

## Robot Positioning and Tracking of Visual Appearance

Shree K. Nayar\*      Hiroshi Murase\*\*      Sameer A. Nene\*

\* Computer Science Dept  
Columbia University  
New York, NY10027  
USA

\*\* NTT Basic Research Labs  
3-1, Morinosato Wakamiya  
Atsugi-shi, Kanagawa 243-01  
JAPAN

**Abstract** The problem of vision-based robot positioning and tracking is addressed. A general learning algorithm is presented for determining the mapping between robot position and object appearance. The robot is first moved through several displacements with respect to its desired position, and a large set of object images is acquired. This image set is compressed using principal component analysis to obtain a low-dimensional subspace. Variations in object images due to robot displacements are represented as a compact parametrized manifold in the subspace. While positioning or tracking, errors in end-effector coordinates are efficiently computed from a single brightness image using the parametric manifold representation. The learning component enables accurate visual control without any prior hand-eye calibration. Several experiments have been conducted to demonstrate the practical feasibility of the proposed positioning/tracking approach and its relevance to industrial applications.

**Key words** Object tracking, pose estimation, principal component analysis, visual learning

## 2次元照合を用いた物体の位置検出と追跡

Shree K. Nayar\*      村瀬 洋\*\*      Sameer A. Nene\*

\*Columbia University  
New York, NY10027

\*\*NTT基礎研究所  
243-01 厚木市森の里若宮3-1

あらまし 本報告では、2次元照合により対象物体の向きや位置を検出しリアルタイムでその方向にロボットアームを移動させる手法について述べる。本手法は学習段階と位置検出段階から構成される。学習段階では、物体を回転させたり、位置を移動させたりすることによる画像の変化を予め蓄える。それらの膨大な画像を固有ベクトル空間上での多様体でコンパクトに記述する(パラメトリック固有空間法)。位置や方向の検出段階では、入力した画像を固有空間に投影し、多様体との位置関係により即座に物体の位置と方向が検出できる。この検出結果をカメラの付いたロボットアームにフィードバックすることによりリアルタイムでアームは物体を追跡することが可能となった。

キーワード 物体追跡、ポーズ推定、主成分分析、視覚学習

本内容は IEEE International Conference on Robotics and Automation (1994年5月, サンディエゴ)においても発表予定である。

## Introduction

For a robot to be able to interact in a precise and intelligent manner with its environment, it must rely on sensory feedback. Vision serves as a powerful component of such a feedback system. It provides a richness of information that can enable a manipulator to handle uncertainties inherent to a task, react to a varying environment, and gracefully recover from failures. A problem of substantial interest and relevance to robotics is visual positioning/tracking; the ability of a robot to either automatically position itself at a desired location with respect to an object, or accurately follow an object as it moves along an unknown trajectory.

This paper presents a new approach to visual positioning and tracking. Before we proceed to describe our approach, a brief review of previous work is in order. Previous tracking methods can be broadly classified into two categories; (a) feature/model based and (b) learning based. The first category uses image features to estimate the robot's displacement with respect to the object. The goal is to find the rotation and translation that must be applied to the end-effector to bring the features back to their desired positions in the image. Image features used vary from geometric primitives such as edges, lines, vertices, and circles [Weiss et al. 87] [Feddemma et al. 91], [Koivo and Houshangi 91] [Hashimoto et al. 91] to optical flow estimates [Papanikolopoulos et al. 91] [Luo et al. 88] and object location estimates obtained using stereo [Allen et al. 92]. The control schemes used to drive the robot to its desired position vary from simple prediction algorithms employed to achieve computational efficiency, to more sophisticated adaptive self-tuning controllers that account for the dynamics of the manipulator. Many of the above methods require prior calibration of the vision sensor's intrinsic parameters (e.g. focal length) as well as its extrinsic parameters (rotation and translation with respect to the manipulator).

The second category of positioning/tracking methods includes a learning component. In the learning stage, the mapping between image feature locations and robot coordinates is generated prior (off-line) to positioning/tracking. This mapping is then used to determine, in real-time, errors in robot position/velocity from image feature coordinates. This is generally accomplished without any explicit knowledge of the object's geometry or the robot's kinematic parameters. In addition, calibration of the vision sensor is not required as long as the sensor-robot configuration remains unaltered between learning and tracking. These methods differ from each other primarily in the type of learning algorithm used. The learning strategies vary from neural-like networks [Kuperstien 87] [Mel 87] [Miller 89] [Walter et al. 90] to table lookup mechanisms such as the cerebellar model articulation controller (CMAC) [Albus 75] [Miller 87].

