Some Experiment With Tic-Tac-Toe

Uri Globus [1], Hiroyuki Iida [2,3]

[1] Graduate School of Science and Engineering, Shizuoka University, Japan
globus@cs.inf.shizuoka.ac.jp

[2] Department of Computer Science, Shizuoka University, Japan

[3] Information and Systems, PRESTO, Japan Science and Technology Agency
iida@cs.inf.shizuoka.ac.jp

Abstract. In this paper we describe the results of our experiment designed to find the relation between playing strength and game length. We used semi-random play as a model for players strength and made self-play experiments between computer programs of equal strength. We later discuss the plausible implications of our results to a broader group of classical games (i.e., chess-like games)

1. Introduction

This research is part of a general study on game informatics. It is based on the previous study [1] suggesting computer analysis as a mean for obtaining insight regarding general characteristics of games.

It is our observation that game designs are evolutionarily changing towards refinement of their entertaining impact. One important clue to a game's entertainment is the game's length. However, game length may not be the same for all players. In this paper we study the relation between playing strength and game length.

In order to model players of various strengths we have used semi-random self-play[2]. The exact definition as well as our seemingly counterintuitive results with Tic-tac-toe are presented and explained in Section 2. These results have encouraged us to slightly modify the original algorithm.

Section 3 presents and discusses our results with our new variant of semi-random self-play that we denote as semi-random die-hard. Lastly, Section 4 is a discussion about some general implications of this research and its plausible future direction.

2. Semi-Random Self-Play

2.1 Definition

We hereby reproduce the definition of [2]:

Definition Semi-random self-play is defined by the two following rules R1 and R2.

R1: Generate all possible moves and try a game-tree search to look ahead by a given search depth. If there is a winning move (by which the max player is able to reach a winning position), then choose it.

R2: Remove losing moves (after which the min player is able to reach his winning position) from the list of candidates at a position considered. If the list is not empty, select a move among the list at random. Otherwise, select a move at random among all possible moves.

We denote Pi $\{i \in N \mid 0 \le i \le 9\}$ the player which looks ahead to depth i. Thus Pi's give a gradual difference of strength ranging from random play at P0 to perfect knowledge denoted as P9.

2.2 Experimental Design

We have performed experiments between equal strength players in two modes. In the first mode we let the game finish and recorded its length. In the second mode we have added an option to stop the game earlier by either a resign or by a draw proposal.

For each category we have performed 10,000 repetitions and gathered statistical data. It is important to note that this algorithm does not use any heuristic evaluation but rather distinguishes only between 4 values: 'win', 'lose', 'draw' or 'unknown'.

2.3 Results and discussion

In Table 1, the three right hand columns are a revision of the tests carried by [2]. The other 4 columns are our new results. For each of the two modes (with stop, no stop) the left side column shows the average length of game and the right hand side column shows the standard deviation respectively.

	With Stop		No Stop		Win Rate	Draw Rate	Lose Rate
	Av (ply)	S. Dev	Av (ply)	S. Dev			
P0vsP0	7.62	1.31	7.62	1.30	58.11	13.08	28.81
P1vsP1	5.97	1.14	6.03	1.25	68.89	4.41	26.70
P2vsP2	6.31	0.86	8.30	0.86	30.72	52.08	17.20
P3vsP3	5.57	0.58	7.86	0.90	52.64	29.82	17.54
P4vsP4	4.33	0.86	7.62	1.70	30.75	54.42	14.83
P5vsP5	3.35	0.50	6.42	1.73	67.43	26.89	5.68
P6vsP6	3.00	0.00	9.00	0.00	0.00	100	0.00
P7vsP7	2.00	0.00	9.00	0.00	0.00	100	0.00
P8vsP8	1.00	0.00	9.00	0.00	0.00	100	0.00
P9vsP9	0.00	0.00	9.00	0.00	0.00	100	0.00

Table 1: Results of semi-random self-play experiments
Av = average length of game, S. Dev = standard deviation of length
Stop can occur by either a resign or a draw proposal.

In the mode with stop option we see that the game length is highly shortened when moving from P0, a random player to P1, a weak player. That happens as simple winning positions are not missed by the P1 player. From P1 to P2 we see an increasing of the game length as the same early winning positions that were reached by P1 are now avoided. From P2

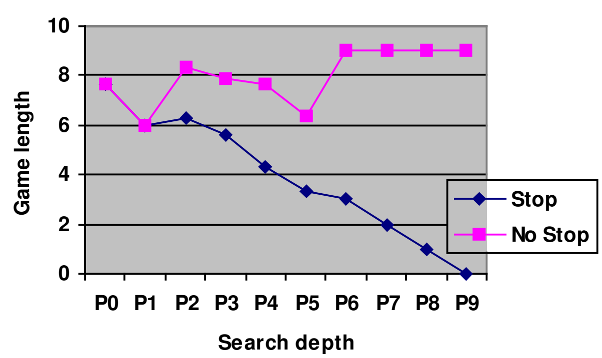

Figure 1: average game length in semi random self play

to P9 players' strength and their average game length are in inverse relation.

