コンピュータビジョン 76－19
(1992. 1. 24)

A Multi–Filtering Approach to Extracting Optical Flow Field

Hsiao-Jing Chen, Yoshiaki Shirai, and Minoru Asada

Faculty of Engineering, Osaka University
Yamadaoka 2-1, Suita-shi, Osaka, 560 Japan
Email : chen@ccmip.ccm.osaka-u.ac.jp

あらまし: オプティカル・フロー抽出の一手法の研究について報告する。本手法は2枚の連続画像に複数のフィルタをかけ、画像中の一点に対して、それらのフィルタの出力によって拘束直線の連立方程式をたて、加重最小2乗法で連立方程式からその点におけるフロー・ベクトルを求める。本手法の特徴としては、フロー場中の不連続の予測に役立つベクトルの信頼度 (confidence measure) をベクトルの抽出と同時に計算すること、高テクスチャ領域では平滑化処理をしなくても正確にベクトルを抽出できること、などがあげられる。

Abstract: We report the preliminary result of our research including a multi–filtering approach based on the gradient scheme to extracting optical flow field of rigid motion. An overdetermined linear system containing several linear equations – equations of constraint lines like that of Horn & Schunck [Horn '81] – about two components of flow vector at a location in image are constructed according to the responses of several spatial filters with different orientations. Two components are then calculated by applying least squares method to the overdetermined linear system. The main characteristics of our approach are the estimation of confidence measure corresponding to each flow vector which can be used as a kind of initial evidence of discontinuities in flow field (or say, motion boundaries), and accuracy of extracted flow field near high-textured region without any regularization processing, etc.

1 Introduction

Development of approaches to analyzing 2-D rigid motion from a sequence of images is an important research field in computer vision. Ability of analyzing motion is necessary for recovering 3-D information about objects and environment which be loosen during projection. Over more than one decade many researchers have reported their results. A survey about these pioneers' contributions to the development of rigid 2-D motion analyzing approaches can be found in [Aggarwal '88]. Here we discuss a gradient-scheme-based approach to the analysis of 2-D apparent motion in image sequence caused by the 3-D relative motion between observer (sensor) and rigid objects in environment.

Within almost all approaches to extracting optical flow field based on gradient scheme, like those of [Horn '81, Nagel '83, Schunck '89], an assumption called *intensity constraint* is preferred which assumes that intensity values of projection of same point in environment will be similar between consecutive image frames. Here we denote the intensity at location (x, y) of the image at time t by $I(x, y, t)$. Intensity constraint can be described as

$$I(x,y,t) = I(x+\Delta x, y+\Delta y, t+\Delta t). \quad (1)$$

In the gradient scheme, by assuming $I(x, y, t)$ is differentiable with respect to x, y, and t, and both of Δx, Δy, and Δt are significantly small, Eq.(1) can be expanded to

$$I_x u + I_y v + I_t = 0 \quad (2)$$

where I_x and I_y denote the partial derivatives of intensity with respect to x and y, respectively, and $u = dx/dt$, $v = dy/dt$ denote the horizontal and vertical components of 2-D flow vector $\boldsymbol{u}(x,y) = (u,v)^T$. It is well known that this Eq.(2) gives a *constraint line* in $u - v$ space.

From Eq.(2), only normal velocity component parallel to gradient vector can be determined, while tangential velocity component perpendicular to gradient vector remained unsolvable. Marr & Ullman [Marr '81] called this the *aperture problem* because a moving edge seen through a circular aperture seems to be moving normal to itself, while the transverse component of the velocity is not perceived. In order to solve the aperture problem, it is necessary to introduce the *smoothness constraint* which assumes that the flow vectors vary smoothly within a small spatial neighborhood region [Horn '81] or on a contour [Hildreth '84]. This constraint has also the effect of propagating the flow vectors extracted near corner points to the homogeneous regions of intensity. Horn & Schunck [Horn '81] expressed the constraint with a stabilizer of first order. Yuille & Grzywacz expended this constraint by expressing it with a summation of stabilizer items including not only lower order but also higher order ones [Yuille '89]. It is well known that the main drawback of the smoothness constraint proposed by Horn & Schunck and Yuille & Grzywacz is the indiscriminate smoothing effects both within homogeneous flow regions and at locations near discontinuities in optical flow field.

