社団法人　情報処理学会　研究報告
IPSJ SIG Technical Report

2005－GI－14（7）
2005／9／5

図形合同分割パズルの自動生成

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概要

図形の分割は幾何的問題で、思考パズルの１種類としても楽しまれている。図形分割は一般的に、アナログ的であり、コンピュータによる処理はそれほど容易ではない。ここでは単位図形（単位正方形）で構成された図形に限り、また分割線も単位図形の境界に限るという条件にし、この問題の１つである図形を二つの合同図形に分割するという問題に取り組んだ。

まず、この分割問題を解くアルゴリズムを設計・実現した。また、これを用いて多量の分割問題を自動生成した。さらにその問題の一部を被験者に解かせ、「面白い」及び「難しい」という人間の感覚が図形のどんな性質に関連するかを解明した。

Automatic Generation of Congruent Figure Dissection Problems

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Abstract

Dissection of figures is variety of geometric problems, and has also been amused as a kind of various thinking puzzles. Figure dissection is not so easy to treat with the computer, because of the difficulty of digital description. In this paper we study this problem in the condition of limiting only the figure composed of unit figures (unit squares) and only the division line to the boundary of unit figures, focusing on the problem of dissection into two congruent pieces.

First of all, we designed and implemented the algorithm that solves the problem. Moreover, a large amount of division problems was generated automatically by the use of this. In addition, we investigated the relation with human sense of “attractive” and “difficult” to the properties of problems, by the experiment where the subjects were made to solve some of the generated problems.

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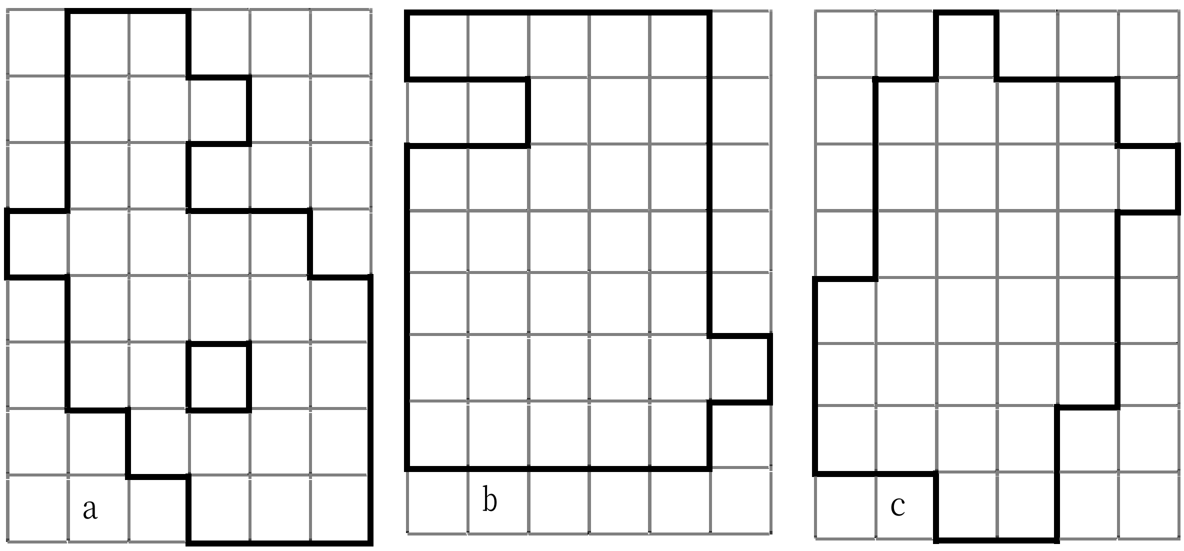
1. Introduction

The “puzzle” is understood as a recreational play similar to the game. There is a figure puzzle group in this. It includes figure dissection puzzles as well as Tangram and Pentomino, etc. are known well. One of them is to dissect a figure into identical pieces. Here, we deal with one of the typical problems.

