Theoretical Analysis of Wiring Modes and Failure Modes for Automatic Wiring Checks

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Abstract

Some efficient automatic checking methods are required for back-planes, printed circuit boards and substraits for integrated circuit wiring before assembling parts. In this paper, wiring modes, failure modes and their numbers for wiring diagnosis are discussed for the preparation of automatic wiring checks. A single wired conductor is represented by a tree graph, and several properties of the tree graph are investigated. As the results, a few rules are found to analyze modes of transmission lines, general tree type wiring and degenerated tree type wiring and their failure modes. These rules will be useful for development of automatic wiring checks and failure exclusion methods.

1. Introduction.

In place of a test method by using bell-check with wiring diagrams and heuristics, a test method by using an automatic wiring test equipment has been mainly used in wiring check. A test algorithm of wiring check equipment used at present is the point-to-point checking method, that is, the method to check connectivity of two terminals by a tester. If no information of wiring pattern is available, $(n - 1)!$ tests are required for wiring check in this method, where n is the number of terminals. If it is available, $(n - 1)$ tests are required in no failure wiring. Therefore, this method has redundancy.

In this paper, theoretical analysis on wiring check for failure location is discussed mainly. Wiring modes (patterns of wiring) and its failure modes are investigated for the following reasons.

(1) The number of wiring tests in a known mode is smaller than one in an unknown mode, and it is desirable to know a wiring mode and to generate their failure modes in automatic wiring check. So these wiring modes and failure modes should be investigated. Redundancy of tests will be reduced by them.

(2) It is expected that test efficiency will be improved by grouping terminals and testing in multiple terminal wiring. So these modes should be examined for grouping.

(3) All wiring modes and the number of wiring modes should be investigated for evaluation of various wiring check methods.

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A single transmission wiring line is represented by a tree-graph as shown in Fig. 1. Wiring modes and their failure modes are investigated for wiring diagnosis by using the tree-graph representing back-plane or printed circuit board wiring.

2. Definitions.

Following definitions are presented to discuss precisely on tree type wiring and to prepare for wiring diagnosis.

(1) Adjacent nodes to input and output. See Fig. 2.

(2) Diverging point node. A node of which degree is more than and equal to three, that is, $\delta(v) \geq 3$ is called to be the diverging point node, where $\delta(v)$ denotes the degree of node v.

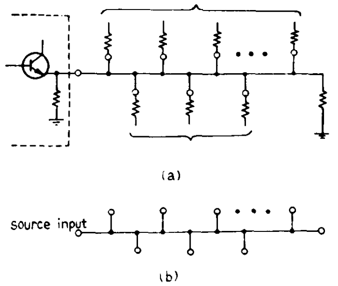


Fig. 1 Transmission line wiring on multilayer boards and its simplified version.

(3) One-input and n-output tree type wiring. See Fig. 3.

(4) Subtree. A subtree is a subgraph of a tree.

(5) Isomorphic tree (in wiring check). Two trees T_1 and T_2 are isomorphic if there exists a one-to-one correspondence between their node sets, and also their edge sets. For example, T_1 and T_2 of Fig. 4 are isomorphic.

(6) Degenerated tree type wiring. See Fig. 5.

(7) Transmission line. A transmission line is degenerated tree type wiring, including two terminal wiring, which forms a chain if all output terminals is eliminated (Fig. 1).

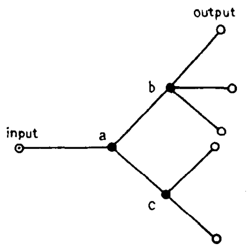
3. Transmission line

3.1 Wiring modes of transmission line

A wiring mode of transmission line is the type as shown in Fig. 1 and the number of total modes for various n terminals is n-1 as a wiring mode is generated in every addition of a terminal.

3.2 Failure modes of transmission line

Failure modes of n-terminal transmission line are considered.



node a : adjacent node to input.
node b,c : adjacent nodes to output.

Fig. 2 An adjacent node to input and adjacent nodes to output.

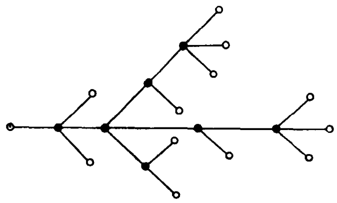


Fig. 3 One-input, n-output tree type wiring.

