Theoretical Value Prediction in Game-Playing

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ABSTRACT

One of the most important characteristic of a game is represented by its theoretical value. However, to find out the theoretical value of a game is time expensive and it is almost impossible to establish it in complex games. Therefore, we propose three different methods to predict the outcome of the game under ideal play. Such methods use information concerning diminishing return, winning rate changing, and score difference obtained by self-play experiments. We apply these methods to some games and we discuss the validity of every single method.

1. INTRODUCTION

The theoretical value of a game represents the outcome of the game under ideal play, i.e., when both players apply their optimal strategies.
Nowadays, the main goal of game researchers is to improve search algorithms in order to make computer players more competitive. In this kind of research, we can distinguish two different aspects. From the one hand, search algorithms are useful in real world applications, e.g., artificial intelligence; on the other hand, it can be interesting to study the nature of the game itself analyzing its characteristics. One of the most important peculiarities of a game is undoubtedly represented by its theoretical value (Iida, 2004; Iida and Yoshimura, 2003).
Every N-player zero sum game has an unique theoretical value. For example, the outcome of Tic-Tac-Toe is always a draw when strong players are involved. Moreover, even in complex games like Chess, Shogi or Go, there exists a unique theoretical value.
However, to establish the theoretical value of a game and the optimal strategy can be a hard task because, e.g., the major part of classical board games belong to the hardest classes of complexity as $\mathcal{NP}$, $\mathcal{PSPACE}$ and $\mathcal{EXP}-\mathcal{TIME}$.
To determine the theoretical value, except for simple games as Tic-Tac-Toe, it is almost impossible for games with huge game-tree. The game of Chess has an average branching factor of 35 and a typical game takes about 80 moves; it follows that the whole game-tree has about 35^{80} nodes, i.e., much more than the estimated number of atoms in the universe (10^{80}).
Section 2 presents the game we use as test-bed for our experiments. In section 3 we introduce our methods that they will be explain in details in section 4, 5, and 6 respectively. Finally, our conclusions and future works.

2. ABOUT SELF-PLAY EXPERIMENTS

The games used as case study in our experiment are Chess, Reversi, and Tic-Tac-Toe. We used computer players for self-play experiments as shown in table 1.

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Game	Authors	Software
Chess	Heinz (Heintz, 2000)	FRITZ6
Reversi	Kita et al.	WZebra (Andersson and Ivansson, 2004)
Tic-Tac-Toe	Kajihara et al. (Kajihara, Sakuta, and Iida, 1999)	Self-made (Semi-random play)

Table 1: Games, authors and software.

3. METHODS

To find out the theoretical value of a game is really time expensive, e.g., solving 6 by 6 Reversi took two weeks of computation to analyze 4×10^{10} nodes of the game-tree (Feinstein, 1993). Table 2 shows game-tree complexity and state-space complexity for some classical board-games (Ishikawa, 2000). Because of their huge complexity, to find out the theoretical value of some games like Go and Shogi is clearly an impossible task from a technical point of view. For this reason, we propose three different methods to predict the theoretical value of a game.

Game	Game-tree	State-space
Checkers	10^{31}	10^{18}
Reversi	10^{58}	10^{28}
Go-moku	10^{70}	10^{105}
Chess	10^{123}	10^{50}
Shogi	10^{220}	10^{80}
Go	10^{360}	10^{172}

Table 2: Game-tree complexity and state-space complexity.

These methods use experimental data obtained by self-play computer games.

- The first one is the method of *diminishing return*. Originally, the term diminishing return was used in economy to describe the phenomenon where if resources increase then profits decrease. In our case, if we increase our computer resources then the difference of performance between players should decrease.
- The second method concerns the analysis of the change of winning rate to predict the theoretical value.
- The third method focus on the regression of the score difference and, of course, we can only apply this method to game which have a score. Therefore, we cannot apply this method to games like Chess and Shogi.

