Properties of Wave Propagation Method for Conversion of Gray Pictures into Line Figures

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ABSTRACT Wave propagation method (WPM) is a new method for converting multi-leveled (gray) pictures into line figures proposed in our previous paper. It can give reasonable line figures corresponding to given gray pictures. In this paper, properties of WPM including characteristics of each mode of raster pair used in the algorithm for WPM, difference between WPM and gray weighted skeleton and a number of times of iteration in the algorithm are discussed in detail.

1. INTRODUCTION

We proposed "wave propagation method (WPM)" for conversion of gray pictures into line figures in the previous paper.(1) This paper presents some basic properties of WPM including characteristics of each mode of a raster pair used in the algorithm for WPM, difference between WPM and gray weighted skeleton (GWS) and the number of times of iteration in the algorithm. Some experimental results are also shown.

2. OUTLINE OF WPM

In this chapter we first introduce some notations used in subsequent parts of the paper, and then give one of the algorithms for performing WPM in order to show an outline of WPM.

We treat here only a quantized image. A quantized image is represented by an array of numbers, i.e. matrix $\mathbb{H} = \{h_{ij}\}$ where h_{ij} is a gray value (density) at a point (i,j) which is located in the i-th row and j -th column. A matrix $\mathbb{F} = \{f_{ij}\}$ derived from $\mathbb{H}$ by eq. (1) is called θ-positive

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function.

$$f_{ij} = \begin{cases} h_{ij}, & \text{if } h_{ij} \geq \theta, \\ 0, & \text{otherwise,} \end{cases} \qquad (1)$$

where θ is a given threshold.

Then, gray weighted distance (GWD) $\mathbb{D} = \{d_{ij}\}$ is defined in the same way as Levy's paper[2] and ours[1] using the θ-positive function.

Various algorithms have been developed for performing WPM. Let us present here one of the fundamental algorithms called 4-neighbor sequential WPM (4n-SWPM). Other types of algorithms are given in another paper.[3]

[4n-SWPM] Let us denote by $\mathbb{F}$, $\mathbb{D}$, and $\mathbb{S}$ a θ-positive function (gray picture to be converted into line figure), GWD, and a working area called S-pattern, respectively. Three modes of scanning are employed which are called mode 1, 2 and 3 raster pair. Each raster pair includes a forward and a reverse raster (Fig. 1).

(I) Phase 1 (Iteration phase)

Step 0: $$d(0)_{ij} = \begin{cases} 0, & \text{if } f_{ij} = 0, \\ N, & \text{otherwise,} \end{cases} \qquad S(0)_{ij} = \begin{cases} 0, & \text{if } f_{ij} = 0, \\ 1, & \text{otherwise.} \end{cases} \qquad (2)$$

where N is a sufficiently large constant.

Step k: $k = 1, 2, \ldots$.

(i) $$d'(k)_{ij} = \min\{d(k-1)_{ij},\ f_{ij} + d'(k)_{i-1,j},\ f_{ij} + d'(k)_{i,j-1}\}, \qquad (3)$$

$$S(k)_{ij} = 1, \text{ only if } d'(k)_{ij} \neq d(k-1)_{ij}. \qquad (4)$$

All points (i,j) are scanned according to mode 1 forward raster.

(ii) $$d(k)_{ij} = \min\{d'(k)_{ij},\ f_{ij} + d(k)_{i+1,j},\ f_{ij} + d(k)_{i,j+1}\}, \qquad (5)$$

$$S(k)_{ij} = 2, \text{ only if } d(k)_{ij} \neq d'(k)_{ij}. \qquad (6)$$

All points (i,j) are scanned according to mode 1 reverse raster.

Operations (i) and (ii) are iterated for $k = 1, 2, \ldots$ until

$d'(k)_{ij} = d(k)_{ij}$ or $d(k)_{ij} = d'(k+1)_{ij}$ holds for all (i,j).

Values of $d'(k)_{ij}$ (or $d(k)_{ij}$) and $S(k)_{ij}$ when the iteration stops give values of d_{ij} and S_{ij}, respectively. These operations are applied only to the inside of the image, so that d_{ij} and S_{ij} are not defined at the points with i = 1 or j = 1.

(II) Phase 2 (Selection phase)

Points (i,j) satisfying all of the following conditions 1, 2 and 3 are selected.

(Condition 1) At least one of the following (i)--(vi) holds.

(i) $s_{ij} = 1$, $s_{i+1,j} = 1$, $s_{i,j+1} \neq 1$, and $d_{ij} \geq d_{i,j+1}$.

(ii) $s_{ij} = 1$, $s_{i+1,j} \neq 1$, $s_{i,j+1} = 1$, and $d_{ij} \geq d_{i+1,j}$.