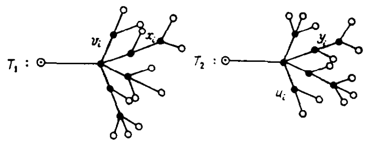


Fig. 4 Isomorphic trees.

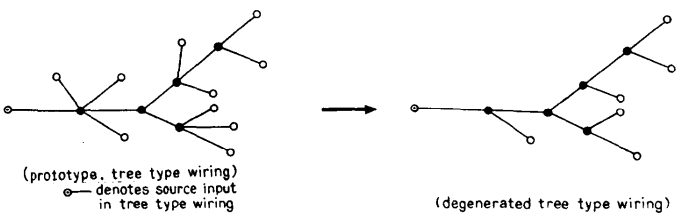


Fig. 5 Degenerated tree type wiring.

All failure modes from two to four terminals are shown in Fig. 6, where double failures as shown in Fig. 7 are indistinguishable from each other and from the triple failures. In general, the following theorem can be given by inductively determining the failure modes of an n+1 terminals from an n terminal line.

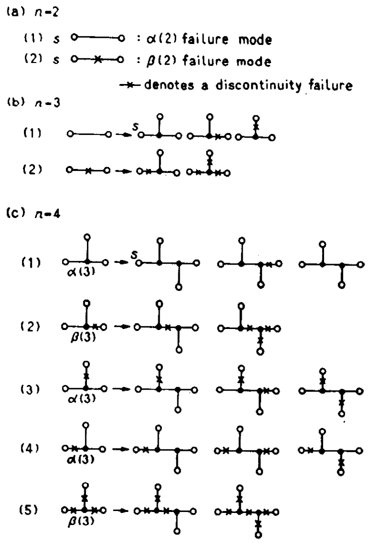


Fig. 6 Failure modes of transmission line.

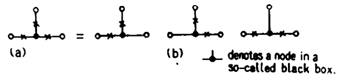


Fig. 7 Indistinguishabll failure modes.

Table 1 The number of failure modes.

n	2	3	4	5	6	7…
$t(n)$	1	3	8	21	55	144…
$d(n)$	1	2	5	13	34	89…
$c(n)$	2	5	13	34	89	233…

<u>Theorem 1.</u> The number of failure mode set elements of transmission line is given as a term of a Fibonacci series, tha is,

$$c(n) = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{2n-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{2n-1}\right\} \qquad (3.\ 1)$$

Table 1 shows t(n), d(n) and c(n) for various of n.

4. Tree type wiring

There exist many wiring modes in general tree type wiring. Therefore their failure modes are so many, and a test by using failure modes is not suitable for tree type wiring.

4.1 Tree type wiring modes

It is considered that there exist linear type, radial branch type and their mixed type in tree type wiring. So tree type wiring modes are classified three types, that is (1) line type, (2) branch type and (3) mixed type. Those type modes are shown in Fig. 8.

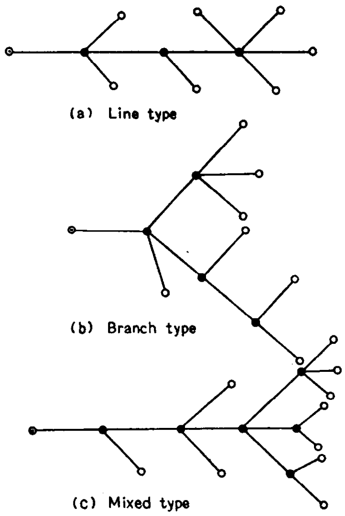


Fig. 8 Types of tree type wiring.

4.2 The number of tree type wiring modes

The number of nodes will be calculated by a correspondence between the number of output terminals and the partitioned number (called a partition number which means each partitioned number here) in partition number theory.

Let the total number of output terminals of partitioned tree type wiring be n, and the number of subtrees in a node which their subtrees having diverging points branch off be i, where the node is called a partition node. If n can be divided as follows;

$$n = n_1 + n_2 + \cdots + n_i,$$

the numbers of output terminals of subtrees are mode correspond with the partition numbers $n_1, n_2, \ldots, n_i$.