4. PREDICTION USING DIMINISHING RETURN

In order to use the method of diminishing return, we make a handicap self-play experiment (Heintz, 2000). Table

Depth	W:D:L/Total	Wins	Draws	Losses	Score	ELO
6 vs. 5	1686:915:399/3000	56.20%	30.50%	13.30%	71.45%	+159
7 vs. 6	1643:1066:291/3000	54.77%	35.53%	9.70%	72.53%	+169
8 vs. 7	1457:1212:331/3000	48.57%	40.40%	11.03%	68.77%	+137
9 vs. 8	1093:1133:274/2500	43.72%	45.32%	10.96%	66.38%	+118
10 vs. 9	434:509:107/1050	41.33%	48.48%	10.19%	65.57%	+112
11 vs. 10	404:539:107/1050	38.48%	51.33%	10.19%	64.14%	+101
12 vs. 11	375:550:125/1050	35.71%	52.38%	11.90%	61.90%	+84

Table 3: Self-play experiment by Heinz.

3 shows the results of self-play experiments made by Heinz. In particular, we pay attention to decreasing of ELO difference. Figure 1 shows the model concerning the relation between diminishing return and strength. Four points, s_0, s_1, s_2, and s_3 show the strength of the players. In the case of computer program, the factor of

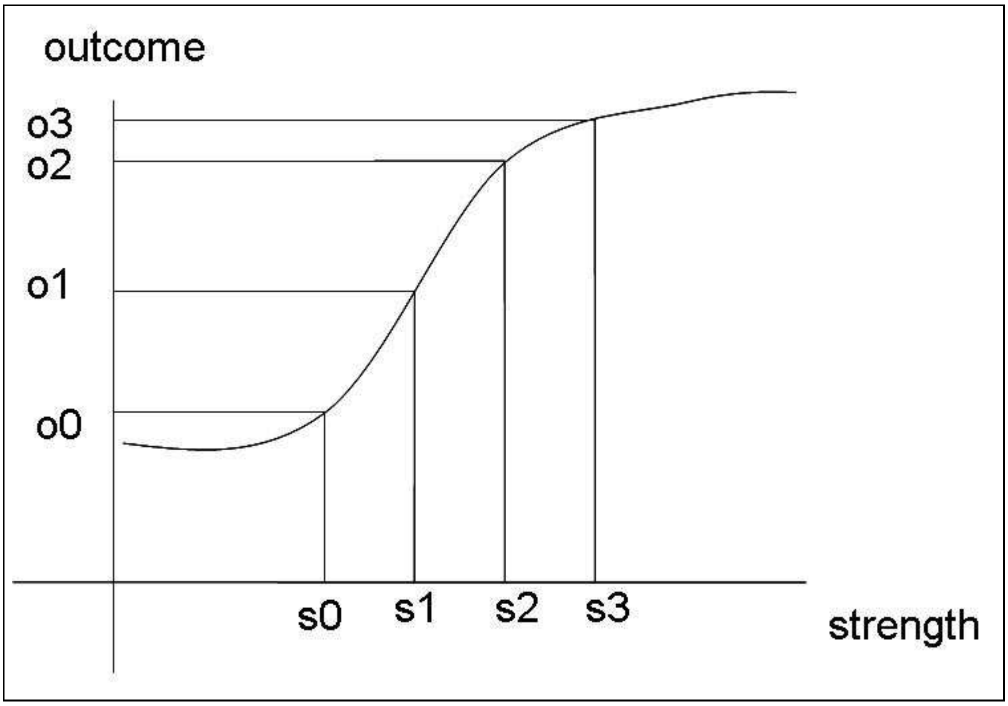

Figure 1: Model of diminishing return and strength.

strength is represented by the depth of search or the correctness of the evaluation function. Moreover, o_0, o_1, o_2, and o_3 represent the probability, during the experiment, to guess the outcome of the game (white wins, black wins, or draw). We consider the model shown in figure 1 as transition as a result of a game. In such model, it is natural to set $o_3 - o_2 < o_2 - o_1$ according to the experiment of Heinz. If a player becomes strong, the difference between o_n and o_{n+1} should decrease. And the player must be the strongest when there exists no more difference between players. It is plausible that the result at that time is equivalent to the game theoretical value. However, to make experiments until there exist no more difference between players, it costs as time as it requires to find out the theoretical value of the game analyzing the whole game-tree. Therefore, it is more important to look for in the diminishing return model the point with a big change. Figure 2 and figure 3 show the draw rate for the game of Tic-Tac-Toe and Chess respectively. It turns out that figure 1 and figure 2 are very similar each other. Comparing figure 1 and figure 3, we guess that, in the case of Chess, s_0 has not been reached yet. Probably, it should be necessary a little more deep search. Even if the model of Figure 1 is right and the theoretical value can be predicted in such way, it is not necessarily helpful because we do not know if players apply the optimal strategy in the game-tree.

