Experimental Evidence for Chaotic Dynamics in Decision-Making without Prior Information

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Abstract

In our previous studies, the non-random performance of decision-making without prior information (Blind Choice) has been indicated. This performance is well described by the Golden Ratio and its numerical properties through out experimental conditions. Therefore we have suggested existence of some common mechanism for the Blind Choice. In this paper, we discuss a new numerical method to study this mechanism. The basic idea behind this method is assumption of dynamical system employed for decision-making. The main result obtained using the method is extraction of a specific space describing the system's structures. Here we focus on the organization of this space's structure which is performed in accordance with interaction of repelling and attracting fixed point deployed across the space. These results suggest that mental information processing underlying the Blind Choice is based on a chaotic system. This system and the Golden Ratio are hypothesized to be contained in Implicit Primordial Knowledge.

Keywords: Blind Choice; the Golden Ratio (GR); GR-based proportions; Implicit Pre-knowledge; GR-based Flower-like Fractal; Chaotic Dynamical System.

Introduction

A particular class of decisions is the selection from a set of options. In the case when no prior information about the options is available, all the options are assumed to be of the same probability of selections.

The simplest example of such a case is coin-flipping. Since the coin's symmetry is usually assumed, the probabilities of head and tail outcomes are equal to the chance, at the level 0.5. This result is obtained using classical probability theory. However, actual human behavior (the frequencies of selection of head and tail) produced by human has yet to be investigated.

Another example of uncertainty providing equal probabilities of selection is the case when people have to select one option from the set of options identical to each other.

The first attempt to study human performance under uncertainty has been done by Lefebvre (1992). Later he has developed the Research of Bipolarity and Reflexivity (2006). In the study he introduced some practical approach to the aforementioned problem from the point of view of human trends towards the different poles. The model of the Autonomous Subject developed in this research describes some fundamentals of the Blind Choice. The key point of this approach is an asymmetry of human choices rather than their symmetry, implying equal probability of selection. Furthermore, the asymmetry is derived from various statistical data of individual human and social choices. The key feature of the asymmetry is the Golden Ratio (GR) ϕ (1.6180339...) which is used as a central ratio. The inverted GR value is suggested to be intrinsic to humans in conditions when an individual has no external (other people's opinion and etc.) or internal (emotions and etc.) pressures. That is, under conditions of so-called Free Will.

In our earlier work (Tarasenko, Inui and Abdikeev, 2006), the evidence for the implicit primordial knowledge (pre-knowledge) possessed by the participants and used during the Blind Choice was provided.

Next, we proceeded to investigate the Blind Choice behavior in greater details. We have found that the human frequencies of selection are good described by the Golden Ratio (GR)-based proportions. In contrast, to the pure proportion of the GR such as inverted GR ($1/\phi$), we have obtained the GR-based proportions in the shape of $(\phi+n)/(\phi^i (n+1)\phi+1)$, where n and i are integers, and its derivatives which can be obtained by decomposition of the enumerator (Tarasenko, Inui and Abdikeev, 2007a and 2007b).

Furthermore, we have presented a special space (Coefficient Space) describing the enumerators of presented GR-based proportions. The origins of this space have been described in our recent work (Tarasenko, Inui and Abdikeev, 2007c).

In this paper, we focus on the structure of Coefficient Space in its part related to the chaos and its relationship with dynamical systems. First, we provide a short overview of experiments. Then we compare the results through out experimental conditions. Next, we provide as short introduction to the Coefficient Space and its origins. Finally, we discussed the ingredient of chaos in the location coefficients in Coefficient Space and fractal structure of both empirical and approximating distributions.

Experiment

Objectives

The purpose of this experiment is to investigate the human decision-making performance under conditions of the Blind Choice.

Method

Subjects The samples for 2, 3, 4, 6 and 7 options of the Blind Choice are 674, 412, 377, 560, 219 and 211 participants, respectively. All the participants had normal or corrected to normal vision.

Stimuli Six columns with 2, 3, 4, 5, 6 or 7 identical squares were presented as the stimulus. The stimuli had the minimal information load (Alvarez and Gavanaugh, 2004).

Procedure At first all subjects read the written instructions. No training was provided for any of the subjects. The stimuli were presented to the subjects on the computer screen.

Each column appeared in the same location: the center of the state column matched with the center of the computer screen.

At the beginning of experiment subjects performed fixation task: they had to fixate a mouse cursor on the circle at the center of the screen.

Subjects had to estimate location of the red circle in a column. To complete this task they had to select one of identical squares in a column (Figure 1).

