain line, the cross pattern tracing method is shown as dotted line, the minimum method is indicated as broken line and the octagonal tracing method is shown as fine line respectively. In the

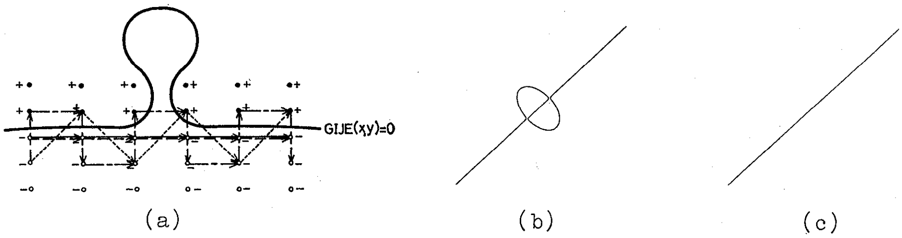


Fig.5 (a); Skipping over at the neck which is narrower than a step. (b); A drawing example of no skipping over. (c); A drawing example of skipping over (drawn in ten times step of (b)'s).

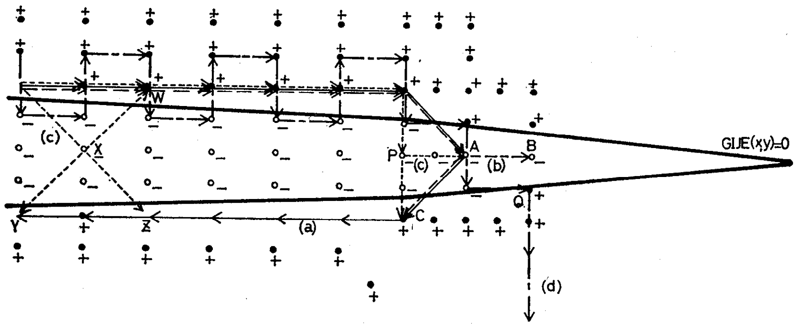


Fig.6 Impossibility of tracing by minimum method, (b), cross pattern method, (c) or staircase method, (d) except octagonal method, (a).

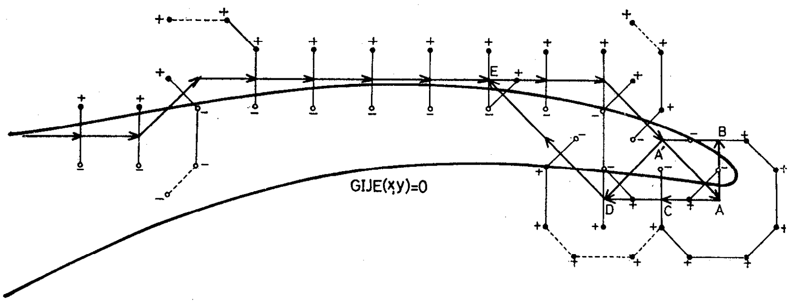


Fig.7-1 Impossibility of tracing at point E without course check.

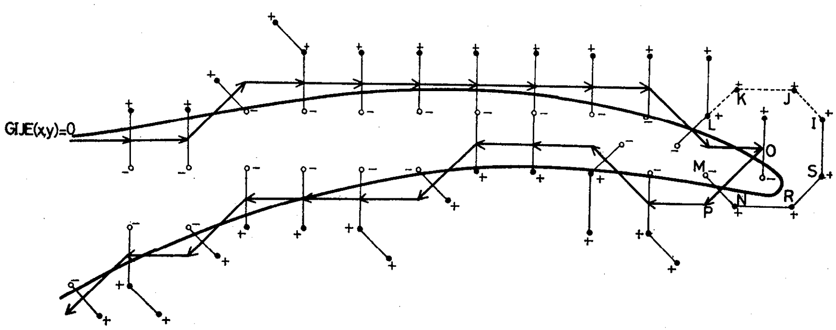


Fig.7-2 Tracing certainly and quickly with course check.