The number of partitioned tree type wiring modes can be calculated step by step from the number of wiring with fewer output terminals by calculating all partition number because of $n>n_1$. In wiring check, failure patterns in Fig. 9(a) are indistinguishable from one another, and these patterns are assumed to be equivalent to a single failure in Fig. 9(b). Therefore the wiring mode of Fig. 9(c) is the simplest pattern on considering the partition number.

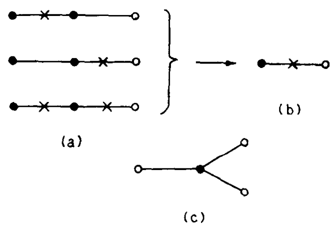


Fig. 9 Indistinguishable failure modes and the minimum wiring pattern.

Let us consider a wiring mode set in partitioned tree type wiring. Let the numbers of output terminals of subtrees connected with the adjacent node to input be the partition numbers k_p, $k_{q'}$, ..., $k_{\ell(e)}$ of output terminals be S_{k_p}, $S_{k_{q'}}$, ..., $S_{k_{\ell(e)}}$ respectively. The following lemma will be given.

<u>Lemma 2.</u> If the number n of output terminals can divided into different natural numbers greater than or equal to two,

$$n=k_1+k_2+\cdots+k_i=k_1'+k_2'+\cdots+k_j'=\cdots$$
$$=k_1^{(e)}+k_2^{(e)}+\cdots+k_l^{(e)}, \qquad (4.\ 1)$$

then the number $\sigma_2(n)$ of elements of the partitioned tree type wiring mode set is given by

$$\sigma_2(n)=\prod_{p=1}^{i} S_{k_p}+\prod_{q=1}^{j} S_{k_q'}+\cdots+\prod_{r=1}^{l} S_{k_r^{(e)}}, \qquad (4.\ 2)$$

If there exist equal numbers in partition numbers, then a part of partitioned tree type wiring become isomorphic. Therefore, it is necessary to classify wiring modes by every isomorphic type and to calculate the number of its classes if it is isomorphic. The following lemma for calculating the number of classes holds.

<u>Lemma 3.</u> Let the number of elements of the subtree wiring mode set be β, where the subtree have m-output terminals, and the number of elements of the α·m-output terminal partitioned tree type wiring mode set be $[\delta_{m,\beta}{}^{\alpha}]$. Then

$$[\delta_{m,\beta}{}^{\alpha}]=\binom{\alpha+\beta-1}{\alpha}, \qquad (4.\ 3)$$

is given, where the α·m-output terminal partitioned tree type wiring mode are composited by combining α's of m-output terminal subtrees at the adjacent node to input.

Table 2 shows $[\delta_{m,\beta}{}^{\alpha}]$ in $\beta=5$, where $[\delta_{m,\beta}{}^{\alpha}]$ is also represented as follows,

$$[\delta_{m,\beta}{}^{\alpha}]=\sum_{r=0}^{\beta-1}[(\delta_{m,\beta}-r)^{\alpha-1}].$$

It is noted that $[\delta_{m,\beta}{}^{\alpha}]$ are given as the partial sum of $(\alpha-1)$ ordered arithmetic progressions. Generally if the number n of output terminals

Table 2 The number $[\delta_{m,\beta}{}^{\alpha}]$ of $\alpha\cdot m$-output tree type wiring modes after classified by isomorphic modes.

	$[(\delta_{4,5}-r)^{\alpha-1}]$					$[\delta_{4,5}{}^{\alpha}]$
α \ r	4	3	2	1	0	
1	1	1	1	1	1	5
2	1	2	3	4	5	15
3	1	3	6	10	15	35
4	1	4	10	20	35	70
5	1	5	15	35	70	126

(where, $\delta_{4,5}=[\delta_{4,5}{}^{1}]=\beta$, $m=4$, $\beta=5$)

can divided into arbitrary natural numbers greater than or equal to two, the next lemma 4 will be given.

