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境界から面の推論

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人間は２次元の線画を見ると、対象の３次元形状を頭の中に描く。その原理の一つは２Ｄ境界を曲率線と解釈し、面の３Ｄ形状の復元を行うことと想われる。本論文では、さらに、面の曲率線の網による表現と、この表現に基づく面の方向の計算法を提案する。最後に、境界から曲率線の網を織る一アルゴリズムおよび実例を示す。

Inferring Surfaces from Boundaries

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ABSTRACT

This paper discusses the problems underlying the inference of surfaces from their boundaries. Humans perceive definte 3-dimensional shapes from 2-dimensional boundaries, though they do not provide sufficient constraints on determining the surfaces they bound. This fact implies that some natural regularities are employed. One such regularity is that if there is no other evidence, then boundary segments are usually perceived as lines of curvature, which impose restrictions on the computational interpretation of the surfaces they bound. A representation of surface shape by a net of lines of curvature inside the boundary is proposed, from which surface orientation can be computed. An algorithm for knitting the net and demonstration examples are presented.

1. INTRODUCTION

As the computer vision research deepens and broadens, the classical problem of interpreting and constructing line drawings is drawing new attention [1,2,3,4,7]. Line drawing is probably the most abstract and efficient means of describing our three-dimensional world in a two-dimensional manner. Therefore, line drawings have very often been used in human communications and is potentially very useful in human-machine interfaces. It is so effortless for us to interpret a line drawing that we rarely pause to ask ourselves how we do. As we try to answer, however, we realize that it is a difficult question.

An intermediate goal of interpreting line drawings is to derive surface orientation from the boundaries[2]. The initial approach to interpreting line drawings is line labeling[5,7,10], which makes clear how the surfaces meet, but not the surface orientaion. Following it, Barrow and Tenenbaum[2] propose a three-step model for interpreting line drawings: classifying lines to extremal and discontinuity boundaries, interpreting 2-D image curves as 3-D space curves, and finally interpolating the surfaces. They propose the smoothness and planarity constraints for interpreting the image curves. They also suggest an interpolation technique that reconstructs surfaces from the known surface orientation along the boundaries. Other related work can be found in Brady and Yuille[3] and Barnard and Pentland[1]. Unfortunately, all these approaches focus on general principles, and produce satisfactory results only in limited special cases.

This paper proposes a new approach to inferring surface shape from a geometrical point of view. The boundaries are usually not arbitrary curves on the surfaces, but the special ones that carry more information than others. Observations indicate that we tend to perceive the 2-D image boundary segments as lines of curvature on the surfaces they bound. This agrees with, and is based on, the fact that we usually use lines of curvature as boundary curves to describe surfaces (see Brady et al. [4]). Lines of curvature are defined as the integrals of the directions of the greatest curvature and the least curvature. They form an orthogonal family of curves covering the surface (excluding umbilic points, where the normal curvature is equal for any direction) simply and without gaps[9]; for example, the meridians and parallels on a surface of revolution, and the straight lines and parallels on a cylindrical surface. Their projection onto the image plane gives a description of the surface. We propose that a net of lines of curvature on the surface is useful as an intermediate representation of the surface, from which surface orientations can be estimated up to a certain degree of accuracy[8]. We prove that a class of surfaces has the property of constant ratio intersection of lines of curvature, and finally propose an algorithm for knitting the net over surface and demonstrate it with implementation examples.

2. THE LINE OF CURVATURE REGULARITY

A boundary is the representative of the surface it bounds. Although it is clearly underconstraining, we perceive a definite 3-D shape (up to a Necker reversal). This fact implies that some natural regularities are employed in the inference process[9,10,12]. Therefore, only if we understand what they are and how they carry surface information, can we recover the surface shape from the image boundary segments. The problem then becomes one of discovering the natural regularities. Various observations indicate that one such regularity is the line of curvature regularity: **In absence of other evidence, the boundary segments in image that can be interpreted as (3-D) lines of curvature on the surface they bound should be interpreted so.**

Note that we intentionally use "curves" to mean 3-D curves and "segments" to mean their projections onto the image plane.