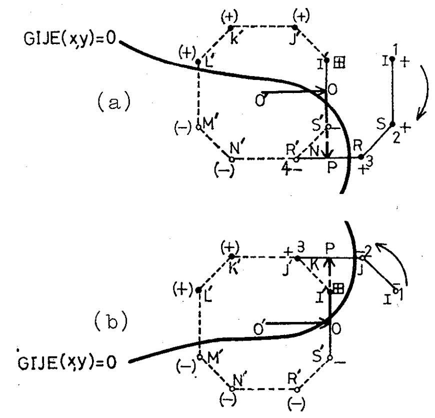


Fig.8 Course check: (a); Clockwise evaluation when [I']·[I]>0. (b); Counterclockwise evaluation when [I']·[I]<0.

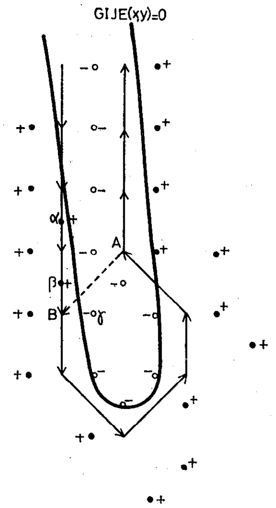


Fig.9 Improved course check

case of (1), skipping over between point O' and point O is occured due to the sign inversion at point I and point I'. In the case of (2) a pointed end is neglected with the octagonal tracing method, but tracing is impossible with other methods. In the case of (3), the difficulty of tracing is unavoidable by each method. It is resulted from neglecting course check for the evaluation sequence. Therefore, let us decide the evaluation sequence as follows: The left-hand point towards the prolonged direction of the vector drawn just before is evaluated in the first time. (See appendix). If the sign at this point is the same as the sign at the left-hand point just before, the right adjacent point is evaluated next and then it is tried succesively in clockwise order until the occurence of the sign inversion. If the sign at the first point is not the same as that, next left adjacent points are evaluated succesively until the occurence of the sign inversion in counterclockwise order as illustrated in Fig.8. The following advantages are resulted from adopting such evaluation sequence: i) The difficulty to trace becomes almost nothing. ii) The frequency of the evaluation decreases. The tracing speed increases due to evaluating only five points at a corner at most. But impossibility of tracing still remains with this course check as shown in Fig.9. When a curve is almost parallel to the x-axis or the y-axis and the interval is almost equal to a tracing square side, on occasion of the tracing center being on point A, since the sign at the first point, α is plus, point β, point γ is evaluated in counterclockwise order and then the tracing point returns back to point B finally. For the purpose of checking this, the course check improved as starting from the right-hand point of the course if vector change occured at the previous evaluation. In final, the case of (3) disappears on account of this improved course check.

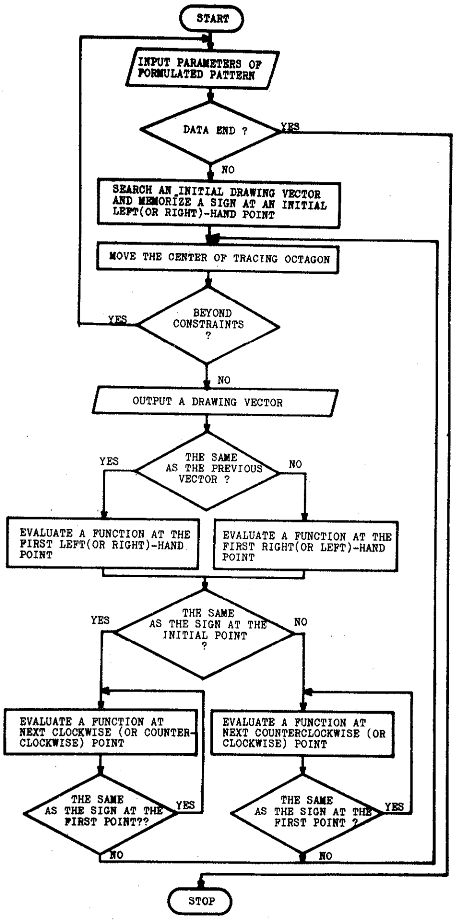


Fig.10 General flowchart.