We present an approach to extracting optical flow field of rigid motion with a kind of gradient scheme. Classical assumptions in optical flow estimation with gradient scheme like those of intensity and smoothness constraints are also used, and flow vector at a location in image is estimated from the outputs of multiple sets of filters which serve as the observing windows. Some details about our approach will be described below.

2 The Basic Idea

2.1 Overdetermined Linear System

In our approach, a new sequence of images $f_i(x,y,t)$ is created by convolving original sequence of images $I(x,y,t)$ with a spatial filter $F_i(x,y)$

$$f_i(x,y,t) = I(x,y,t) * F_i(x,y) \quad (3)$$

where $*$ denotes a convolution. We assume that the intensity constraint like that of Eq.(1) will not be violated in $f_i(x,y,t)$. This constraint is rewritten as

$$f_i(x,y,t) = f_i(x+\Delta x, y+\Delta y, t+\Delta t). \quad (4)$$

A constraint line equation

$$f_{ix}u + f_{iy}v + f_{it} = 0 \quad (5)$$

can be obtained from Eq.(4) like that of Eq.(2). On the other hand, we can create another sequence of images with a spatial filter $F_j(x,y)$ which is different from $F_i(x,y)$, and also assume that the intensity constraint like that of Eq.(1) will not be violated. Therefore, another constraint line can also be obtained. The flow vector $\boldsymbol{u} = (u,v)^T$ at location (x,y) in original image $I(x,y,t)$ can be calculated by solving simultaneous equations of these two constraint lines.

Due to the calculation of convolution expressed in Eq.(3), Eq.(4) holds only when all pixels within the domains of filters $F_i(x,y)$ and $F_j(x,y)$ have the similar velocity. This is a kind of smoothness constraint expressed implicitly like that of Heeger's [Heeger '87]. It is evident that if filters overlap motion boundaries, Eq.(4) would be violated and the estimation of flow vector would become erroneous.

Srinivasan presented a generalized gradient scheme in which six filters – partial derivated versions of two different spatial-temporal filters with respect to x, y, and t – are used to extract optical flow field [Srinivasan '90]. His idea is based on using two filters to generate two constraint lines and solving simultaneous equations to calculate two components u and v of flow vector. A drawback of the approach proposed by Srinivasan is its impossibility for estimating the degree of how calculated vector fits the observed image data, while this degree plays important roles at the stages of discontinuity detection, segmentation of flow field, and regularization process, etc.

Instead of calculating flow vector from only two linear equations, approach proposed here calculates vector at each location from an overdetermined linear system which contains more than two equations of constraint lines. Classical least squares method is usually used to solve the system. It is well known in the theory of data modeling that a covariance matrix corresponding to estimated parameters (components of flow vector here) can be obtained from the overdetermined system. We can get a confidence measure according to the covariance matrix as the rating value of estimated flow vector at each location.

2.2 Multi-Orientation Filtering

As described above, a filter can generate a constraint line at each location in image. In order to obtain an overdetermined linear system containing several constraint lines for the estimation of flow vector and its confidence measure, more than two filters are necessary. Furthermore, differences between slopes of constraint lines generated by filters should be as large as possible. Ideally, all of these lines will intersect at a single point within $u-v$ space. When processing real image, however, it is difficult, if not impossible, to expect accurate intersection at a point due to several sources of noise. At this situation, least squares method can be used to obtain unique solutions about two components of flow vector. A merit of using multiple filters is that solution's rating value, which can serve as confidence measure, will be obtained based on the residuals between least-squared solutions and several constraint lines. This measure will play important roles at the stages of detection of discontinuities, segmentation of flow field, regularization processing, etc.

Srinivasan described a constraint on the selection of two spatio-temporal filters which simply limits the filters to be different only. Here we append an additional constraint to filters as that they should be able to create a set of constraint lines as that of Eq.(5)'s so that at locations near discontinuities within high-textured regions the residuals will become significantly large.

Usually, the orientation-selective spatial filters will satisfy the constraint of filters described above. Orientation-selective filter can serve as a kind of observing window through which "normal" component of flow vector parallel to the "gradient vector" measured by that filters is observable. Near a corner point, normal components observed by several window are compatible, means that the summation of squared residuals is small. At locations near discontinuities within high-textured regions, normal components observed by several window become no longer compatible, means that the summation of squared residuals will become larger.