In the mode without stop we have wrongly anticipated more fluctuations of the game length in the middle range (P3-P5). Our anticipation was based on the following observation. When search depth increases from odd numbers to even numbers, more losing positions are avoided resulting theoretically in longer games. However, when search depth increases from even to odd more winning positions are found, resulting theoretically in shorter games. Therefore, the results of P3, P4 and P5 seemed suspicious at first look. They do not fluctuate nicely according to theory. Moreover, when we looked at actual play we observed that in some cases, playing strength decreased when search depth increased.

Take a look at the two Tic-tac-toe positions illustrated in Figures 2 and 3.

A	b	**O**
C	d	e
X	f	**X**

Figure 2

a	b	c
d	**X**	E
f	**O**	**X**

Figure 3

In the following two positions P0 and P1 both select randomly any move on an empty square (a-f). In case of P1, that means 5/6 probability the game will finish on the next move as P1 will definitely complete a line if it is possible. P2 and P3 have only one choice: 'f' in figure 2 and 'a' in figure 3. P4 and P5 behave like P0 in this situation. Both select a random move among 'a' to 'f' resulting in a 5/6 probability the game will end on the next move. The reason for this behavior can be found in the algorithm. P4 and P5 already recognize these positions as lost ones and according to the algorithm play randomly.

Table 2 presents the rate of winning at a certain move number. The rate of winning on the 5th move is related to the probability that such positions like those presented in Figures 2 and 3 (that are not unique) are formed. We observe that P5 can already deliberately form such positions on the 3rd move if possible. The indifferent behavior (i.e., playing randomly) of this algorithm in losing positions has encouraged us to create and test the following variation.

Move Pi	5th	6th	7th	8th	9th
P0	9.48	8.94	26.49	19.87	22.14
P1	47.86	21.71	17.55	4.99	3.48
P2	0.00	0.00	26.23	17.20	4.49
P3	0.00	0.00	48.03	17.54	4.61
P4	21.38	12.00	7.00	2.83	2.37
P5	54.52	4.63	12.35	1.05	0.56

Table 2: Rate of winning in a certain move (ply)

3. Semi-Random-Die-Hard

3.1 Definition

Semi-random-die-hard self-play is defined by the three following rules R1-R3:

R1: Generate all possible moves and try a game-tree search to look ahead by a given search depth. If there is a winning move (by which the max player is able to reach a winning position), then choose it.

R2: Remove losing moves (after which the min player is able to reach his winning position) from the list of candidates at a position considered. If the list is not empty, select a move among the list at random.

R3: In losing positions, select a move that prolongs the game as much as possible.

3.2 Tic-tac-toe results and discussion

Table 3 shows the results of semi-random die-hard self-play experiment and Figure 4 illustrates them. With a stop option the algorithm behaves in the same way as the semi-random self-play algorithm and therefore those results were omitted from Table 3.
It can be seen that the average game length fluctuates from P0 to P6 and then reaches stability. There is a correlation between search depth and game techniques. P2 for example recognizes naïve threats and defends against them. P5 is already capable of deliberately forming double threats. P6 is a level that though it has no complete knowledge its performance is perfect.

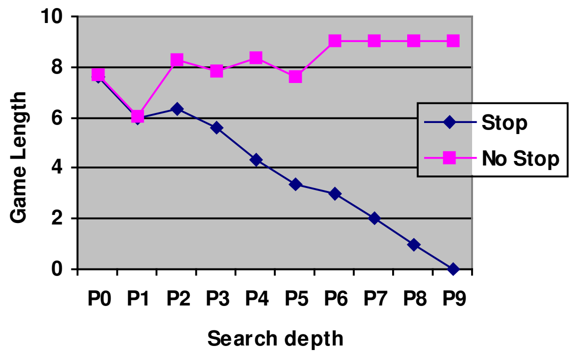

Figure 4: Search depth and game length with semi-random-die-hard self-play

	No Stop	
	Av (ply)	S. Dev
P0vsP0	7.65	1.29
P1vsP1	6.04	1.25
P2vsP2	8.30	0.85
P3vsP3	7.87	0.90
P4vsP4	8.33	0.86
P5vsP5	7.61	0.89
P6vsP6	9.00	0.00
P7vsP7	9.00	0.00
P8vsP8	9.00	0.00
P9vsP9	9.00	0.00

Table 3: Results of semi-random die-hard

3.3 Results on a 4x4 board

Tic-tac-toe's theoretical outcome is a draw. We wanted to find out what is the relation between playing strength and game length in a game that has a non-draw theoretical outcome (i.e., win for one of the players). Tic-tac-toe is one game of a larger class of games called MNK-games [4] in which M by N boards are used and a player can win by putting K pieces in a raw. In a MNK perspective, Tic-tac-toe is a (3,3,3) MNK-game. We have chosen the (4,4,3) MNK-game as it's theoretical outcome is "win for the first player".