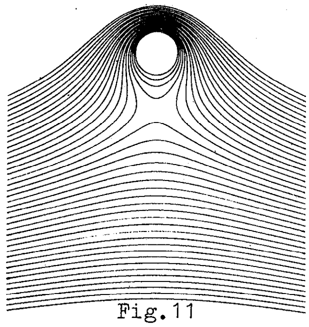


Fig.11

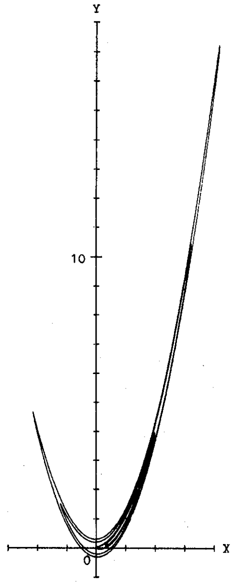


Fig.12

3.5 Comparative Discussion of Tracing Efficiency and Approximation

Concerning tracing speed, comparing this method with the staircase tracing method, the latter has the same efficiency as the former at most. The minimum method is slowest due to the neccesity of evaluating all eight points. The cross pattern tracing method must evaluate four points per an output of the drawing vector. On the other hand, the octagonal tracing method requires to evaluate two or three points at the portion where the radius of curvature is greater than a half of a tracing square side and also requests to evaluate five points at an acute angle at most . Concerning the approximation, this method and the minimum method are best. If x-axis component and y-axis component exist together after searching several steps forward, in the case of the staircase tracing method or the cross pattern tracing method, although it is necessary to compose both components for the purpose of getting better approximation, this method never necessitates these processes due to the consideration to minimize the drawing error on establishing the tracing algorithm.

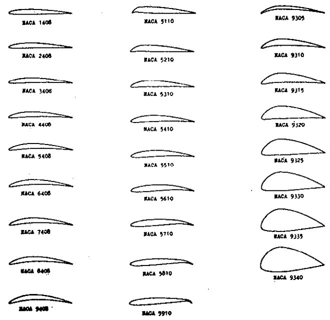
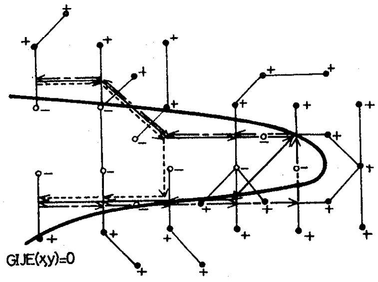


Fig.13

Fig.14



The general flowchart is shown in Fig.10. Examples as in Fig.11, 12, 13 were drawn by FORTRAN program of about 150 steps with OKITAC-4300 core memory size 4kw, on a directly coupled digital XY plotter, WATANABE WX-530. Fig.11 is the two-dimensional stream line when a cylinder exists in the ideal fluid. Contours of Rosenbrock's equation were drawn as shown in Fig.12. Fig.13 is the drawing of NACA aerofoil sections.

IV CONCLUSION

This method can be applied not only for drawing two-dimensional pattern used in CAD system, but also for graphically solving implicit functions. For the purpose of the latter, to find the crossing points or the branching points is rather important when curves cross or branch. On tracing in minimum drawing stroke, of course the traceability and the approximation are never neglectful, but the efficiency is essential also. Especially, to decrease the frequency of the evaluation as well as possible is necessary.

APPENDIX

It makes no great difference to evaluate the left-hand point or the right-hand point at the beginning. When the right-hand point is evaluated, the evaluation sequence is decided as contrary against the right-hand point evaluation. The occurence of a little difference is resulted, although it is negligible practically, from the difference between evaluating at the right-hand and the left-hand, or the difference of the initial tracing direction. In Fig.14, if the drawing course is directed from upper left towards the right, the fine line and the dotted line are obtained when the evaluation point is on the left-hand side or on the right-hand side respectively. On the other hand, if it is directed from the lower left towards the right, the chain line is obtained under no influence of the left or the right.

REFERENCES

[1] N. Okino, Computer Aided Design. Nikkan-kogyo press, 1967.

[2] T. Saito et al, "Fundamental study for formulating two-dimensional pattern by raster scan sampling", in Proc. Conf. IPSJ, 1968.

[3] T. Kamae and S. Murakami, "Graphical representation of implicit 2 variable function", Information Processing, vol.12 no.9.