Another merit of using multiple orientation-selective spatial filters come from the reason of that the geometry of real image is so complex that it is impossible to predict such image features as orientations of edges and the neighbor region of a pixel, etc. We define the "optimal" filters for a point as those ones by which more than one reliable normal components (constraint lines) with different directions (slopes) could be obtained. For instance, for a corner point composed of two edges, the optimal filters will be two ones each of which orientation is completely the same as the directions of edges. Because of the difficulties in predicting which filters can be used as optimal observing windows, flow vector can not be estimated accurately with only two filters of fixed orientations under almost all situations. Therefore, it is necessary to use a set of filters with several different orientations.

As a summary of this section, we would like to claim that multiple filters with different orientations are necessary for the detection of motion boundary and the accurate estimation of flow vector. In the next section, we describe some computational details about how the estimation of flow vector field can be obtained from output of multiple filters.

3 Computational Details

3.1 Solution with WLS Method

A set $\mathcal{L} = \{L_n, n = 1, \ldots, N\}$ of constraint lines

$$L_n: \quad f_{nx}u + f_{ny}v + f_{nt} = 0$$

can be obtained from the output of a set of $N\,(N > 2)$ filters of different orientations with same size. These lines can be used to construct an overdetermined system for solving two components u and v of flow vector $\boldsymbol{u}$. In this section, we simply describe the estimation of flow vector by applying the WLS – *Weighted Least Squares* – method to overdetermined system.

Solving the overdetermined system with least squares method is equivalent to find a solution $\bar{\boldsymbol{u}} = (\bar{u}, \bar{v})^T$ which minimizes the weighted summation of squared residuals. This problem can be expressed as

$$\text{find a solution } \bar{\boldsymbol{u}} \text{ which minimizes } J = \sum_{n=1}^{N} w_n r_n^2 \quad (6)$$

where r_n^2 denotes the squared residual of constraint line L_n defined as

$$r_n^2 = \frac{(f_{nx}\bar{u} + f_{ny}\bar{v} + f_{nt})^2}{f_{nx}^2 + f_{ny}^2}, \quad (7)$$

and w_n denotes the relative weight of L_n within $\mathcal{L}$ which can be calculated as a function of intensity contrast $\mathcal{C}_n$ as follows

$$w_n = \frac{\mathcal{C}_n}{\sum_{i=1}^{N} \mathcal{C}_i}, \qquad \text{where } \mathcal{C}_n = \sqrt{f_{nx}^2 + f_{ny}^2}.$$

There are several methods to solve Eq.(6). In our approach, we select the SVD – *Singular Value Decomposition* – method for the least-squares solution. Some details about the SVD method can be found in [Press '86]. Although the SVD method requires a higher cost of calculation, it has several persuasive characteristics : (i) This method can diagnosis the singularity of overdetermined system, and (ii) In some cases, it can give the correct solution even though the system is nearly singular.

There are three kinds of solutions for flow vector obtained from overdetermined system as: $\{a\}$ Reliable Solution – If the overdetermined system is not singular, (variation of gradient directions is significant within observing windows,) the estimated flow vector will be reliable. This case occurs near corner points or high-textured region; $\{b\}$ Normal Component Solution – If the overdetermined system is singular or near singular, only normal component of flow vector can be estimated. This case occurs when a straight edge is longer than the size of filters and there is not any texture near the edge; and $\{c\}$ Unreliable/Unsolvable Solution – The flow vector can not be calculated if the spatial gradient is zero. Or the flow vector will become erroneous if spatial gradient is nearly zero. This case will occur in the homogeneous regions, and the solutions of flow vectors in these regions are ambiguous.

We use the CN – *condition number* – of overdetermined system to identify the cases of $\{a\}$ and $\{b\}$. A definition of condition number can be found in [Kearney '87], and the method to calculate it with SVD can be found in [Press '86]. The CN will be large if the set of constraint lines is nearly singular, and will become infinite when it is singular. Therefore, the identification of estimated vector can be performed by limiting the value of CN. The third case $\{c\}$ can simply be identified by thresholding the intensity contrast.