A. Interpretation as Lines of Curvature Is Reasonable

Suppose that we try to describe a surface by line drawing. There are two types of boundaries; one is the extremal boundaries and the other is the discontinuity boundaries along "cut" by us. To describe the surface effectively, we never choose an arbitrary path of "cut". Rather, we choose special curves that are descriptive. These special curves are usually lines of curvature, and sometimes asymptotes on parabolic surfaces(as in the case of helicoid[4]). The extremal boundaries are determined by the surface itself and the viewing direction (we assume the orthogonal projection in this paper). Still, there exist many surfaces whose extremal boundary curves are inherently lines of curvature, no matter where the viewpoint is (e.g., the surfaces of revolution). Since the curvature variation on a surface is limited by the greatest and least curvatures, it is postulated that the lines of curvature carry more information about the surface than any other curves on the surface. This may explain why we tend to perceive the boundary segments as lines of curvature; it is because we require the boundary convey as much information about the surface as possible. A strict proof of the postulation remains an open question.

B. Boundary Segments as Lines of Curvature

It is easily understood from the definition of lines of curvature that we can bound a surface (excluding umbilics) with four curves, if they are all lines of curvature, except for the case of a conical surface where one of them degenerates to a point. Some examples are illustrated in Fig. 1. All the segments can be interpreted as lines of curvature, and the surfaces are perceived as a cylindrical, an elliptic and a parabolic surfaces, respectively. The four segments are grouped into two pairs, A to B, and C to D; one pair is of lines of greatest curvature and the other of lines of least curvature.

It is generally not hard to divide a closed boundary into segments in image, if the boundary is truly composed of lines of curvature. It is impossible for two lines of curvature to bound a surface, because there exists at most one line of curvature through two points (not umbilics) on a surface. If a surface is bounded by more than four lines of curvature, the problem can be simplified by dividing the surface along line(s) of curvature into smaller ones. If a surface bounded by three segments is given, we interpret it as a conical surface bounded by three lines of curvature; one pair consists of a segment and a point. If we do not know which of the four segments degenerates to a point, then amboguities will appear in the surface interpretation. The boundary shown in Fig. 2(a) has three interpretations as a conical surface.

Of cource, not all boundary curves are lines of curvature. To know that a segment is not a line of curvature, we need some evidence suggested by the neighboring surfaces. In fact, the surface depicted in Fig. 2 is more likely perceived as a part of a cylindrical surface. Adding another segment makes it explicit (Fig. 2(b)). The surface bounded by the segments D and A is perceived as a circle or an ellipse. The interpretation as an ellipse is more likely because the adjacent two surfaces suggest a circular cylinder which is more likely than an elliptic cylinder. Thus, in summary, we need information interflows across surface intersections and higher level cooperation among the surfaces to determine which boundary segments are lines of curvature and which are not.

3. A NET REPRESENTATION

Lines of curvature form an orthogonal family of curves that covers the surface simply and without gaps like a piece of cloth. If sparsely sampled, it is like a net (in the sense of both two-dimensionality and three-dimensionality). Such a net is originally used by Stevens[8], who studys the interpretation of contours across surfaces. He suggests that the "parallel" contours be interpreted as lines of curvature on a cylindrical surface. They and the recovered straight lines, the rulings (also lines of curvature), form a net over the surface.

For an arbitrary smooth surface, lines of curvature are generally not "parallel", but smoothly deformed. A boundary curve can be thought of as obtained by moving the curve on its opposite side along the way guided by the other pair of curves. This is essentially a problem of interpolation. The general principle one should follow is "No news is good news"[6]. Thus the deformation should be kept as smooth and planar as possible; we call smoothness and planarity together "least variation" in both position and orientation. Recall that the only source of information is the boundary; the shape information that can be recovered must be reflected by the boundary.