Lemma 4. If the number n of output terminals can divided into natural numbers in which the smallest number is two,

$$\begin{aligned} n &= k_1+k_2+\cdots+k_i = k_1'+k_2'+\cdots+k_j' = \cdots \\ &= k_1^{(s)}+k_2^{(s)}+\cdots+k_{j-1}^{(s)} \\ &\quad +k_j^{(s)}+k_j^{(s)}+\cdots+k_j^{(s)} \\ &\quad +k_{j+1}^{(s)}+k_{j+2}^{(s)}+\cdots+k_l^{(s)}, \end{aligned} \tag{4. 4}$$

then

$$\begin{aligned} \gamma_2(n) &= \prod_{p=1}^{i} S_{k_p} + \prod_{q=1}^{j} S_{k_q'} + \cdots \\ &\quad + [\delta_{k_j^{(s)},s^*}] \cdot \prod_{r=1}^{j-1} S_{k_r^{(s)}} \cdot \prod_{r=j+1}^{l} S_{k_r^{(s)}}, \end{aligned} \tag{4. 5}$$

The following theorem holds for tree type wiring under lemma 2 ~ lemma 4.

Theorem 5. Let the total number of elements of the tree type wiring mode set be S_n, and the numbers of elements of the line type mode set, the branch type mode set and the mixed type mode set be ℓ_n, b_n, m_n respectively. Then

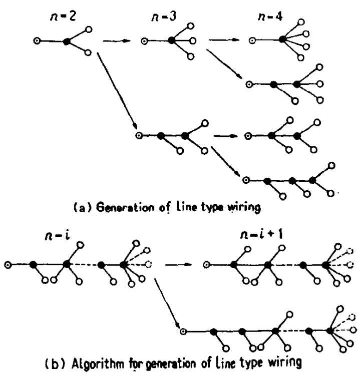
$$S_n = \ell_n + b_n + m_n \tag{4. 6}$$

where

$$\ell_n = 2^{n-2} \tag{4. 7}$$

$$b_n = b_{n-1} + \gamma_2(n) \tag{4. 8}$$

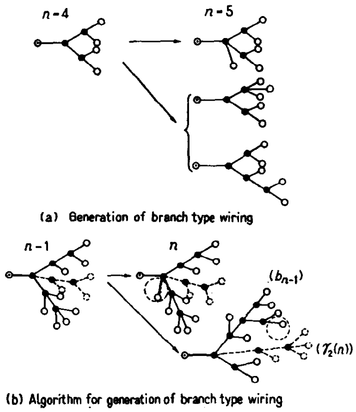
$$m_n = \sum_{i=1}^{n-4} 2^{i-1} b_{n-i} \tag{4. 9}$$



(a) Generation of line type wiring

(b) Algorithm for generation of line type wiring

Fig. 10 Generation of line type wiring and its algorithm.



(a) Generation of branch type wiring

(b) Algorithm for generation of branch type wiring

Fig. 11 Generation of branch type wiring and its algorthm.

Table 3 shows the number of elements of the tree type wiring mode set.

Table 3 The number of tree type wiring modes.

n	2	3	4	5	6	7	8	9	10
$\gamma_2(n)$	0	0	1	2	9	24	81	244	780
l_n	1	2	4	8	16	32	64	128	256
b_n	0	0	1	3	12	36	117	361	1,141
m_n	0	0	0	1	5	22	80	277	915
S_n	1	2	5	12	33	90	261	766	2,312

5. Degenerated tree type wiring

As the above results, the number of tree type wiring modes increases rapidly with the increase of the number n of output terminals, and it is complicated for considering wiring check algorithm.

5.1 Degenerated tree type wiring modes

Classification of tree type wiring modes can be applied for degenerated tree type wiring modes as it is. Lemma 2~Lemma 4 in tree type wiring can be also applied for the number of degenerated tree type wiring. So let S, $\sigma_2(n)$, δ, $\gamma_2(n)$ be S', $\sigma_2'(n)$, δ_2', $\gamma_2'(n)$ in equations (4. 2)~(4. 5). The following theorem holds for degenerated tree type wiring.

<u>Theorem 6.</u> Let the total number of elements of the one input and n output degenerated tree type wiring mode set be S_n', and the numbers of elements of the line type mode set, the branch type mode set and the mixed type mode set be ℓ_n', b_n', m_n' respectively, then

$$S_n' = \ell_n' + b_n' + m_n' \qquad (5.\ 1)$$

where $\ell_n' = 1$, $b_n' = \gamma_2'(n)$ and $m_n' = \sum_{i=1}^{n-4} b_{n-i}'$

Table 4 shows ℓ_n', b_n', m_n' and S_n' for various of n. It is noted that there exists large difference between Table 3 and Table 4.