5. PREDICTION USING WINNING RATES

This is simplest method we present in this paper. We process data about winning rates using the least-squares method and predict the theoretical value from the results. However, applying this method it comes out a problem as shown in the game of Reversi. In this case at 60-th move the outcome is not clear.

We expose some ideas concerning the interpretation of this results.

1. Even though we put in correlation depth search and strength, these two factors are not proportional in fact. Are depth-search and strength correctly proportional? Probably, they are not. Moreover, there is a problem of diminishing return.
2. White has a winning strategy under ideal play because he/she has the highest winning rate.
3. A draw with the largest increasing rate (actually White wins and Black wins both decrease) is the theoretical value.

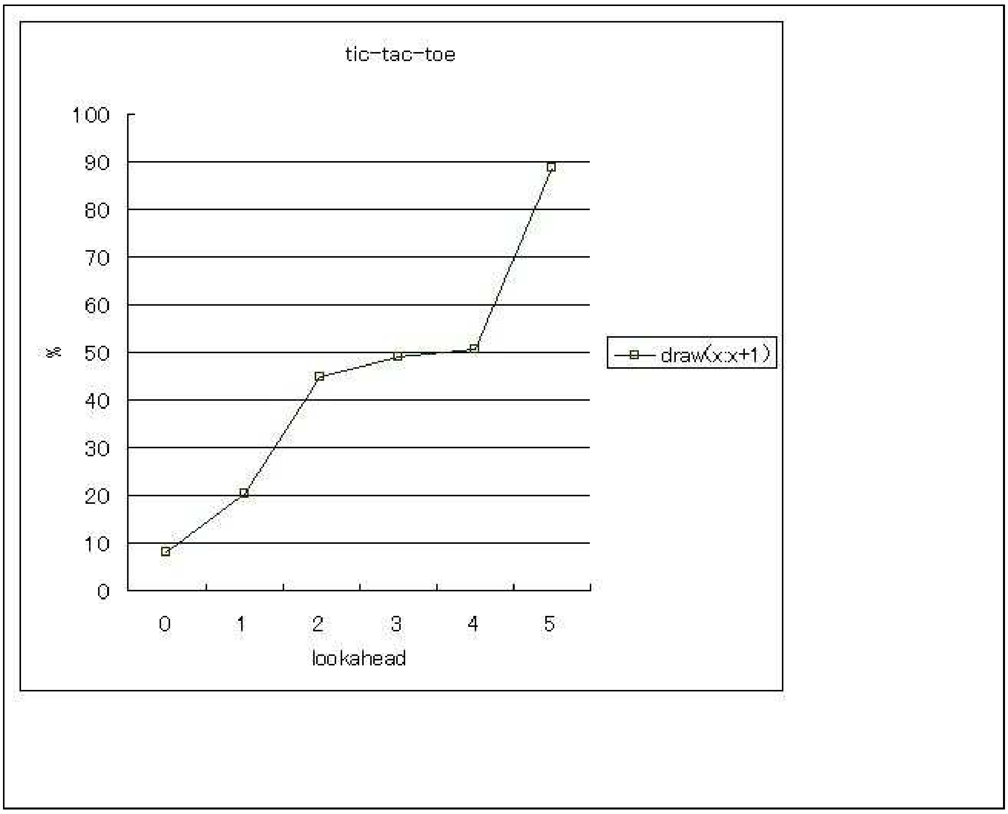

Figure 2: Results for Tic-Tac-Toe.

4. The theoretical value of the game is not represented by this graph.

The second and third ideas are very similar each other. In order to confirm which is the most plausible, we show the example of Tic-Tac-Toe. Tic-Tac-Toe is a solved game. The result of Tic-Tac-Toe resembles the result of Reversi very much. Therefore, it seems that our third hypothesis is the most plausible.
The fourth idea is completely different from the other three ideas.