The net representation is a bridge

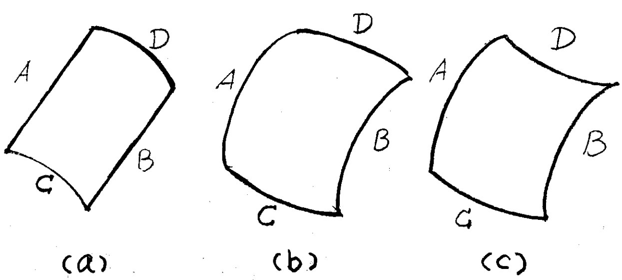


Fig. 1 A cylindrical, an elliptic and a parabolic surfaces

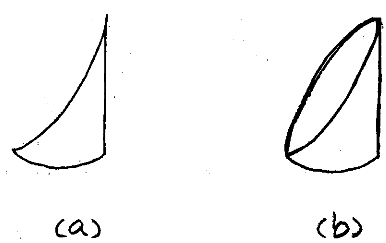


Fig. 2 Possible ambiguous interpretations

between 2-D boundary and 3-D surface orientation. Once the net is knit in image, the surface orientation can be estimated. The lines of greatest curvature and the lines of least curvature intersect each other at a right angle in space, but they are foreshortened to obtuse angles in image. Stevens[8] proposes the bisector method to estimate the surface orientation at each intersection which is expressed by tilt and slant. The more obtuse the angle, the more accurate the estimation. If the obtuse angle reaches two rigth angles, the surface orientaion is uniquely determined. The slant is a right angle, and the tilt is the normal angle of the image segment.

Stevens[8] also proposes a method of propagating the surface orientation along the parallels on a cylindrical surface, from places where it is determined accurately to places where it is not. In image, a parallel intersects the straight rulings, which have a constant orientation, at different angles. As shown in Fig. 3, when the angle $\beta 1$ changes to $\beta 2$, we have the equation

$$-\tan\tau 1 \tan\beta 1 = \tan\tau 2 \tan\beta 2,$$

where $\tau 1$ and $\tau 2$ are the tilts. If $\tau 1$, $\beta 1$ and $\beta 2$ are known, $\tau 2$ can be calculated.

However, for an arbitrary smooth surface, both the lines of greatest curvature and the lines of least curvature change their orientations. As shown in Fig. 4, suppose that when we move along a line of curvature from an intersection to the next one, **V** turns to **V'** and **U** turns to **U'**. Provided that the changes are not too great, we can modify the method proposed by Stevens into a two-step approximation method. Assume $\Delta\beta 2 > \Delta\beta 1$ (without loss of generality). We first fix **U** (let $\Delta\beta 1 = 0$) and calculate a new tilt due to only $\Delta\beta 2$. Then we fix **V'** and calculate a new tilt due to $\Delta\beta 1$. The obtained tilt is the required one. Once the tilt is determined, the slant can also be calculated. Therefore, after the surface orientation at the most obtuse angle is determined, we can propagate it along the lines of curvature to all the other intersections by the method just described.

4. AN ALGORITHM FOR KNITTING THE NET

Since how to knit the net depends on how the lines of curvature "flow" over the surface, we first present a theorem that states the property of constant ratio intersection of lines of curvature over a class of surfaces, then propose an algorithm for knitting the net of lines of curvature in image, and finally give examples and discussions.

A. Theorem of Constant Ratio Intersection

Before we present the theorem, we first fix notations and give preparatory explanations.

A surface can be expressed by using the lines of curvature as the parametric lines. Suppose that the intersections are P, $\mathbf{x}(u1,v1)$, Q, $\mathbf{x}(u2,v1)$, R, $\mathbf{x}(u2,v2)$, and S, $\mathbf{x}(u1,v2)$ (Fig. 5). The four boundary segments are then

$\mathbf{x}=\mathbf{x}(u,v1)$, $u1 \le u \le u2$,
$\mathbf{x}=\mathbf{x}(u2,v)$, $v1 \le v \le v2$,
$\mathbf{x}=\mathbf{x}(u,v2)$, $u1 \le u \le u2$,
$\mathbf{x}=\mathbf{x}(u1,v)$, $v1 \le v \le v2$.