	With Stop		No Stop		Win Rate	Draw Rate	Lose Rate
	Av (ply)	S. Dev	Av (ply)	S. Dev			
P0vsP0	9.98	2.35	9.78	2.43	60.20	0.00	39.80
P1vsP1	6.02	1.24	5.99	1.14	69.30	0.00	30.70
P2vsP2	6.31	1.99	8.44	1.95	58.10	0.30	41.60
P3vsP3	3.76	1.20	5.80	1.30	85.80	0.20	14.00
P4vsP4	3.37	1.78	7.45	1.78	70.20	0.00	29.80
P5vsP5	1.00	0.00	7.00	0.00	100.00	0.00	0.00
P6vsP6	1.00	0.00	7.00	0.00	100.00	0.00	0.00
P7vsP7	1.00	0.00	7.00	0.00	100.00	0.00	0.00
P8vsP8	1.00	0.00	7.00	0.00	100.00	0.00	0.00
P9vsP9	1.00	0.00	7.00	0.00	100.00	0.00	0.00

Table 4: Results of semi-random die-hard self-play for 3 in a row on a 4x4 board.
Av = average length of game, S. Dev = standard deviation of length

Figure 5 illustrates the results presented in Table 4. We can see that with a no-stop option the average game length fluctuates and gradually converges to the length of a minimal principal variation (which may not be unique). The same fluctuations occur in the case of a stop option, but the average game length is a decreasing function of the playing strength. Because this game's theoretical outcome is a "win for the first player", the first player always plays the first move, after that his opponent may resign depending on his strength.

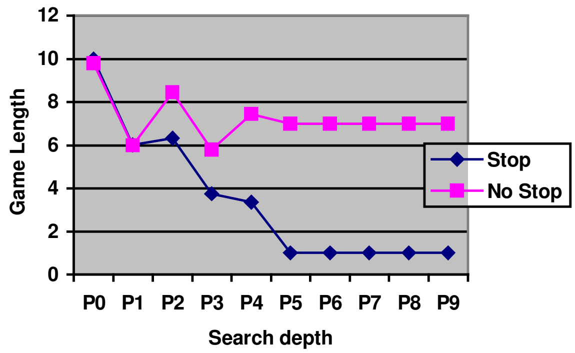

Figure 5: Search depth and game length with semi-random-die-hard self-play results in (4,4,3) game

Take a notice on the difference between the no-stop lines in Figure 4 and Figure 5. Before reaching a constant level both lines fluctuate. However, in Figure 5 fluctuations are very clear in comparison. This difference can be explained by the relatively very low rate of draws in the (4,4,3) game.

4. General Discussions

4.1 The relation between playing strength and game length

No matter how strong the players play, the position balance gets upset sooner or later. Tying (i.e., draw) is the only exception. Although every game may start out evenly in the beginning, at certain point, the position balance is upset. We call this point the "game equilibrium". The equilibrium point can be approximated by the number of moves required to end a game, i.e., the game length. We conjecture that such equilibrium depends on the playing strength. Moreover, the statistics on the average number of possible moves and game length, obtained from master games in classical games such as chess, shogi and Go, will converge at a certain value. Therefore, when the group of players is fixed, the statistic on the game length obtained from their games might be unique to each game.

Note that different kinds of seesaw may occur during a game. Weak players tend to miss strong moves when playing each other, thus a game is well balanced. On the other hand, good players tend to play optimal moves, and so the seesaw of a game is preserved longer. Our experiments with Tic-tac-toe show that under the condition of no stop (resign or draw proposal), the game length is a decreasing function of playing strength at some weak levels (P0-P1), whereas it is basically an increasing function of playing strength at some strong levels (P1-P6). Under the condition of stop, it is a decreasing function of playing strength at every level except P2. In the future, we plan to study the scope and applicability of this theory to more complicated games.

4.2 The limit of human capacity

The players at some level somewhere between P4 and P5 in our experiments with Tic-tac-toe may correspond to masters in classical games like chess. This means that the game length may suggest the limit of human capacity of game playing skill. Iida *et al.* [3] pointed out that refined games such as chess, shogi and Go, which have a long history, take a close value of the root square of the average possible moves over the game length.