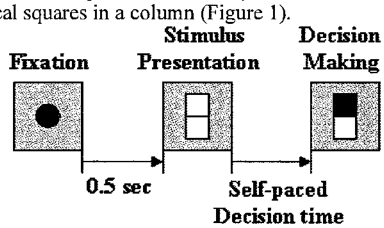

Figure 1: The Experimental task.

Selection was indicated by a blue (black in Figure 1) square inside the chosen cell.

Results and Discussion

For the purpose of the experimental data analysis the squares in the column were enumerated with natural numbers in increasing order, downwards starting from the top square of the column.

An analysis of the frequencies of different responses under the Blind Choice condition showed a statistically significant difference between the frequencies of the response choices and corresponding chance levels under all the experimental conditions.

In the case of only two options, the Binomial test revealed a significant difference between empirical frequencies and the chance level of 0.5 (n = 674, $p < 0.001$)(Figure 2, a).

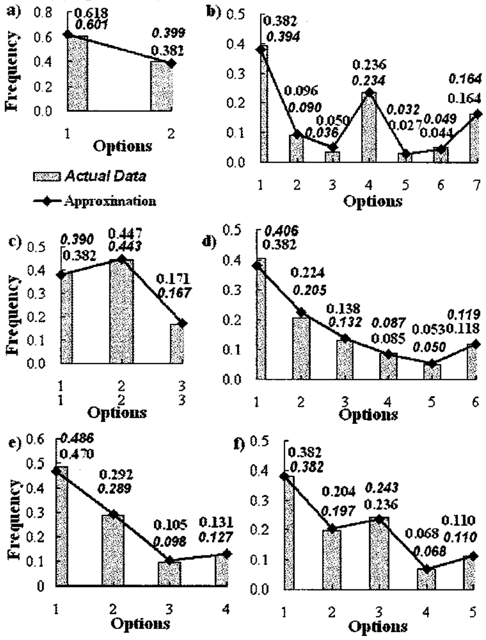

Figure 2: The frequencies of various options selection in the case of 2, 3, 4, 5, 6 and 7 options.

For 3, 4, 5, 6 and 7-option conditions, the Chi-square test revealed a significant difference between the empirical frequencies and corresponding chance levels $\chi^2(2, 412) = 52.937$, $p < 0.001$; $\chi^2(3, 377) = 143.350$, $p < 0.001$; $\chi^2(4, 560) = 176.982$, $p < 0.001$; $\chi^2(5, 219) = 108.260$, $p < 0.001$ and $\chi^2(6, 211) = 185.947$, $p < 0.001$, respectively. The distributions are presented in Figure 2.

Next, we analyzed the empirical frequencies in greater details. The main purpose of this analysis is to determine whether there are common rules describing the frequencies in all the experimental conditions.

Detailed analysis of 3 and 5-option conditions. For the purpose of the further analysis, we introduce a frequency of selection as *fr(p,c)*, where *p* is number of a particular option and *c* is the total number of options, while *fr*(p,c)* is approximation of the corresponding frequency. The *(p,c)* notion is used to identify a particular option. For example, the frequency of the option (5,5) in the case of 5 options is $fr(5,5)$ = 0.11 (Figure 2, f).

In our previous paper (Tarasenko, Inui and Abdikeev, 2007b), it has been shown that response frequencies were well described by the pure GR and GR-based proportions. The pure GR proportions are based on various negative integer degrees of the GR, which can be obtained by identity (1)(id. (1)):

$$1/\phi^{n} = 1/\phi^{n+1} + 1/\phi^{n+2} \quad (1),$$

where n is real number (Gazale, 1999).

Under the GR-based proportions were understand the proportions obtained using combination of the GR and real numbers. These proportions can be obtained using id. (2):

$$\frac{n+\phi}{(n+1)\phi+1} = \frac{1}{\phi} \quad (2),$$

where n is real number. This formula has been first presented by Tarasenko et al (2007c).

The extension of ids. (1) and (2) is id. (3), which represents general GR-based proportions:

$$\frac{n+\phi}{((n+1)\phi+1)\phi^{i-1}} = \frac{1}{\phi^{i}} \quad (3)$$

where n and i are real numbers (Tarasenko et al, 2007c).

In this part of the paper, we present the way the approximations presented in our previous studies has been obtained.

In the case of 2 options, the approximating frequencies are obtained using id. (1) with n=0: $fr^*(1,2) = 1/\phi$ and $fr^*(2,2) = 1/\phi^2$.