Here, we propose a new framework for learning-based visual positioning/tracking. Our approach differs from previous ones in two significant ways; (a) the method uses raw brightness images directly without the computation of image features, and (b) the learning algorithm introduced is based on *principal component analysis* rather than a large input/output mapping network. During the learning stage, a sizeable image window is selected that represents the appearance of the object when the robot is in the desired position. A large set of object images is then obtained by incrementally perturbing the robot's end-effector (hand-eye system). The image set is compressed using principal component analysis [Oja 83] to obtain a

low-dimensional subspace, called eigenspace. Variations in object images due to robot displacements are represented in the form of a parametrized manifold in this eigenspace. The advantages of using this representation are discussed in the paper.

In the positioning or tracking application, each new image is projected to the eigenspace and the location of the projection on the parametrized manifold determines the robot displacement (error) with respect to the desired position. Positioning and tracking are achieved without prior knowledge of the object's geometry or reflectance, the robot's kinematic parameters, and the vision sensor's parameters. We conclude with experiments conducted using an Adept robot. The accuracy and efficiency of the proposed method are demonstrated using three sample applications; (a) the assembly of electronic devices on a circuit board, (b) the insertion of a peg in a hole; and (c) the tracking of a manufactured part moving on a turntable.

## The Approach

Figure 1 shows the *hand-eye* system we have used to demonstrate visual positioning and tracking. The manipulator used is a 5 degree-of-freedom Adept robot that is interfaced with a Sun IPX workstation. A CCD camera is mounted adjacent to the robot gripper and provides images of the tracked object.

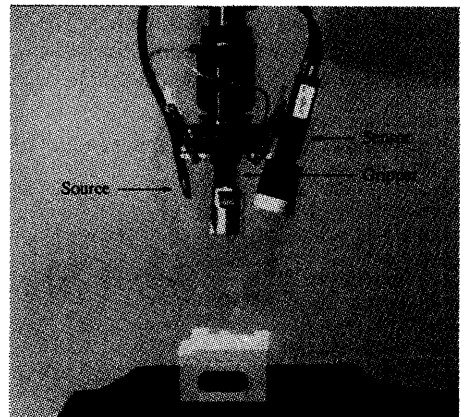


Figure 1: The hand-eye system used for visual positioning and tracking. The end-effector includes a gripper, an image sensor, and a light source.

The end-effector's position (translation and rotation) can be described in any frame of reference attached to the robot. Without loss of generality, we denote the end-effector position by the generalized coordinates:

$$\mathbf{q} = [q_1, q_2, \dots, q_m]^T \quad (1)$$

where  $m$  represents the end-effector degrees of freedom (DOF) used in the positioning or tracking application. The imaging optics is selected such that the tracked object occupies a large section of the image. The image area used as visual input is a fixed window, within the complete image, which includes sufficient object detail (for example, see Figure 2)<sup>1</sup>. This image window is written as a vec-

<sup>1</sup> Alternatively, the contents of several windows of fixed sizes and shapes, scattered in the image, can be concatenated and treated as a single window.

tor  $i$  by reading brightness values from it in a raster scan fashion:

$$i = [i_1, i_2, \dots, i_N]^T \quad (2)$$

Our objective is to compute off-line the mapping between the robot end-effector coordinates and object images. The brightness image  $i$  for any given robot position  $q$  depends on the three-dimensional shape of the object, its reflectance properties, the illumination conditions, and the end-effector coordinates with respect to the object. Shape and reflectance are intrinsic properties of a rigid object that do not change during positioning or tracking. In order to overcome the effects of possible illumination variations, we have used a light source that is also mounted on the end-effector. In our setup (see Figure 1), the source is one end of a fiber-optic cable connected to a strong light source at the other end. This *hand-source* is the dominant source of object illumination, and minimizes the effects of ambient illumination.

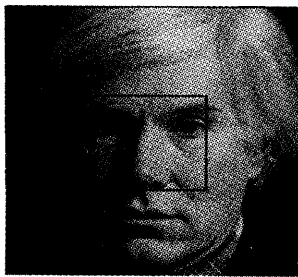


Figure 2: Each image vector  $i$  is obtained by reading pixel brightness values from an image window (black box) of fixed size and position.