It is not easy on computer to solve figure dissection problems, because of the permission for free cutting dissection. Even some solutions has fractal dissection. Here we simplify to allow only the figures and their dissected pieces to be composed of unit squares, which is called Polyomino(1). This make it possible to process digitally. The problem of treating here is defined as follows: “to dissect polyomines into a pair of congruent subpolyominoes”

The examples of the problem are shown below.



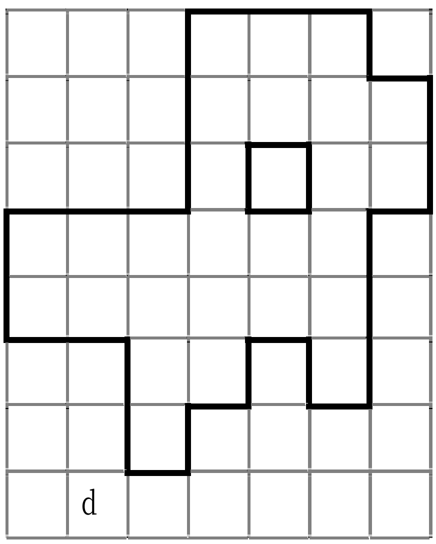


Fig 1. Examples of Congruent Dissection Problems. (a, b, c, d)

The word “congruent” means that the size is the same and the shape is identical. Identical pieces can be mirror image each other (unless allowing mirror image, we have another problem which can be investigated).

The dissected subfigure should be connected on the edge, not but the corner of unit square.

2. The algorithm which solves congruent dissection problems

The algorithm was designed, based on the idea to search the state space , branching by choosing one of the congruent subfigures for each unit square in the figure.

1. to fix a pair of starting points, which are corresponding point of the two congruent subfigures.
2. to fix the direction of one congruent subfigure to the other, and to fix whether they are mirror image to each other, or not.
3. for each unit square in the figure, make it belong to one of the subfigure and its corresponding square belong to the other.
4. repeating 3, determine the two subfigures, if all the unit square belong to either. If failed, to backtrack.
5. if succeeded, checking the connectivity of the subfigure, to show the result.

The algorithm was made based on this idea. The outline is as follows:

```
make the top leftmost unit square of the problem be the start point o1 of the
   subfigure 1;
for(the start point o2 of the subfigure 2 ∈ the problem){
        for(the rotation degree between the subfigure 1 and 2∈{0,90,180,270}){
                for(the relation of the subfigure 1 and 2
                                          ∈{identical, mirror image}){
                        search();
                }
        }
}
```

The search procedure is as follows:

```
search(){
       find the unit square s which belongs to neither;
       If(s does not exist any more){
              If(subfigures are connected figures){
                     record the result;
                     return;
              }
      }
      calculating the bias of s to o1, find the unit square s' that has the
      same bias to o2;
      if(s' belongs to either the subfigure 1 or 2)return;
      make s belong to subfigure 1 and s' belong to subfigure 2;
      search();
      make s belong to subfigure 2 and s' belong to subfigure 1;
      search();
      make s and s' belong to neither;
}
```

This algorithm can find all the solutions. In the implementation, it stops when the second solution is found. It is because the figure cannot be a right problem, if there are two solutions or more.

3. The generation of candidates and selection of problems

The figures for the candidate of the problem are generated in the random process. The first method repeats to add a unit square on the surface of the target or the medium figure, after placing the center unit square. As a result, connected figures of various shapes are generated. The second method is almost the same as the first except that it has some holes, which may increase the difficulty and the amusement of the problems.

These two methods are used in the experiment to make subjects solve it. Here symmetric candidates whose symmetry causes symmetric dissection. It is the case that the center of the symmetry is on the corner or on the edge. The size of the problem has been controlled by changing the number of arranged unit squares.

Besides this, we are trying the method of generation with a little cutting down the large square outside.

The problem generation is done in order by the following procedure.

1. The problem candidate is generated.
2. The algorithm try to solve it.
3. Repeat from 1 if there is no solution, or are two or more.
4. It outputs it as a problem if it was not a trivial solution.