In the case of the 3 options, $fr(3,3)/fr(2,3) \approx \phi^2$ and $fr(2,3)+fr(3,3) \approx 1/\phi$. Then denoting $fr^*(3,3)=x$, we obtain equation $x + (\phi^2)x = 1/\phi$, where x is any real number, and its solution is $x = 1/(3\phi + 1)$. Therefore the sum $fr(2,3)+fr(3,3)$ can be described using id. (2) with $n=2$ as $(2+\phi)/(1+3\phi)$. Consequently, $fr^*(2,3)$ equals $(1+\phi)/(1+3\phi)$.

Summarizing the application of ids.(1) and (2) together, we found the following approximations: $fr^*(1,3)=1/\phi^2$, $fr^*(2,3)=(1+\phi)/(1+3\phi)$ and $fr^*(3,3)=1/(1+3\phi)$ (Figure 3 c). The difference between the empirical distribution and its approximation as evaluated by the Kolmogorov-Smirnov test turns out to be insignificant (n = 412; DN = 0.333; K-S= 0.408; p = 0.996).

For the further analysis, we highlight two key points related to the empirical frequencies and their approximations.

There are no equal proportions among the empirical frequencies. This is the first key point. Absence of the equal proportions guarantees that uncertainty will not arise in the next possible steps of decision-making.

The second key point refers to when id.(2)(in general, id.(3)) should be used instead of id.(1). Using ids. (1) and (3), it is possible to provide an iterative procedure for acquisition of approximating frequencies.

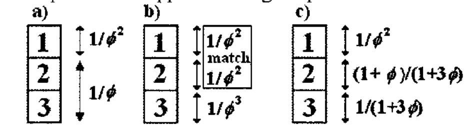

Figure 3: Acquisition of frequencies for 3 options.

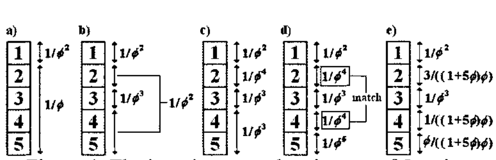

Figure 4: The iterative procedure in case of 5 options.

acquisition of the frequencies is performed in an iterative manner. The example of such an iterative procedure is presented in Figure 3. In the first step $n=0$ and id.(1) is used to obtain frequencies of the options (Figure 3 a)). In the second step $n=1$ and id.(1) is used again (Figure 3 b)). However, two options have the same frequency, which is in conflict with the first key point. Therefore, in the third step id.(1) is substituted for id.(3) with $n=2$, $i=1$. Consequently, we assume that the coincidence of two frequencies in the complete set of frequencies induces the re-division in accordance with id.(3).

Next, to illustrate the iterative assignment of frequencies, we employ an integer counter n and ids (1) and (3) to obtain approximations for frequencies in the case of 5 options as follows:

1) id.(1) with $n=0$: $1=1/\phi+1/\phi^2$ (Figure 4 a));
2) id.(1) with $n=1$: $1/\phi=1/\phi^2+1/\phi^3$ (Figure 4 b));
3) id.(1) with $n=2$: $1/\phi^2=1/\phi^3+1/\phi^4$ (Figure 4 c));
4) id.(1) with $n=3$: $1/\phi^3=1/\phi^4+1/\phi^5$ (Figure 4 d)).

It appears that all the options are assigned with the frequencies obtained using id.(1). But there are two options with the same frequencies ($1/\phi^4$) obtained using this equation. Therefore id.(3) is used in the next step;

5) id.(3) is used with $n=4$, $i=2$: $1/\phi^2=(4+\phi)/((5\phi+1)\phi)$. The numerator of right-hand side of the equality is separated into the components 3, 1 and ϕ. The location of the frequencies given in Figure 4 e).

The resultant distribution is presented in Figure 4 e. The Kolmogorov-Smirnov test revealed the difference between the empirical distribution and its approximation to be insignificant (n = 560; DN = 0.200; K-S = 0.316; p = 0.999).

We have shown that this procedure allows the evaluation of the frequencies of options, and, consequently, it can be used in principle for the exploration of the inherit mechanism of the Blind

Choice. On the other hand, this procedure based on ids. (1) and (3) provides the GR-based proportions.

Detailed analysis of 7-option condition. The frequency fitting iterative procedure proposed in the previous sections works perfectly for in the case of 3 and 5 options. However, implementation of the proposed procedure for the case of 7-option condition did not provide good enough fitting of the empirical frequencies as the procedure of consecutive acquisition of frequencies failed.

Here, we provide a modified procedure of the empirical frequencies acquisition. Since the consecutive frequencies' acquisition failed, we looked for similar properties for empirical distributions of 3-, 5- and 7-option conditions.