By constant ration intersection of the lines of curvature u=const., we mean

$$\frac{d}{dv}\left(\int_{u1}^{u1+k(u2-u1)}\sqrt{\frac{d\mathbf{X}}{du}\frac{d\mathbf{X}}{du}}\,du \Big/ \int_{u1}^{u2}\sqrt{\frac{d\mathbf{X}}{du}\frac{d\mathbf{X}}{du}}\,du\right)=0,\quad 0<k<1;$$

i.e., a line of curvature $u=u1+k(u2-u1)$ divides the arc length of any line of curvature v=const. that it intersects at the same ratio. Similarly, we can define constant ratio intersection of the lines of curvature v=const..

THEOREM:

(1) If the surface patch (that is bounded by lines of curvature and without umbilics on it) is taken from a surface of revolution, then a meridian intersects any parallel at a constant ratio, and a parallel intersects any

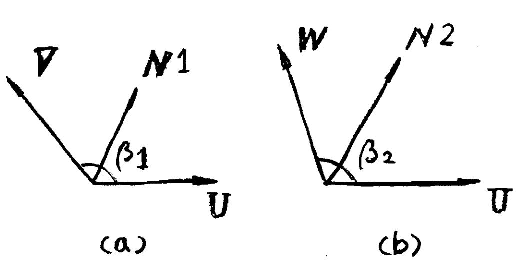


Fig. 3 Tilt variation from intersection angle variation on a cylindrical surface

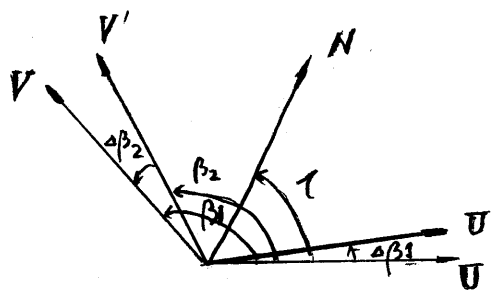


Fig. 4 Tilt variation from intersection angle variation on a doubly-curved surface

meridian at a constant ratio inside the boundary;

(2) If the surface patch is taken from a generalized cone whose axis is straight and normal to the planar cross section, then a fluting intersects any skeleton at a constant ration inside the boundary; and

(3) If the surface patch is taken from a developable surface whose Gaussian curvature is positive, then a line of curvature other than a ruling intersects any ruling at a constant ratioinside theboundary. **[end]**

A proof of the theorem is given in the appendix.

B. Algorithm

The algorithm reconstructs the lines of greatest curvature and the lines of least curvature inside the boundary. We first describe it and then give explanations.

As shown in Fig. 6, the segments a0-a1, b0-b1, c0-c1 and d0-d1 bound a surface. The points a2, b2, c2 and d2 are the center points that divide the segments into equal chord lengths. The points a3, a4, b3, b4, c3, c4, d3 and d4 are the center points of the new segments. We can go further until the unit segment is sufficiently small. We first find a point having both an equal distance to the points a2 and b2, and an equal distance to the points c2 and d2, which we regard as the intersection of the segments a2-b2 and c2-d2.

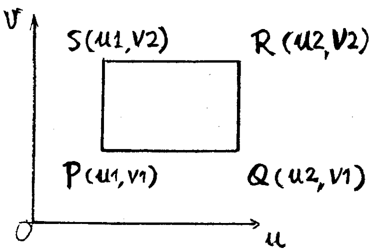


Fig. 5 Parameterized coordinate system

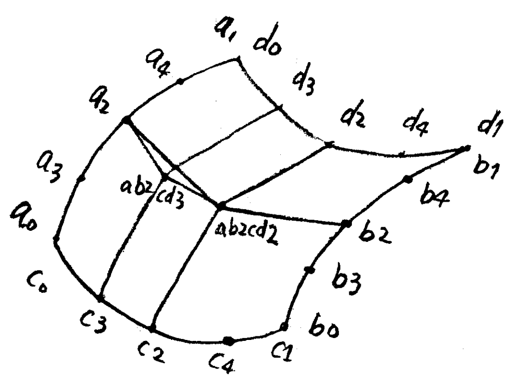


Fig. 6 A net-knitting algorithm

Similarly, we can then find the intersection of the segments a2-ab2cd2 and c3-d3. Repeating this process, we have two point sets that are dense enough to approximate the segments a2-b2 and c2-d2, two lines of curvature. Interestingly, they divide the surface into four subsurfaces, to which we can apply the algorithm described above until the net is fine enough to approximate the surface. Note that the algorithm needs only a small modification, when one of the four boundary curves degenerate to a point.