6. PREDICTION USING SCORE DIFFERENCE

This method resembles the method discussed in section 5; moreover, this method can predict both the outcome of the game and the score difference between players. The bad news is that only a few games can be analyzed with this method. Figure 6 shows the prediction of theoretical value by regression from the score difference. When depth search is 60, the score difference is 0 exactly therefore, we suppose that the outcome is a draw.

7. CONCLUSION AND FUTURE WORKS

We proposed three different theoretical value prediction methods using data obtained by self-play experiment: diminishing return, winning rate change, and score difference. We discussed advantage and disadvantage of each method. In the future we will try to apply our methods to others games in order to establish their validity.

8. REFERENCES

Andersson, G. and Ivansson, L. (2004). WZebra. http://radagast.se/othello/.

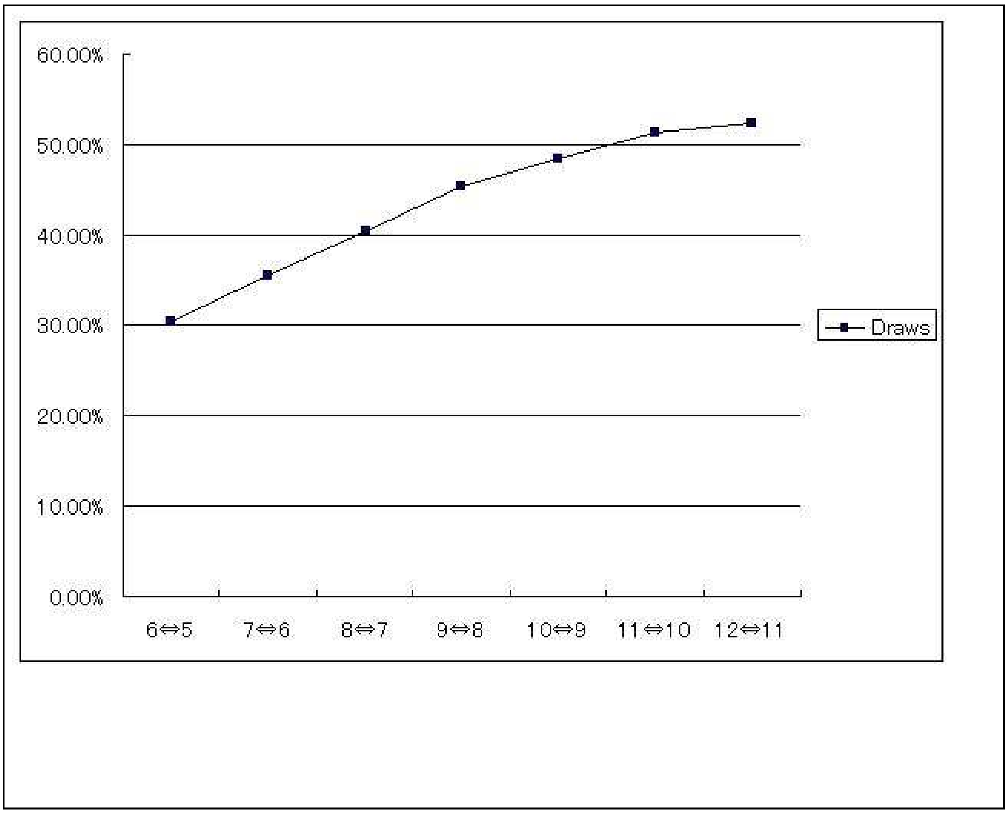

Figure 3: Results for Chess.

Feinstein, J. (1993). The perfect play line in 6x6 Othello. http://www.maths.nott.ac.uk/personal/jff/othello/6x6sol.html.

Heintz, E. (2000). A New Self-Play Experiment in Computer Chess. Technical Report MIT-LCS-TM-608.

Iida, H. (2004). Game Equilibrium. Technical report, IPSJ SIG 2004-GI-12. in Japanese.

Iida, H. and Yoshimura, J. (2003). A Logistic Model of Game's Refinement. Technical report, Shizuoka University.

Ishikawa, D. (2000). Game programing reserch and Reversi. http://dais.main.jp/reversi/bit.html.