3.2 Confidence Measurement

Calculations of the covariance matrix and confidence measure of the estimated flow vector $\bar{\boldsymbol{u}}$ are very important for the estimation of optical flow fields. Covariance matrix has been used in incremental estimation [Matthies '89, Singh '91], and confidence measure can be used as a kind of weight assigned to estimated flow vector when smoothing process is performed. Anandan [Anandan '89] and Singh [Singh '90] used this kind of measures in their stages of regularization. In our approach, as described in Sec. 2 and Sec. 2.2, the confidence measure is specially important for the initial prediction of discontinuities in optical flow field.

In our approach, we simply apply the typical method of error analysis excluded the assumption about Gaussian distribution. The weighted summation of squared residuals ε and a matrix $\boldsymbol{D}$ are defined as

$$\varepsilon = \sum_{n=1}^{N} w_n r_n^2, \qquad \boldsymbol{D} = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$$

where w_n and r_n^2 are whose described in Section 3.1, and d_{ij} are represented as

$$d_{11} = \sum_{n=1}^{N} w_n f_{nx}^2, \qquad d_{22} = \sum_{n=1}^{N} w_n f_{ny}^2,$$
$$d_{12} = d_{21} = \sum_{n=1}^{N} w_n f_{nx} f_{ny}.$$

From ε and matrix $\boldsymbol{D}$, covariance matrix of $\bar{\boldsymbol{u}}$ can be calculated as

$$\boldsymbol{Cov} = \frac{\varepsilon}{|\boldsymbol{D}|} \begin{pmatrix} d_{22} & -d_{21} \\ -d_{12} & d_{11} \end{pmatrix} \tag{8}$$

Confidence measure $\mathcal{R}$ is determined as the inverse of larger eigenvalue of matrix $\boldsymbol{Cov}$. This kind of determination of confidence measure is reasonable because it is evident from Eq.(8) that $\mathcal{R}$ will be large only when the ε is small, and determinant of matrix $\boldsymbol{D}$ is large. Larger ε will appear mainly near motion boundaries, and smaller determinant of matrix $\boldsymbol{D}$ will appear at where aperture problem occurs. All of these two situations would drop down the value of confidence measure $\mathcal{R}$.

From the definition of $\mathcal{R}$ we can see that this measure can be used as an initial prediction of discontinuity in flow field. It can also be used as a kind of weight value during smoothing (regularization) stage.

4 Implementation

In our implementation, we select the spatial orientation-selective Gaussian filters as those described above. This selection is preferred based on that: (1) Gaussian filters have the averaging effect and therefore noise in observed intensity of original image can be eliminated, (2) They are low-pass filters and therefore can be used to eliminate high frequency component at step edge, and (3) They can serve as observing windows through which information about motion can be obtained, and information is weighted inversely with respect to distance from center of window.

In Fig.1, a set of orientation-selective filters with eight different orientations are shown in brightness.

5 Experimental Result

In this section we show two experimental results applied our approach to sequences of symmetry images. The first frame in original consecutive images of first example is shown in Fig.2(a). "Background" in symmetry image is a sine grating plate added with random noise. "Object region" shown in center is a real image of magazine. "Background" is shifted to left by one pixel, and "Object region" to right-down direction by one pixel in horizontal and vertical axis, respectively. Extracted optical flow field containing "Reliable Solutions" is shown in Fig.2(b), and "Normal Component Solutions" is shown in Fig.2(c). Inverse values of confidence measures is shown in brightness in Fig.2(d).

Consecutive images used in the second example are the same as the first one, but image motions of "Background" and "Object region" are different. In the second example, "Background" remains stationary, while "Object region" is rotated within the image plane between two frames of onsecutive images. Extracted optical flow field containing "Reliable Solutions" is shown in Fig.3(a), and "Normal Component Solutions" is shown in Fig.3(b).

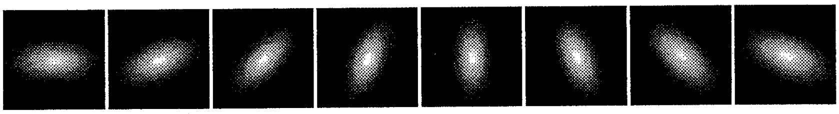

Figure 1: A set of eight filters shown in brightness.