The algorithm is based on three assumptions: (1) both the lines of greatest curvature and the lines of least curvature have the property of constant ratio intersection; (2) the arc center of each segment in image is the projection of the arc center of the corresponding curve; and (3) the arc center of each segment has the same chord length to the two end points.

If the assumption (1) holds, then we can draw a line of curvature through the arc centers of the boundary curves of each pair. As stated in the theorem, a broad class of surfaces possesses the property of constant ratio intersection of lines of curvature. Empirically, if the boundary curves of each pair are similar, the lines of curvature on surfaces which do not strictly satisfy the assumption (1) can still be treated as they do.

If the assumption (2) holds, then the segments drawn through the arc centers of the boundary segments of each pair are the projections of the lines of curvature through the arc centers of corresponding boundary curves. Because of the foreshortening, this assumption does not always hold. However, if the curves of each pair are foreshortened to the corresponding segments in nearly the same way, then we can still draw lines of curvature through the arc centers of the boundary segments of each pair, though they are no longer the projections of the lines of curvature through the arc centers of the corresponding curves.

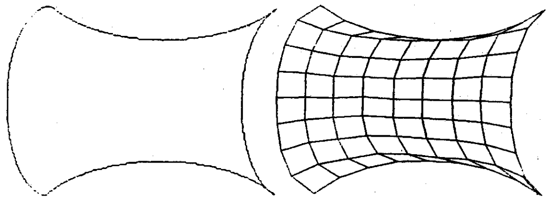
If the assumption (3) holds, then we can find the arc centers of the lines of curvature inside the boundary before the complete segments are reconstructed. Provided that the segments are curved in a similar way, being based on this assumption does not introduce error, even if it does not hold strictly.

In summary, if these assumptions do not hold, the obtained net only approximates the surface. However, if the curvature of the boundary segments is not too great, the error is within tolerance.

C. Experimental examples and Discussions

The algorithm has been implemented on a Lisp machine. Two examples are shown in Fig. 7 and Fig. 8. The output nets are intuitively satisfactory. The work to do yet is to search for the largest (or the smallest) intersection angle and to compute the surface orientations at the intersections.

The algorithm can work well for cylindrical surfaces; a generalized example is that shown in Fig. 9, very similar to that used by Stevens[8]. Given two parallel straight line segments and two parallel curved segments as the bounadry, the algorithm will knit an accurate net of lines of curvature inside the boundary, which can be identical with the net constructed by Stevens. For cylindrical surfaces, the assumption (1) holds strictly, while the other two do not. The obtained nets are accurate because the boundary segments of each pair are very similar. Note that the input boundary segments can be either extremal or discontinuity edges. Note that we can also apply the assumption that boundary segments intersect at a right angle in space to planar surfaces. A parallelogram as shown in Fig. 10 is always interpreted as a rectangle.



(a) (b)

Fig. 8 An example (a) boundary (b) constructed net

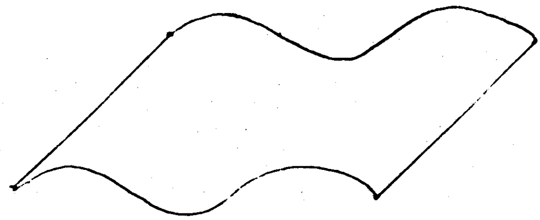
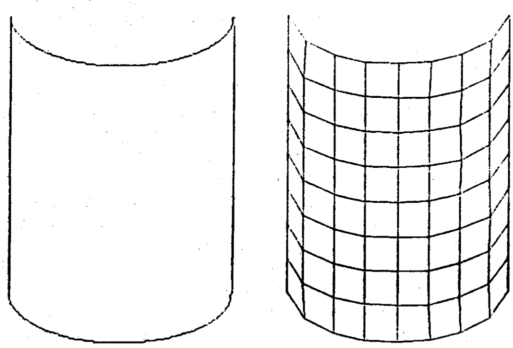