For this purpose we analyzed the 3-D diagram of frequencies of all odd-number options conditions. The distributions are merged together to match the center options. This means that $fr(2,3)$, $fr(3,5)$ and $fr(4,7)$ are in the same line. This line is the axis of symmetry. The diagram is presented in Figure 5.

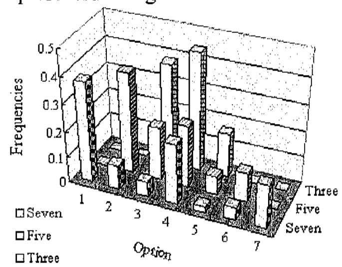

Figure 5: The frequencies of odd-numbered-option conditions.

The empirical distributions are merged by the center-option frequencies, which are $fr(1,3)$, $fr(3,5)$ and $fr(5,7)$. In the new coordinate space $fr(1,3)$, $fr(3,5)$ and $fr(5,7)$ will correspond to the points $(4,Three)$, $(4,Five)$ and $(4, Seven)$, respectively (Figure 5). It can be inferred from Figure 5 that $fr(1,3) \approx fr(1,5 \approx fr(1,7)$ and $fr(3,5) \approx fr(4,7)$. We consider these approximate equalities to highlight the specific part of option 1 and the middle option.

It can be inferred from the above iterative procedure, that a particular value for variable n used as input for id.(3) was $(c-1)$, where c is the total number of options.

Besides, the proposed procedure assumes the acquisition of the frequencies for 3 and 5 options using id.(3). In the case of 3 options, id.(3) was used with $n=2$ and $i=1$: $(2+\phi)/(1+3\phi)$ and the frequencies were obtained using decomposition into proportion $(n-1)/\phi^2$: $(1+\phi)/(1+3\phi)$ and $1/(1+3\phi)$. While in the case of 5 options, the frequencies for three options were acquired using id.(3) with $n=4$ and $i=2$: $(4+\phi)/((1+5\phi)\phi)=3/((1+5\phi)\phi)+1/((1+5\phi)\phi)+\phi/((1+5\phi)\phi)$. The corresponding proportions are $(n-1)/1$ and $1/\phi$.

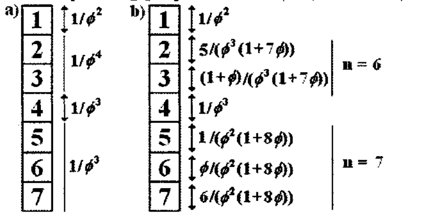

Figure 6: The acquisition of frequencies for 7-option condition.

Using the common features of the empirical distributions and elements of the proposed iterative procedure, we introduce a modified procedure bridging the previous gaps.

In the case of the 7 options, we assume $fr^*(1,7)=1/\phi^2$ and $fr^*(4,7)=1/\phi^3$. Therefore the cumulative unassigned value $Unfr^*$ is $(1-fr^*(1,7)-fr^*(4,7))=1/\phi^2$. According to the location of the options 1 and 4, all 7 options are separated into two groups with unassigned frequencies. Group 1 contains options $(2,7)$ and $(3,7)$. Group 2 consists of options $(5,7)$, $(6,7)$ and $(7,7)$. We consider that the $Unfr^*$ is distributed between theses groups in $1/\phi$ proportion. In this case, the greater part $1/\phi^3$ is assigned to Group 2 as it contains more options than Group 1. Consequently, the cumulative frequency of Group 2 is $1/\phi^4$ (according to id. (3)).

Then setting $n=6$ and $i=4$, id.(3) is used to obtain frequencies for options $(2,7)$ and $(3,7)$: $(6+\phi)/(\phi^3(1+7\phi))=1/\phi^4$. The denominator of the fraction is divided in proportion $(n-1)/\phi^2$, therefore the resulting frequencies are $fr^*(2,7)=5/(\phi^3(1+7\phi))$ and $fr^*(3,7)=(1+\phi)/(\phi^3(1+7\phi))$.

Next, we use equation (5) with $n=7$ and $i=3$ to obtain frequencies for options $(5,7)$, $(6,7)$ and $(7,7)$: $(7+\phi)/(\phi^2(1+8\phi)) =1/\phi^3$. The frequencies follow the proportions given in case of 5 option: $fr^*(5,7)=1/(\phi^2(1+8\phi))$, $fr^*(6,7)= \phi/(\phi^2(1+8\phi))$ and $fr^*(7,7)=6/(\phi^2(1+8\phi))$. The acquisition of the frequencies in the case of 7 options is presented in Figure 6 b).