This leaves us with the position and orientation of the end-effector with respect to the object. Each robot position  $q$  produces an image  $i$  (Figure 3). It is this relation, or mapping  $i(q)$ , that we wish to learn. To this end, we define the nominal or *desired image  $\hat{i}$  as the one produced when the robot is in the desired position  $\hat{q}$  with respect to the object. For convenience, we define all robot positions with respect to  $\hat{q}$ , i.e. the coordinate system with  $\hat{q} = 0$ . Once the mapping  $i(q)$  has been learned, the robot displacement  $q$  with respect to the desired position can be determined from any image  $i$  by *inverse mapping*. As we will see shortly, this inverse mapping is continuous and can be achieved using raw images without the computation of image features, such as, edges, lines, corners, circles, or optical flow estimates.*

Several advantages result from this approach. (a) The three-dimensional shape and reflectance properties of the object need not be known or computed. The effects of shape and reflectance are embedded in the raw brightness images. (b) Robot displacements are computed using images rather than image features. This not only saves computations but also avoids detection and localization errors introduced by feature extraction algorithms. (c) The extrinsic and intrinsic parameters of the camera are not used. Therefore, it is not necessary to calibrate the camera with respect to the hand or any other coordinate system; a process that is known to be cumbersome. All that is required is that the camera be positioned on the hand such that a descriptive image of the object is available.

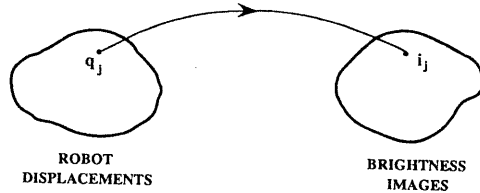


Figure 3: Estimation of the robot's displacement from its desired position is based on the observation that every displacement produces a unique brightness image.

## Learning Displacement-Image Mapping

In this section, we present the learning approach used to determine a direct mapping between the generalized robot coordinates and object images. Later, we show how this mapping can be effectively used for positioning and tracking purposes.

### Acquiring Learning Images

All images of the object taken by varying the robot position are of the same size, the size of the image window used (Figure 2). The window is selected such that it remains on the object even while robot coordinates are varied during the learning process. This ensures that the positioning and tracking stages are insensitive to the background of the object, avoiding the use of any segmentation algorithm. It is reasonable to assume that in a positioning or tracking application errors in robot position are relatively small. Hence, the range of discrete manipulator positions used to obtain the image set may be confined to a small area around the desired position  $\hat{q}$ . We denote the image corresponding to the discrete position  $q_j$ , as  $i_j$ .

Prior to learning, the imaging sensor is linearized using a simple calibration procedure. This ensures that image brightness is proportional to scene radiance. We would like our learning process to be unaffected by variations in the intensity of illumination or the aperture of the imaging system. This can be achieved by normalizing each acquired image, such that, the total energy contained in the image is unity:  $\hat{i} = i / \|i\|$ . Let the number of discrete samples obtained for each DOF  $l$  be  $R_l$ . Then the total number of images is  $M = \prod_{l=1}^n R_l$ . The complete image set can be written as:

$$\{\hat{i}_1, \dots, \hat{i}_2, \dots, \hat{i}_M\} \quad (3)$$

The image  $\hat{i}$  corresponding to the desired robot position  $\hat{q} = 0$  is also included in this set.

### Computing Eigenspaces

Images in the above set tend to be correlated to a large degree since end-effector displacements between consecutive images are small. Our first step is to take advantage of this correlation and compress the large set to a low-dimensional representation that captures the key appearance characteristics of the object. A suitable compression technique is based on principal component analysis [Oja 83], where the eigenvectors of the image set are computed and used as orthogonal bases for representing individual images. Though, in general, all the eigenvectors of an image set are required for perfect reconstruction of any particular image, only a few are sufficient for positioning or tracking applications. These eigenvectors constitute the dimensions of the *eigenspace*, or image subspace, in which

object appearance is represented as a function of robot displacement.