(iii) $s_{ij} = 1$, $s_{i+1,j} \neq 1$, and $s_{i,j+1} \neq 1$.

(iv) $s_{ij} = 2$, $s_{i-1,j} = 2$, $s_{i,j-1} \neq 2$, and $d_{ij} > d_{i,j-1}$.

(v) $s_{ij} = 2$, $s_{i-1,j} \neq 2$, $s_{i,j-1} = 2$, and $d_{ij} > d_{i-1,j}$.

(vi) $s_{ij} = 2$, $s_{i-1,j} \neq 2$, and $s_{i,j-1} \neq 2$.

(Condition 2) $d_{ij} \geq \theta_1$, where θ_1 is a given threshold.

(Condition 3) $$P_{ij} = \max_{\{Q_{rs}\}} \{(d_{rs} - d_{ij})/f_{rs}\} \leqq \theta_2 \tag{5}$$

where $\{Q_{rs}\} = \{(s, r) \mid |r-i| + |s-j| = 1\}$. If $f_{rs} = 0$, then $(d_{rs} - d_{ij})/f_{rs}$ is assumed to be one. θ_2 is a predetermined threshold.

(III) Phase 3 (Mixing phase)

Phase 1 and 2 are performed using mode 2 and 3 raster pairs. Suffixes in eqs. (3) and (5) and condition 1 of phase 2 are also modified so that the same relationship as in the case of the mode 1 raster pair may be kept between the direction of a scan and a set of points considered in eqs. (3), (5) etc. (Fig. 1). The set of all points extracted by the above procedure gives a line figure converted from θ-positive function $\mathbb{F}$.

3. CHARACTERISTICS OF EACH RASTER PAIR IN SWPM

Each mode of raster pair has its own directional characteristic. As a result, there exists a line difficult to be extracted corresponding to each raster pair. For example, a line at 45° to a horizontal axis cannot be extracted by the procedure using mode 1 raster pair. Other examples are shown in Fig. 2. This difficulty is avoided by combining three modes of raster scan as in phase 3. But, in practical applications, mode 3 raster pair can be often neglected without any serious effect on an obtained line figure.

4. COMPARISON BETWEEN WPM AND GWS

WPM seems to be similar to gray weighted skeleton (GWS) in that WPM uses gray weighted distance (GWD). But there are some important differences between them, which are shown in this chapter.

Let us define GWS here as follows. [GWS (4-neighbor GWS)] 4nGWS for a function $\mathbb{F}$ is a set of points at which P_{ij} given in eq. (5) is less than one.

This definition is slightly different from the one given by Levy et al.(2) and is considered as a straight forward extension of Rosenfeld's definition(4) to a gray picture. Major differences between WPM and GWS are as follows.

(1) A particular point and its 4-neighbor are never included in GWS at the same time. Therefore the intersection (and its neighborhood) of two lines meeting at right angles cannot be extracted by GWS. This is possible by WPM with $\theta_2 = 1$ in eq. (5).

(2) Width of GWS is hardly less than two points. Therefore GWS can seldom give completely thin lines. On the other hand, WPM gives thin lines with one point width almost everywhere (Fig. 3).

(3) Let us denote by $\mathcal{S}_W$ a set of points extracted by WPM and by $\mathcal{S}_G$ a set of points contained in GWS. Then neither $\mathcal{S}_W \subseteq \mathcal{S}_G$ nor $\mathcal{S}_W \supseteq \mathcal{S}_G$ holds.

(4) Information concerning the minimal path(1) is not used in GWS, while it is very important in WPM.

Generally speaking, GWS is not so suitable for extracting line figures from gray pictures as WPM. Examples of GWS and a result of WPM are shown in Fig. 4.

5. NUMBER OF TIMES OF ITERATION IN PHASE 1 OF WPM

Computing time of WPM depends strongly on the number of times of iteration in phase 1 (iteration phase). In this chapter, some comments are given concerning the number of times of iteration.

(1) Let us assume that a length of minimal path* to a point (i,j) is k*. Then in the case of parallel type WPM(PWPM),(1) $d(n)_{ij} = N$ for all $n<k^*$ and $d(n)_{ij} = d_{ij}$ for all $n \geq k$. In the case of SWPM, the number of times of iteration necessary for giving d_{ij} cannot be determined uniquely from the value of k* only.(1)

* See the footnote in the next page.

(2) In the case of SWPM, $d(k)_{ij}$ gives an approximate value of d_{ij} , even if the iteration is truncated at a relatively small value of k. Strictly speaking,

$d(n_1)_{ij} \geqq d(n_2)_{ij} \geqq d_{ij}$ if $n_1 \leqq n_2$. In fact, truncation at k = 2 or 3 gave satisfactory results in most of our applications[5].

On the other hand, truncation is meaningless in PWPM.