The Kolmogorov-Smirnov test revealed no significant difference between the empirical distribution of the frequencies and its approximation (n = 211; DN = 0.143; K-S = 0.267; p = 0.999).

Detailed analysis of even-number of options. Here we employ a 3-D histogram for the analysis of the even-numbered conditions presented in Figure 7. The distributions are left-skewed in all cases. The skewness value for the case of 4 and 6-option conditions are -0.956 and -0.902, respectively.

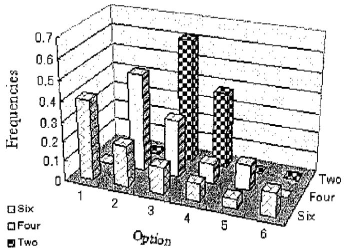

Figure 7: The frequencies of even-numbered-option conditions.

The even-numbered-option conditions are characterized with an imaginary line of symmetry rather than a particular option as in the case of odd-number of options. The cumulative frequencies of options above the imaginary line of symmetry are 0.775 and 0.744 for 4 and 6 options, respective. These frequencies are scattered around the value 0.764, which is $2/\phi^2$. The Relative Error (RE) is calculated as

$$RE = |fr - fr^*|/fr^* \qquad (4)$$

where fr is actual cumulative frequency and fr^* is approximation. The REs for the cases of 4 and 6 options, are 0.014 and 0.026, respectively.

Therefore in the both cases, the cumulative frequencies are distributed in the same manner. In the case of 4 options, the proportions of frequencies above and below the line of symmetry are $fr(2,4)/fr(1,4) \approx 0.595$ and $fr(3,4)/fr(4,4) \approx 0.772$, respectively. Therefore it is assumed that $fr^*(2,4)/fr^*(1,4) = 1/\phi$ and $fr^*(3,4)/fr^*(4,4) = 2/\phi^2$. The approximating distribution is $fr^*(1,4)=0.47$, $fr^*(2,4)=0.292$, $fr^*(3,4)=0.105$ and $fr^*(4,4)=0.131$. The Kolmogorov-Smirnov test revealed no significant difference between the empirical distribution of the frequencies and its approximation (n = 377; DN = 0.25; K-S = 0.354; p = 0.999).

In the case of 6 options, besides the aforementioned similarity with 4 options distributions, the distribution shares the similarity with the distribution of frequencies of 3 options. In fact, the empirical frequencies are $fr(1,3)= 0.39$, $fr^*(2,3)=0.443$ and $fr(3,3)=0.167$. Now we distribute 6 options among three groups: the first group consists of only option $(1,6)$, the second group contains options $(2,6)$, $(3,6)$ and $(4,6)$; the third group consists of option $(5,6)$ and $(6,6)$. The corresponding cumulative frequencies of given groups are 0.406, 0.425 and 0.169, respectively. Since the Kolmogorov-Smirnov test revealed no significant difference between these distributions (n_3=412; n_6=219; DN=0.333; K-S=0.408; p=0.996), the distributions of cumulative frequencies can be approximated in the same manner as the distribution in case of 3 options (Figure 3 c)). The next step is to distribute the cumulative frequencies among the within-group options.

From the empirical data it follows that for the second group $fr(3,6)/fr(2,6) \approx 0.65$ and $fr(4,6)/fr(3,6) \approx 0.64$. Since this proportions belong to confidence interval (1), we consider that $fr(3,6)/fr(2,6) = fr(4,6)/fr(3,6) \approx 1/\phi$. The approximated values of the given frequencies are $fr^*(2,6)=0.224$, $fr^*(3,6)=0.138$ and $fr^*(3,6)=0.085$. Finally, the proportion of the frequencies in the third group is $fr(5,6)/fr(6,6) \approx 0.42$. We assume that this ratio can be approximated with $(1+\phi)/(3\phi+1)$. Thus, the approximated values of the frequencies are $fr^*(5,6)=0.053$ and $fr^*(6,6)= 0.118$. The statistical difference between the empirical distribution and its approximation turns out to be insignificant (the Kolmogorov-Smirnov test: n=219; DN=0.167; K-S=0.289; p=0.999). Considering the increase in the number of options to be a scaling operation, it shows the self-similarity in the case of 3 and 6-option conditions.

Analysis summary. Summarizing the above given results, we can conclude that the GR-based frequencies are intrinsic for the Human Blind Choice of identical options located in a vertical line.

In the case of the odd-number of options, it was possible to propose the iterative procedure of the frequencies acquisition. This procedure is based on ids. (1) and (3). This illustrates that the location of a particular option relatively to the others is of great importance. Furthermore, the proposed procedure renders the frequencies uniformly, thus providing a strict description of its mutual similarity.