Kajihara, Y., Sakuta, M., and Iida, H. (1999). Semi-Random Play in Game Playing. *Proc. of JICAST'99*, pp. 205–208.

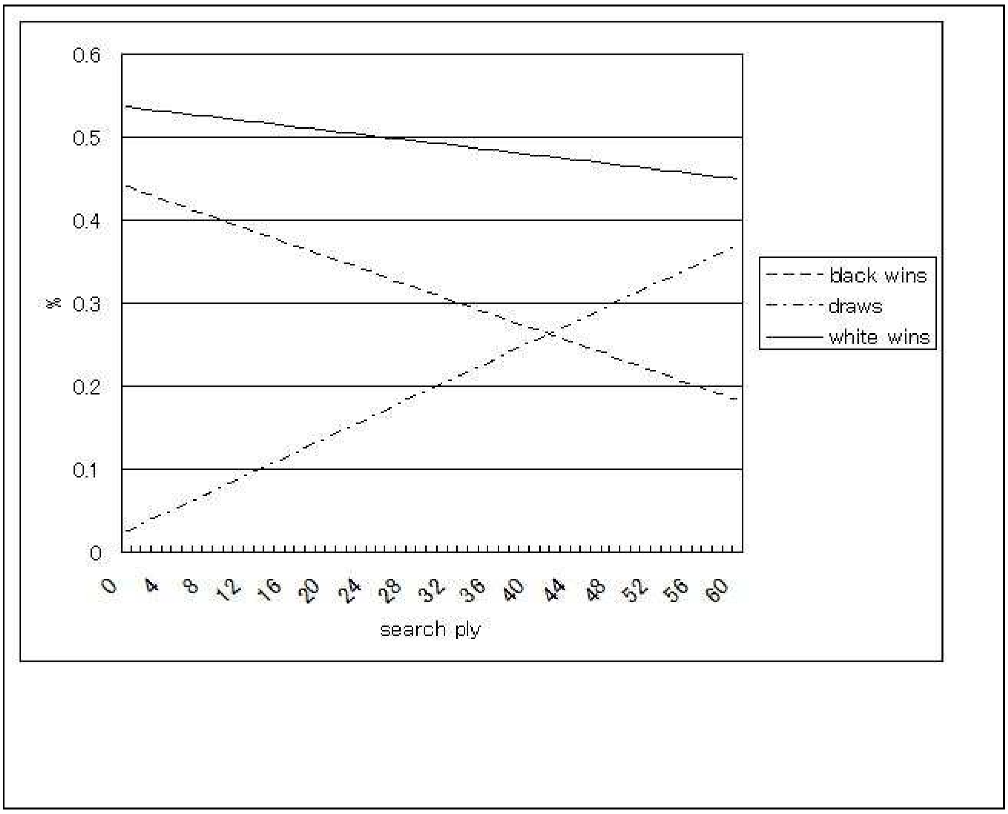

Figure 4: Results for Reversi.