Table 4 The number of degenerated tree type wiring modes.

n	2	3	4	5	6	7	8	9	10
l_n'	1	1	1	1	1	1	1	1	1
b_n'	0	0	1	1	4	6	17	33	82
m_n'	0	0	0	1	2	6	12	29	62
S_n'	1	1	2	3	7	13	30	63	145

5.2 Failure modes of degenerated tree type wiring

The number of elements of the failure mode set in wiring shown in Fig. 12 is investigated for a preparation of the calculation method.

(1) Let the number of output terminals in Fig. 12 section (a) be n_a, where an input terminal is not contained. The number c_a of elements of the failure mode set in Fig. 12 section (a) is given by

$$c_a = c(\ n_a + 2\ ) \qquad (5.\ 2)$$

from the equation (3. 4).

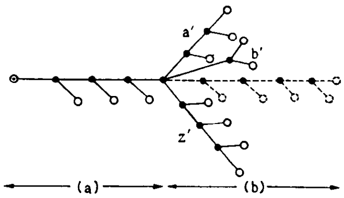


Fig. 12 A degenerated tree type wiring for calculating the number of elements of the failure mode set.

(2) Let the numbers of output terminals of subtrees a', b', ···, z' be n_1, n_2, ··· , n_k respectively. The number c_b of elements of the failure mode set in Fig. 12 section (b) is given as follows by using the equation (3. 4) and lemma 3.

a. If n_1, n_2, ···, n_k are different from one another, then

$$c_b = c(n_1 + 1)c(n_2 + 1) \cdots\cdot c(n_k + 1) \qquad (5.\ 3)$$

b. If n_1, n_2, ···, n_k are equal together,

$$c_b=[\delta_{n,\beta}{}^{k}]=\binom{k+\beta-1}{k} \qquad (5.\ 4)$$

where β is the number of elements of each failure mode set.

c. If the numbers of subtree output terminals are $n_1, n_2, \cdots, n_i, \underbrace{n_j, \cdots, n_j}_{\alpha\text{'s}}, n_{j+1}, n_{j+2}, \cdots, n_k\ (n_i \neq n_j)$

$$c_b=c(n_1+1)\cdot c(n_2+1)\cdot \cdots \cdot c(n_i+1)\cdot \times[\delta_{n_j,\beta}{}^{\alpha}]\cdot c(n_{j+1}+1)\cdot \cdots \cdot c(n_k+1). \qquad (5.\ 5)$$

Thus, calculation of the number of failure modes in general degenerated tree type wiring is performed by the following procedure.

Step 1. Disjoin subtrees (containing a partition node) with an input terminal (considering as an output node is annexed in input side wiring) as the farthest partition node from an input terminal (called separating operation). Repeat this separating operation and separate completely the tree into subtrees consisted as shown in Fig. 12.

Step 2. In the subtree set disjoined by separating operation and the subtree of which contains the input terminal (an input subtree), calculate the number of elements of the failure mode set by using (1) and (2) above.

Step 3. Let the numbers of elements of the subtree set and the number of the failure mode set of the input subtree be $c_{s_1}, c_{s_2}, \cdots, c_{sl}, c_p$ respectively. If there exist no isomorphic subtree wiring modes assuming the adjacent node to be fixed,

$$c_d = c_p \cdot \prod_{r=1}^{l} c_{sr}. \qquad (5.6)$$

where c_d is the total number of elements of the failure mode set; otherwise regard c_{s_i} of an isomorphic part as in lemma 3, and calculate the total number by applying lemma3.

Table 5 shows the total number of elements of the failure mode set obtained by this mathod.

Table 5 The number of failure modes of degenerated tree type wiring.

n	2	3	4	5	6
line type	5	13	34	89	233
branch type	0	0	30	130	892
mixed type	0	0	0	75	520
total number of failure modes	5	13	64	294	1,645

6. Conclusion

It becomes important to check backplane wiring and printed circuit board wiring before assembling parts in accordance with higher complex system composition.

In this paper, wiring modes and their failure modes are discussed theoretically for more efficient wiring diagnosis procedures. The wiring modes are classified into transmission line type, branch type and mixed type and the numbers of their modes are calculated by applying several rules. The failure modes of them are also investigated precisely for future application of failure detection and location procedures by using wiring test equipments.

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