4.3 The relation between the outcome of games and fairness

Competitive games are designed as to conclude who is best. To establish their goal they are refined over time as to improve fairness. In most classical board games (such as chess, Go, Othello, and Tic-tac-toe) the first player has a big advantage. There are some mechanisms for balancing this advantage and achieving fairness. The most fundamental of these mechanisms is having a theoretical draw outcome. Tic-tac-toe is a good example. However, we can see from the results of our experiment that it only achieves fairness for perfect players, i.e., players P6-P9 in our experiment. For other levels of play, the first and second players cannot maintain fairness in the sense of winning percentage. Namely, The first player has a big advantage. Tic-tac-toe can have a draw only at the last possible move (9^{th} ply). It may support the relation between the playing strength and game length depicted in Figure 1 and 4. For a game for which the ultimate outcome is draw, the game length might be an increasing function of playing strength when it is played until the very end of the game (i.e. no resigning or draw proposal). For games that have a theoretical outcome other than draw, (viz. win for either player) it is hypothesized that as playing skill increases a game's average length will converge to the length of a shortest principle variation.

Consider the more general relation between the outcome of games and fairness. Take for example the chess-like games: chess, Chinese chess and Shogi. In chess and Chinese chess, for some standard level players, 30% to 40% of all games end up in a draw. This statistics increases as the playing strength becomes higher. In the two games where draws occur frequently, unless a winning player has a large enough advantage gap, the game usually ends up in a draw. Therefore, the ability of widening small advantages is very important. In games with frequent draws, the equilibrium is often upset early on in the game. Hence, the first player is more likely to have an advantage. On the other hand, games for which equilibrium can easily becomes upset early in the game, maintain fairness by adopting draws, as well as increasing their chance of occurrence.

In fact, statistics show that in master games of chess and Chinese chess, there is a slightly higher probability of draws than in similar Japanese chess games. This indicates that in chess and Chinese chess, despite the significant advantages that the first player has, the games maintain their fairness by having the possibility of draw.

Draw is like a gift to the second player (handicap to the first player) who does not have the initiative of the game [4]. However, if the advantage at the beginning of the game is too great to the point that even a draw is not possible, then this "gift" is meaningless. Other games use other kind of gifts (handicap) to the second (first) player in order to maintain fairness. The kind of gifts range from simple, like Komi (points given to the second player) in Go, to more complicated ones, like the opening procedure of Renju. These mechanisms refine the balance of the game thus allowing and prolonging the seesaw of the game.

5. Conclusions

In order to find the relation between players' strength and their average game length we have conducted self-play experiments with two different algorithms. We have used the original semi-random self-play [2] and a modified version, namely, semi-random die-hard, to simulate play between players of equal strength. For each level of play 10,000 matches where conducted in two modes: one in which stopping of the game by either a resign or a draw proposal is allowed, and another in which play must continue until a concrete outcome was reached (i.e., win, draw, lose).

Our results with Tic-tac-toe show that with a stop option the relation between average game length and playing strength is a decreasing function except for some weak level (viz. P2). In the case that play has to continue until a concrete outcome was reached, the two algorithms behave differently. The original semi-random self-play algorithm [2] plays randomly in losing positions and so shows a seemingly pathological behavior, namely, a deeper search renders a shorter game (i.e., P5's average game length is shorter than that of P2). The modified version, namely, semi-random die-hard, avoids losing for as much as it can. With this algorithm, the average game length is an increasing seesaw function of the playing skill. The average game length clearly fluctuates between odd and even search depths until it reaches perfect play (P6-P9).

We have conducted another experiment to check the relation between game length and playing skill in a game that have a non-draw theoretical outcome. We used 3 in a raw game on a 4x4 board with the semi-random die-hard algorithm. In the no-stop mode the average game length is a seesaw function that converges to the length of a minimal principal variation. In the mode with stopping (by either resign or draw proposal) the average game length is a decreasing seesaw function of the playing strength.

It is still an open question to what scope can these results be generalized. As well, do other games have a different relationship between playing strength and their average game length?

6. References

1. Iida, H., Hashimoto, T., Sasaki, N., Uiterwijk, J.W.H.M., Herik, H.J. van den. : A Computer Analysis: Towards a Classification of Games, Boardgames in Academia III, Italy (1999).
2. Kajihara, Y., Sakuta, M., Iida, H.,: Semi-Random Play in Game Playing. Proc. JICAST99, Zhejiang University, China pages 205-208 (1999).
3. Iida, H., Takeshita, N., and Yoshimura, J.: A Metric for Entertainment of Boardgames: its implication for evolution of chess variants, Proc. IWEC2002. pages 65-72 (2003).
4. Uiterwijk, J. W. H. M., and Herik, H. J. van der.: The advantage of the initiative, Information Sciences 122 (1) pages 43-58 (2000).