Finally, Fig. 5 shows one of the experimental results concerning the number of times of iteration.

6. CONCLUSION

Wave propagation method (WPM) for extracting line figures from a gray picture was proposed in the authors previous paper (1), (2). In this paper, some properties of WPM, that is, properties of raster pairs used in the algorithm, comparison with a gray weighted skeleton and a number of times of iteration in the iteration phase of the algorithm etc. were discussed in detail.

WPM is better than other methods for thinning in that global information concerning spatial distribution of gray values of an input image is made use of and that the algorithm for WPM is very suitable for computer processing. Its usefulness was established by an application to pattern recognition of chest x-ray images.[5]

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* The minimal path is the path from an arbitrary point with the grey value 0 to the point (i,j) with the positive grey value. The sum of grey values along this path gives the value of d_{ij}.[1],[2]

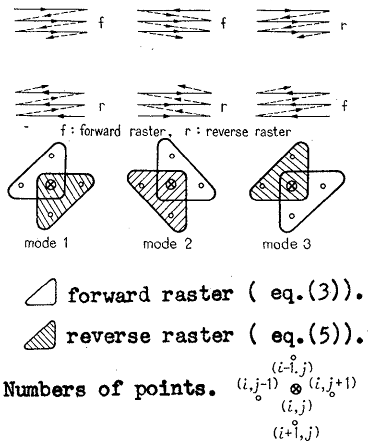
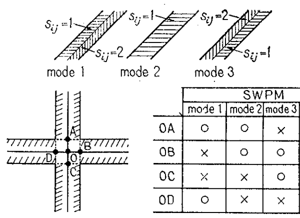


Fig.1 Raster pairs used in SWPM.

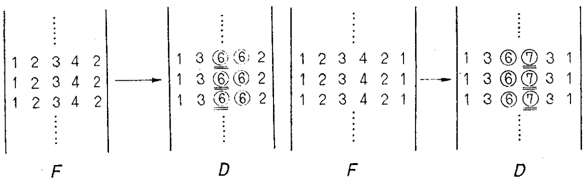


	SWPM mode 1	SWPM mode 2	SWPM mode 3	SWPM SWPM	PWPM
OA	○	○	×	○	×
OB	×	○	○	○	○
OC	×	×	○	○	○
OD	○	×	×	○	○

○ : detectable

×: undetectable

Fig.2 Relationships between modes of raster pairs and direction of lines to be extracted.



(○: GWS ═ : results of WPM)

Fig.3 GWS and results of WPM.

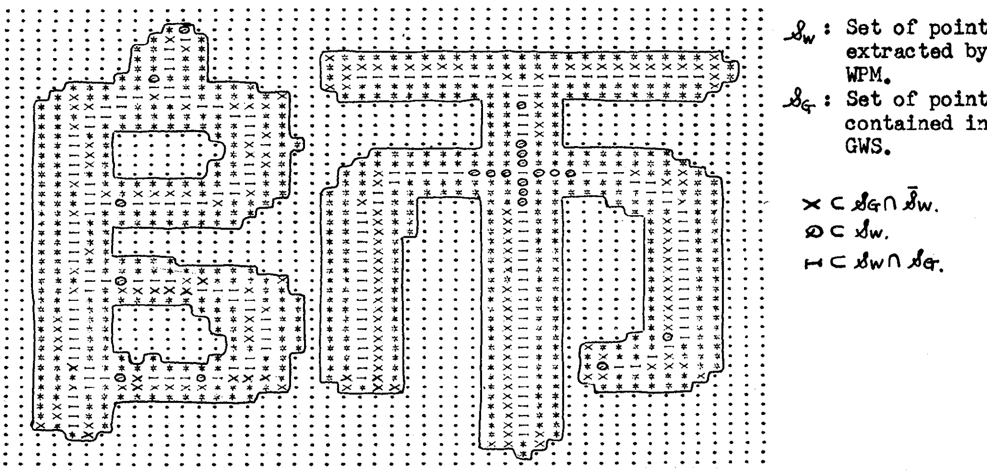


Fig.4 GWS and result of SWPM. (Application to the thinning of printed Chinese characters)

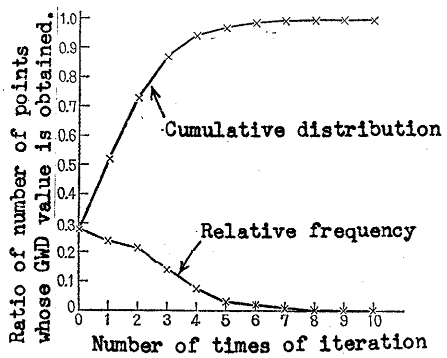


Fig.5 An example of the number of times of iteration (iteration phase of PWPM).