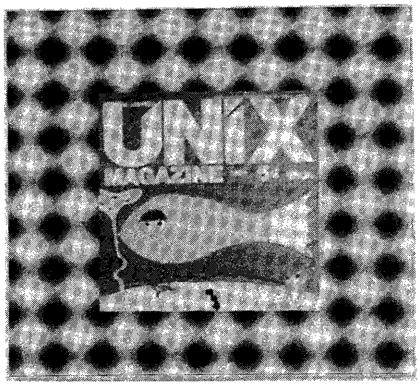

Fig.2(a)

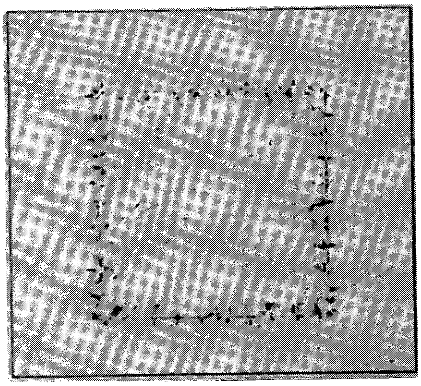

Fig.2(d)

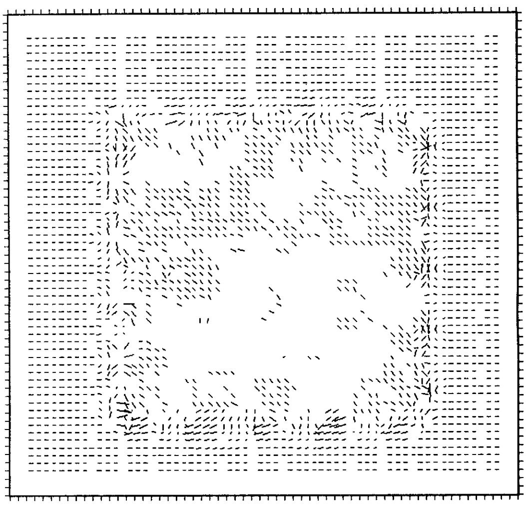

Fig.2(b)

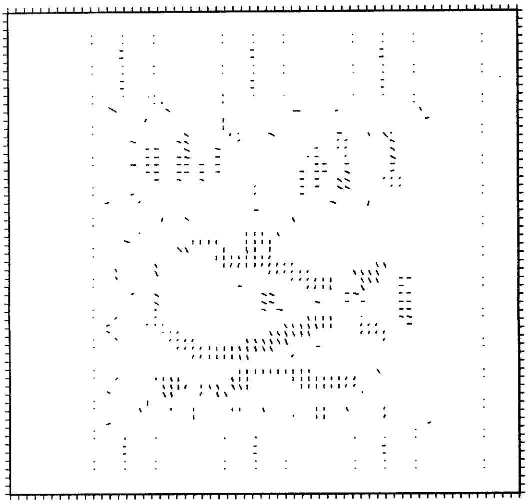

Fig.2(c)

Figure 2: The first experimental result (see text).

Inverse values of confidence measures is shown in brightness in Fig.3(c).

From these two experimental results we can see that (1) At motion boundaries the inverse values of confidence measures become larger comparing to other locations, and (2) Although the regularization processing was not performed, the estimated optical flow field is accurate except at that locations near discontinuities.

6 Discussion

In our preliminary result of research, there is not yet any type of regularization processing. Only normal vectors can be extracted at the locations lying on extended straight lines, and flow vectors at the locations within homogeneous regions of intensity are still remained unsolvable. It is necessary for us to develop a method of regularization for the purpose of propagating the "Reliable Solutions" extracted near corner points to those locations lying on extended straight lines and within homogeneous regions.

Poggio *et. al.* presented a generalized theory for the regularization problem in several computer vision tasks [Poggio '85]. Yuille & Grzywacz presented their motion coherence theory for solving that problem in the extraction of optical flow field [Yuille '89]. Applying these theories to regularizing optical flow field generated by a relative motion between sensor and a continuous 3-D surface whose projection to sensor occupies the whole image plane, flow field will become smooth and therefore well-posed. If the 3-D surface is not continuous, however, discontinuities will occur in flow field, and theories proposed by Poggio *et. al.* and Yuille & Grzywacz etc. will no longer valid for the solution of ill-posed problem.