The similarity in structure of odd-number of options distributions can be inferred from Figure 5. The distributions are separated into two parts: to the left and right from the center frequencies. The magnitudes of frequencies decrease from both outer options towards the center ones. This can be illustrated as follows: $fr(1,7) > fr(2,7) > fr(3,7)$; $fr(5,7) < fr(6,7) < fr(7,7)$ and $fr(1,5) > fr(2,5); fr(5,5) > fr(4,5)$.

Meanwhile, for the even-numbered-option case, the decreasing dynamics is characterized by one direction towards the lowest option: $fr(1,4) > fr(2,4) > fr(3,4)$ and $fr(3,4) < fr(4,4)$ (Figure 7).

Furthermore, taking into consideration the illuminated self-similarity, we assume the general similarity for all the distributions.

Object with self-similar structure are called fractals. Fractals are of two types: either regular or irregular. The regular fractals share the same rule through out, while irregular fractals (or multi-fractals) have a set of rules which varies for different patterns. The common method to present the fractal structure is the Iterated Function System (Falconer, 2003).

We consider that the given description of similarities in structures of empirical distributions provides the evidence for these distributions to be multi-fractals. Furthermore, ids. (1) and (3) are suggested to be the Iterated Function System describing these multi-fractals.

Evolution of the Blind Choice performance (1): ingredients of chaos. To analyze human performance, the new method for response frequency analysis has been proposed. Its gist is the following representation of the approximation frequencies $fr^*(p,c)$:

$$fr^*(p,c) = k^*(p,c)D(c) \quad (4)$$

where $k^*(p,c)$ is coefficient corresponding to option p and $D(c)$ is normalizing factor equals to the inverted sum of all the $k^*(p,c)$.

The eq.(4) allows for study of the coefficients derived for the different conditions in the same coordinate space – the Coefficient Space. An abscissa of the Coefficient Space is number of a particular option (p), while ordinate is value of a coefficient of corresponding options ($k^*(p,c)$).

In the recent paper Tarasenko et al. (2007c), the structure of this space has been discuss in the case of 2, 3, 4 and 5 options. The main result obtained from the study of the Coefficient Space is that particular coefficients are connected with the imaginary straight lines, whose parameters are either stable or functions of the number of options. For instance, it was found that coefficients of top ($k^*(1,c)$) and bottom options ($k^*(c,c)$) are connected with the *y-lines*, whose equation is

$$y = -\phi x + (c\phi + \phi + 1) \quad (5)$$

for $c \geq 2$, where *y (ordinate axis)* is real continuous function of auxiliary continuous variable *x (abscissa axis)*.

Therefore, the coefficients $k^*(1,c)$ and $k^*(c,c)$ are obtained as intersections of these lines *(y-lines)* with lines x = 1 and x = c, respectively. Therefore, the general equations of these coefficients are

$$k^*(1,c) = c\phi + 1 \quad (6)$$

$$k^*(c,c) = \phi + 1 \quad (7), \text{ for } c \geq 2.$$

Similar equations has been found for coefficients $k^*(1,c)$ and $k^*(2,c)$*(z-lines)*, $k^*(c-1,c)$ and $k^*(c,c)$*(w-lines)*. Another family of imaginary *γ-lines* is used to connect $k^*(c/2,c)$ and $k^*(c/2+1,c)$, for even c, or $k^*((c-1)/2-1,c)$ and $k^*((c-1)/2+1,c)$, for odd *c*. These line families are valid for c > 3. The general equation of the *γ-lines* is

$$\gamma = -\phi x + c\phi \quad (8)$$

for $c \geq 2$, where *γ (ordinate axis)* is real continuous function of auxiliary continuous variable *x (abscissa axis)*.

Using *y-line* and *γ-line*, it is possible to provide new approximations for the 4-option distribution in the Coefficient Space: coefficients $k^*(1,4)$ and $k^*(4,4)$ are obtained using eqs. (6) and (7), while $k^*(2,4) = 2\phi$ and $k^*(3,4) = \phi$ are obtained as intersection of *γ-line* with x= 2 and x=3, respectively. The Kolmogorov-Smirnov test revealed no significant difference between the empirical distribution and approximation (n = 377; DN = 0.250; K-S = 0.354; p = 0.999). Therefore, this technique renders 2-, 3-, 4- and 5-option conditions to be of the same nature.

Moreover, the Coefficient Space allows us to explore the relationship between approximating frequencies' distributions through out experimental conditions more explicitly as it has been done in the previous sections.