4. Solving experiment of congruent dissection problems

The experiment of solving was made. Thirty two subjects solved 20 problems of different types and sizes, in 20 minutes. They are of middle and old age (average 49.2 years old), fond of puzzles, and male except a few.

Obtained information is the following three points. The first is whether the problems were solved correctly, or not. The incorrect answer and the unfilled answer are classified as unsolved. Secondly the difficulty they felt in the problems was filled in at three grades. Thirdly, attraction or amusement was made to be filled in at three grades.

The measures of characteristics of each problem are determined by the above information as:

(1)Difficulty 1(difficulty whether actually solved or not)

can be solved (-1). ----- cannot be solved (1)

(2)Difficulty 2(difficulty which subjects feel)

easy (-1) -- medium (0) -- difficult (1)

(3)Attraction and Amusement

trivial (-1) -- medium (0)-- interesting (1)

In the experiment result, the average of difficulty 1 was -0.206(it means 39.7% of the correct answer rate). The average of difficulty 2 was -0.206(the filling was 41.7% in ratio). The average of amusement was -0.184(the filling ratio was 31.4%).

All the extreme values existed in some specific problems. That is, there were problems that all members were able to solve and a problem that all members were not able to solve, too. The evaluation was in the same manner.

5. Estimation of difficulty and amusement from the geometric values of problems

We estimated the difficulty 1, 2 and amusement : the characteristics of congruent dissection problems.

It was done by determining linear expression by using the least mean square method for this. The following five were used as figure attribute.

(1) size
(2) surface length of the problem
(3) length of dissection line
(4) the angle of rotation between subfigures is 90 degree
(5) the subfigures are mirror image of each other

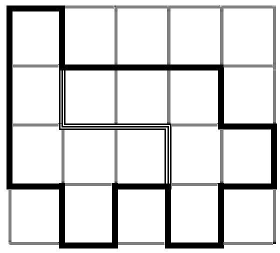


Fig. 2.
figure attribue

The figure 2 has the five attribute values 7, 10, 4, 1, 1. The value (1) is the number of unit squares in a subfigure and has the range 5 to 17 in the experiment. The value (2) means length of the surface of a subfigure, and has the range 3 to 19. The values (4) and (5) has the rage 0 and 1. If the condition holds, the value is 1.

These have been prepared as the parameter possible to relate to the difficulty and amusement. We guessed that 90 degree rotation and mirror image relation cause human perception difficulty, because it means the change of x axis and y axis of two dimensional plain.

The result formulas by the least square method are shown, where the X1, X2,...,X5 are the values of the above-mentioned figure attributes.

$$\text{difficulty1} = 0.300\,X1 + 0.010\,X2 - 0.102\,X3 + 0.224\,X4 - 0.091\,X5 - 2.445$$

$$\text{difficulty2} = 0.136\,X1 + 0.060\,X2 - 0.019\,X3 + 0.377\,X4 - 0.047\,X5 - 2.105$$

$$\text{amusement} = 0.020\,X1 + 0.089\,X2 - 0.002\,X3 + 0.172\,X4 - 0.293\,X5 - 1.274$$

As for the composition of difficulty 1, the size of the problem takes a large part. This is also because most of the subjects avoid big-sized problems. Surface length is not so related. It is uncertain why shorter boundary line causes difficulty. The 90 degree rotation has brought the difficulty of the problems. Mirror image is not so related to the difficulty1.

The difficulty 2, which means human sense, is not so related to the size, as the difficulty 1. The 90 degree rotation brings much difficulty 2. As for amusement or being enjoyable, the size is not so related. Surface length is related considerably. The 90 degree rotation also contributes to amusement. The mirror image contributes oppositely. It is understood that there is plus correlation between the pair of difficulty and the amusement.