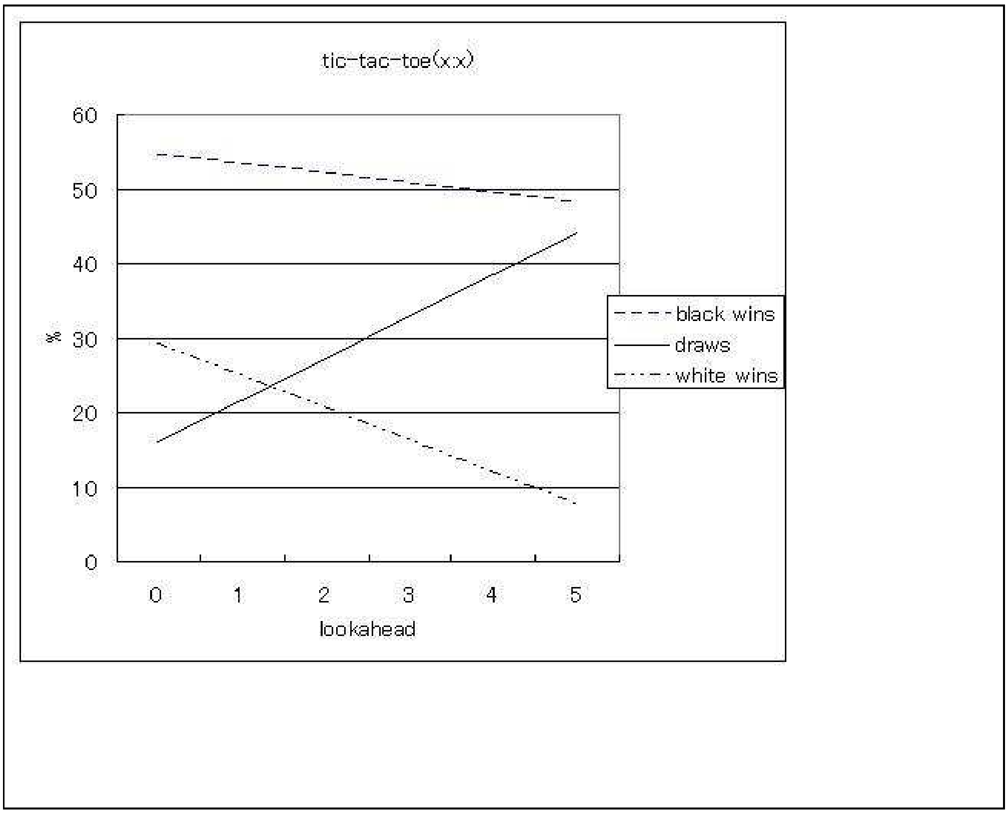

Figure 5: Results for Tic-Tac-Toe.

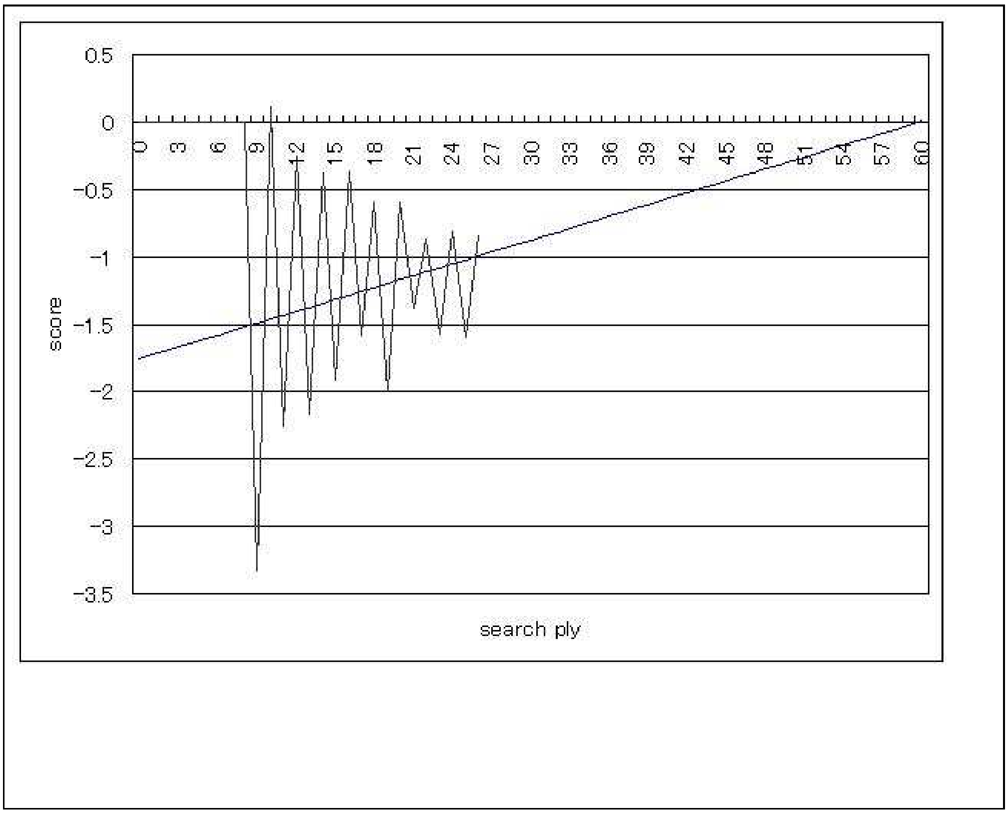

Figure 6: Results for Reversi.