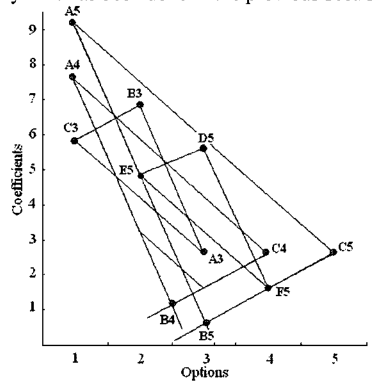

Figure 8: Comparison of the imaginary triangles for 3, 4 and 5 options.

However, it was yet possible to extend this technique for greater number of options. In the case of 4 and 5 options, we consider self-similar triangles A4B4C4 and A5B5C5, where B4 and B5 are imaginary points; A*i* and C*i* (i = *4, 5*) are $k^*(1,i)$ and $k^*(i,i)$, respectively. However, in the case of 3 options, point B3 is real and coincide with $k^*(2,3)$ (Figure 8). However, triangle A3B3C3, where A3 and C3 are $k^*(1,3)$ and $k^*(3,3)$, is similar to aforementioned two triangles. Therefore, under all three conditions the structure of the approximating distributions is self-similar. Therefore, it is possible to use linear operators of rotation and scaling in order to obtain the approximating distribution of 4 options from the approximating distribution of 3 options. Besides, only scaling operator is need to perform transition from triangle A4B4C4 to triangle A5B5C5. However, triangle D5E5F5 is not similar to other three ones. Therefore, linear assumption provides approximations only for four coefficients out of five.

To explain the occurred non-linearity, we suggest a chaos-based analysis of the Coefficient Space.

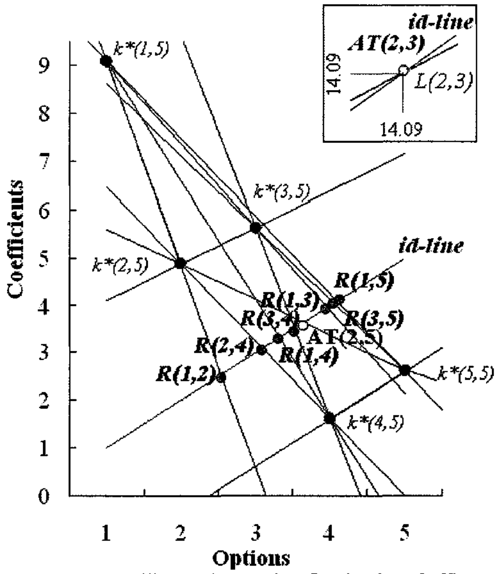

Figure 9: Repelling and attracting fixed point of affine mappings in the case of 5 options.

Here we discuss the iterator mappings. The idea behind is that the previous value of the functions is taken as an argument for the next step. Let we have $f(x)$ mapping and starting point x_0. Then $x_1 = f(x_0)$, $x_2 = f(x_1)$... The important notions is $f'(x_0)=f(f(f(...(x_0))))$. We consider the imaginary lines connecting various coefficients to be these iterator mappings. In fact, such a class of mappings is referred as the affine mapping $f(x) = ax + b$, $a \neq 0$, $b \neq 0$.

Recently, the piecewise models of chaotic attractors have been presented (Amaral et al., 2006). Therefore, we consider that it is possible to use affine mappings for the preliminary description of basic characteristics of dynamics hidden beyond the location of coefficients in the Coefficient Space.

Next, we analyze fixed point of affine mapping. To obtain a fixed point, one should solve equation (9):

$$f(x) = x \qquad (9),$$

where y = x is a bisector line or *id-line*. Therefore, both coordinates of a fixed point x^* are the same.

It is known from that if absolute value of iterator mapping derivative value at the fixed point is less than unit ($|df(x^*)/dx|>1$, x^* - fixed point), this fixed point is repelling fixed point. In contrast to the repelling fixed point, the attracting fixed point occurs in the case when $|df(x^*)/dx|<1$, x^* - fixed point. For the affine mapping, the derivative $df(x^*)/dx$ is constant ($df(x^*)/dx = a$) for any real x.

Therefore, in the case of 2, 3 and 4 options, only repelling fixed point have been found to be deployed across the Coefficient Space.

However, in the case of 5 options, fixed points of both types (repellors and attractors) have been found. Besides, this case is characterized by the occurrence of non-linearity. Therefore, we discuss the case of 5 options in greater details.