Several researchers have proposed their approaches to solving the ill-posed problem with discontinuity treatment. Adiv proposed an approach to segment optical flow field by assuming that the optical flow field is generated by several 3-D patches and then estimating the motion parameters of patches with Hough transformation from a given noisy flow field [Adiv '85]. Hartley [Hartley '85] segmented flow field with the method of pyramid linking. Thompson *et. al.* [Thompson '85] de-

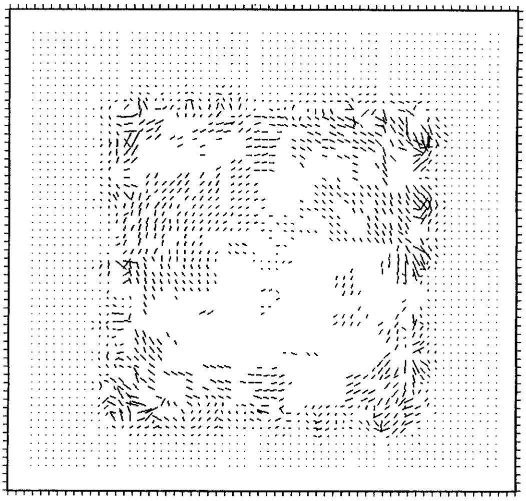

Fig.3(a)

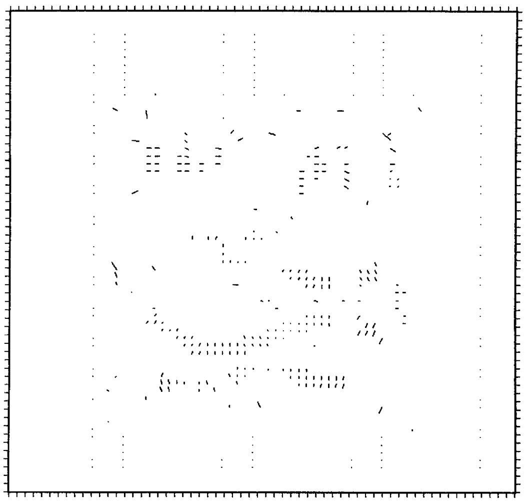

Fig.3(b)

Fig.3(c)

Figure 3: The second experimental result (see text).

tected discontinuities by applying the zero-crossing principle to their sparse displacement field extracted with the method of [Barnard '80]. Schunck [Schunck '89] also used the same idea to segment dense flow field. Murray & Buxton [Murray '87] applied the methods of stochastic annealing and "line process" proposed by Geman & Geman [Geman '84] to a given optical flow field. Spoerri & Ullman described their histogramic approaches to detecting discontinuities [Spoerri '87]. Dengler [Dengler '91] and Darrell & Pentland [Darrell '91] proposed their methods respectively for the segmentation of flow fields based on the MDL criterion. Hutchinson *et. al.* applied the deterministic annealing and "line process" to optical flow field [Hutchinson '88]. Peng & Medioni [Peng '88] proposed an approach which be able to predict and detect occlusion and disocclusion based on the image motion estimation method presented by Bolles *et. al.* [Bolles '87].

There are several limitation in approaches described above. The main limitation in stochastic and deterministic annealing methods, and also in methods using MDL criterion, is their higher computational cost. Limitation in the zero-crossing methods is, e.g., their accuracy on locating motion boundary. Approach of pyramid linking needs a prior knowledge about the number of objects (regions) with different motion parameters.

We have described in Sec. 3.2 that near the discontinuities, the confidence measures will become lower. This decreases of confidence measures can be used to predict the evidences of discontinuities in optical flow fields. For instance, a constraint could be preferred in the stage of discontinuity detection which assumes that the detected discontinuities should appear near that locations having lower values of confidence measures. We believe that introducing confidence measures to the processing of boundary detection will affect the accuracy and efficiency of detection.

7 Conclusion

We have proposed an approach to extracting optical flow field with gradient scheme. Characteristics of our approach are, e.g., (1) Beside the motion boundaries, confidence measures of estimated vectors will decrease compared to those inside the region with almost constant velocity. This relative comparison of confidence measures can be utilized to detect boundaries; (2) With this approach, flow vector can be estimated accurately at that location where the variance of intensity gradients observed through a surrounding window is significant; and (3) Orientation-selective Gaussian filters are utilized as observing windows with which the errors coming from sensor, digitizing process, and intensity differentiation, etc., can be deduced. They can also serve as kinds of windows by which information under observing is weighted inversely with respect to the distance to window center.

We will put the emphasis of our future work on the regularization of optical flow field and low level vision task of effective detection of motion boundaries with lower computational cost.

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