Actually the problems a, b, c, and d of Figure 1 are the ones of the first place and the second to the three estimation values difficulty 1, difficulty 2 and amusement. As for difficulty 1, 1st place was b, and 2nd place was c. As for difficulty 2, 1st place was b, and 2nd place was a. As for amusement, 1st place was a, and 2nd place was d. The answers for the problems are shown in the appendix. Only a few subjects solved these problems, because they are considerably difficult. Therefore, they were hardly answering difficulty 2 and amusement. However, the evaluation can be done by the expression of presumption above.

6. Summary

The research and the result are concluded as follows. First, the algorithm that solved the problems of congruent dissection into two pieces was designed and was used to implement the system that solved it. Secondly, the system that generated variety of problems by the solving system was made. Thirdly, the experiment to make subjects solve it was conducted, and it was investigated what geometric characteristics determine difficulty and amusement of problems.

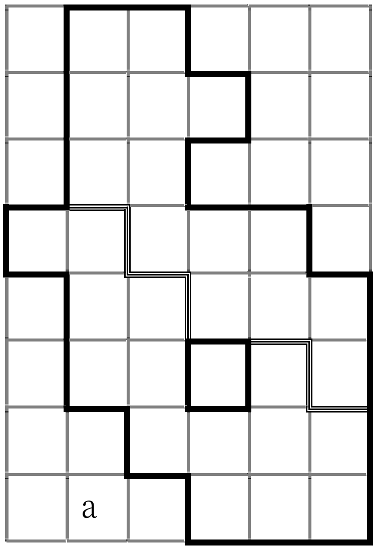
This method can be applied to other figure problems as a problem in the future. It can be directly applied to the problems of congruent dissection into three or more pieces, similar dissection problems, dissection problems dissected into other than the unit square, etc. .

Reference

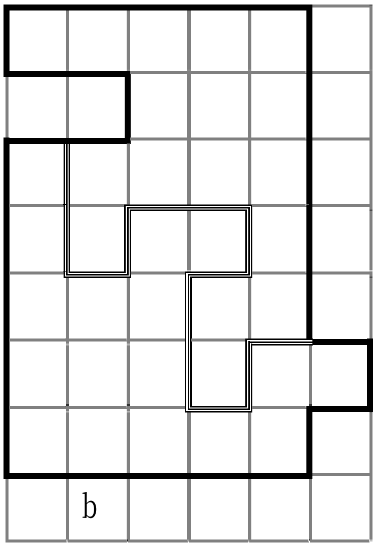
(1) Solomon W. Golomb, Polyominoes, Princeton University Press, 1994.

Appendix

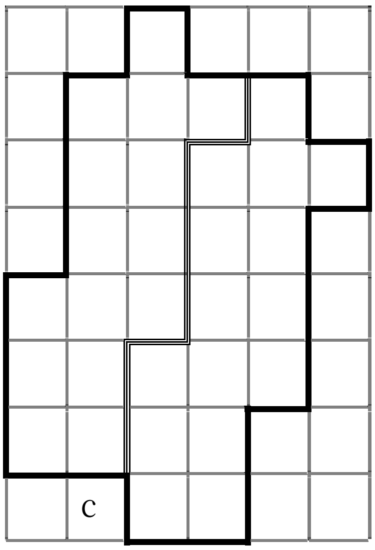
Solutions of Figure 1



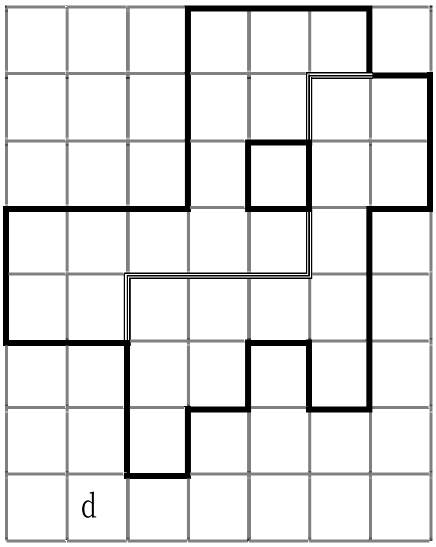
a



b



c



d