Fig. 9 Boundary of a cylindrical surface



(a) (b)

Fig. 7 An example (a) boundary (b) constructed net

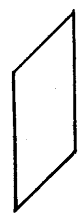


Fig. 10 A parallelogram interpreted as a rectangle in space

5. CONCLUSIONS

We have introduced a natural regularity employed in human inference of surfaces from boundaries; namely, in absence of other evidence, the boundary segments that can be interpreted as lines of curvature should be interpreted so. We have also proposed a representation for surface shape by a net of lines of curvature inside the boundary, from which surface orientations at the intersections can be estimated. Finally we presented an algorithm for knitting the net and demonstrated some examples. It is considered that if the world is constrained to objects of regular shapes, such as the man-made objects, the described algorithm will work satisfactorily, on both line drawings and edge images. However, it is also evident that humans utilize more knowledge, not only what has just been unveiled, in the perception of line drawings of natural scenes. Therefore, there is still a long way ahead towards the complete understanding of line drawings. We hope that this paper successfully makes it one step closer.

APPENDIX

[Proof] (1)When the z-axis is taken as the axis of revolution of the curve z=f(v), the resulting surface (Fig. 11) can be given by

$$\mathbf{x} = (\ v\cos u,\ v\sin u,\ f(v)\ ).$$

The lines of curvature are the meridians and the parallels, given by u=const. and v=const., respectively. For the meridians, the integral

$$\int_{u1}^{u1+k(u2-u1)} \sqrt{\frac{dX}{du}\,\frac{dX}{du}}\,du \Big/ \int_{u1}^{u2} \sqrt{\frac{dX}{du}\,\frac{dX}{du}}\,du = k$$

is independent of v. For the parallels,

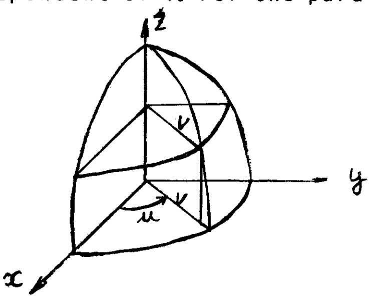


Fig. 11 Surface of revolution

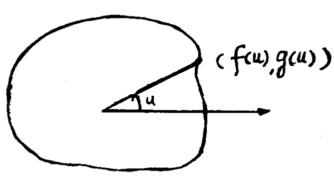


Fig. 12 Cross section of generalized cylinder

the integral

$$\int_{v1}^{v1+k(v2-v1)} \sqrt{\frac{dX}{dv}\,\frac{dX}{dv}}\,dv = \int_{v1}^{v1+k(v2-v1)} \sqrt{1+[f'(v)]^2}\,dv$$

is independent of u.

(2)If the planar cross section curve (Fig. 12) is given by

$$x = f(u),\ y = g(u),$$

the z-axis is taken as the v-axis of the generalized cone, and the expansion function is h(v), then the resulting surface can be expressed by

$$\mathbf{x} = (\ h(v)f(u),\ h(v)g(u),\ v).$$

The lines of curvature are the flutings and the skeletons, given by u=const. and v=const., respectively. The integral

$$\int_{u1}^{u1+k(u2-u1)} \sqrt{\frac{dX}{du}\,\frac{dX}{du}}\,du = \int_{u1}^{u1+k(u2-u1)} \sqrt{[f'(u)]^2+[g'(u)]^2}\,du$$

is independent of v.

(3)When a developable surface has positive Gaussian curvature, one set of the lines of curvature is the straight lines. The surface can be rewritten as

$$\mathbf{x}(u,v) = \mathbf{A}(v)u + \mathbf{B}(v).$$

The straight lines are then given by v=const. and the other set by u=const.. The integral

$$\int_{u1}^{u1+k(u2-u1)} \sqrt{\frac{dX}{du}\,\frac{dX}{du}}\,du \Big/ \int_{u1}^{u2} \sqrt{A(v)\ A(v)}\,du = k$$

is independent of v. **[end]**

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