In Figure 9 and Table 1, we present all possible straight lines and their equations connecting any two of coefficients $k^*(p,5)$: $L(p_1,p_2)$-*line*, where p_1 and p_2 are numbers of options, connects $k^*(p,5)$. The repellors $R(p_1,p_2)$ and attractors $AT(p_1,p_2)$ of these $L(p_1,p_2)$ lines are marked with grey and white circle, respectively.

Table 1: Equations of lines $L(p_1,p_2)$

Line notion	Equation
$L(1,2)$	-4.2361x + 13.326
$L(1,3)$	-1.7361x + 10.826
$L(1,4)$	-2.4907x + 11.581
$L(1,5)$	-1.6180x + 10.708
$L(2,3)$	0.7639x + 3.3268
$L(2,4)$	-1.6180x + 8.0902
$L(2,5)$	-0.7454x + 6.3448
$L(3,4)$	-4.0000x + 17.618
$L(3,5)$	-1.5000x + 10.118
$L(4,5)$	1.0000x – 2.382

Therefore, the location of the coefficients in the case of 5 options is explained by influence of both repellors and attractors. We suggest that the interaction of repellors and attractors causes the non-linearity to occur.

Evolution of the Blind Choice performance (2): GR-based flower-like fractal. The possible fractal properties of human performance under various experimental conditions has been already discussed in this paper. The reason for discussion was appearance of iterative procedure and inside-group and between-group similarities of even- and odd-numbered-option distributions.

In this section, we state a hypothesis about the structure of the fractal which describes distributions of response frequencies under various conditions. We present two operations describing evolution of this fractal's shape under various conditions.

The first operation is Spiking operation. It is shown in Figure 10 a). This operation describes occurrence of the pick (maximum value) while transferring form 2-option condition distribution to the one of 3 options.

The second operation is Unfolding operation which is characterized by shift of left and right lines forming a pick in the Spiking operation. This causes an additional "bottom"-line (connecting $k^*(2,4)$ and $k^*(3,4)$, Figure 10 b)) similar to the one in the case of 2 options to appear (Figure 10 b) and c)). This procedure reminds the unfolding of the flower blossom out (Figure 10 b)).

Finally, in the case of transition from 4-options distribution to the one of 5 options, both Spiking and Unfolding operations take place (Figure 10 c)). As it follows from empirical data for 6 and 7 options, these operations occur as well under those conditions.

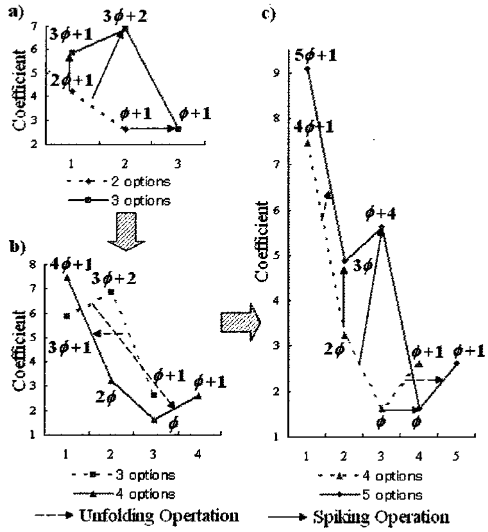

Figure 10: GR-based Flower-like Fractal: Spiking (solid arrows) and Unfolding (dashed arrows) operations in growing structure of Blind Choice distributions.

Therefore, we assume that this is evidence for the fractal nature of the human performance during the Blind Choice. According to the similarity with a flower growing and basis of this fractal is the GR, we call this growing fractal to be a *GR-based Flower-like Fractal.*

General Discussion and Conclusion

In this paper, we have illustrated how the GR-based proportions presented in the previous studies have been derived. We have also highlighted the matter of similarity of both empirical and approximating distributions.

An analysis of such a similarity using the Coefficient Space has shown explicitly an assumed fractal nature of human Blind Choice behavior. This fractal is a growing GR-based flower-like structure.

Besides we have discussed the location of the coefficients in the Coefficient Space which is influenced by the repelling and attracting fixed points of the presented iterator mappings.

The occurrence of such an influence and interaction between repellors and attractors causes appearance of non-linear phenomenon starting from the 5 options, while under lower number of options, human performance can be explained using linear operators of scaling and rotation.

Therefore, we suggest that there is some non-linear chaotic dynamical system underlying human Blind Choice performance.

Finally, we consider that the human Blind Choice behavior refers to the matter of the Free Will (Lefebvre, 2006). Therefore, aforementioned properties of the Blind Choice behavior can be automatically applied to the human Free Will uncovering its sacred structure hidden from our sight before.

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