First, the average  $c$  of all images in the set is subtracted from each image. This ensures that the eigenvector with the largest eigenvalue represents the subspace dimension in which the variance of images is maximum in the correlation sense. In other words, it is the most important dimension of the eigenspace. An image matrix is constructed by subtracting  $c$  from each image and stacking the resulting vectors column-wise:

$$P \triangleq \{ \hat{i}_1 - c, \hat{i}_2 - c, \dots, \hat{i}_M - c \} \quad (4)$$

$P$  is  $N \times M$ , where  $N$  is the number of pixels in each image and  $M$  is the total number of images in the set. To compute eigenvectors of the image set we define the *covariance matrix*:

$$Q \triangleq P P^T \quad (5)$$

$Q$  is  $N \times N$ , clearly a very large matrix since a large number of pixels constitute an image. The eigenvectors  $e_k$  and the corresponding eigenvalues  $\lambda_k$  of  $Q$  are to be determined by solving the well-known eigenstructure decomposition problem:

$$\lambda_k e_k = Q e_k \quad (6)$$

Algorithms for eigenvalue decomposition produce a set of eigenvalues  $\{\lambda_k \mid k = 1, 2, \dots, K\}$  where  $\{\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_K\}$ , and a corresponding set of orthonormal eigenvectors  $\{e_k \mid k = 1, 2, \dots, K\}$ . Note that each eigenvector is of size  $N$ , i.e. the size of an image. These  $K$  eigenvectors constitute our eigenspace; it is an approximation to the complete eigenspace with  $N$  dimensions. In our experiments we have used eigenspaces of less than 20 dimensions.

### Parametric Eigenspace Representation

We now represent the appearance of the object as a function of the robot coordinates  $q$ . The result is a manifold in the  $K$ -dimensional eigenspace computed above. This representation is called the *parametric eigenspace*<sup>2</sup>.

Each learning sample  $\hat{i}_j$  in the image set is projected onto the eigenspace by first subtracting the average image  $c$  from it and finding the inner product of the result with each of the eigenvectors (dimensions) of the eigenspace. The result is a point  $f_j$ :

$$f_j = [e_1, e_2, \dots, e_K]^T (\hat{i}_j - c) \quad (7)$$

By projecting all the learning samples in this manner, a set of discrete points are obtained in eigenspace. Since consecutive object images are strongly correlated, their projections in eigenspace are close to one another. Hence, the discrete points obtained by projecting all the learning samples can be assumed to lie on a manifold that represents all possible object appearances for all possible manipulator coordinates. The discrete points are interpolated to obtain this manifold. In our implementation, we have used a standard quadratic B-spline interpolation algorithm [Rogers 90]. The resulting manifold can be expressed as:

$$f(q) = f(q_1, q_2, \dots, q_m) \quad (8)$$

This manifold is in a low-dimensional space and therefore is a compact *continuous* representation of object appearance

<sup>2</sup>The parametric eigenspace representation was introduced in [Murase and Nayar 93] for object recognition and pose estimation.

as a function of manipulator coordinates  $q$ . In practice, the number of end-effector DOFs used for positioning and tracking can vary.

The above eigenspace representation has an important property. Consider two images  $\hat{i}_r$  and  $\hat{i}_s$  that belong to the image set used to compute an eigenspace. Let the points  $f_r$  and  $f_s$  be the projections of the two images in eigenspace. It is well-known in pattern recognition theory [Oja 83] [Murase and Nayar 93] that the distance between the two points in eigenspace is an approximation to the correlation between the two brightness images:

$$\| \hat{i}_r - \hat{i}_s \|^2 \approx \| f_r - f_s \|^2 \quad (9)$$

The closer the projections are in eigenspace, the more similar are the images in  $I^2$ . Therefore, the eigenspace is optimal for computing the correlation between images. It is this property that motivates us to develop a learning methodology based on principal component analysis.

### Visual Positioning

We now discuss the automatic positioning of a manipulator at its desired coordinates  $\tilde{q}$  with respect to the viewed object. A brute force solution would be to compare an unknown input image with all images corresponding to different discrete learning coordinates. Such an approach is equivalent to exhaustive template matching. Clearly, this is impractical from a computational perspective given the large number of learning images obtained. Further, the input image may not correspond exactly to any one of the learning images, i.e. the current displacement may lie in between the discrete ones used for learning.

The parametric eigenspace representation enables us to accomplish image-displacement mapping in a very efficient manner. Since the eigenspace is optimal for computing the correlation between images, we can project the current image to the eigenspace and simply look for closest point on the manifold. Also, since the manifold is continuous, displacements that are not exactly the ones used for learning can also be estimated.

Let the robot's current position be  $q_c$  and the corresponding normalized image be  $\hat{i}_c$ . The average  $c$  of the learning set is subtracted from  $\hat{i}_c$  and the resulting vector is projected to eigenspace to obtain the point:

$$f_c = [e_1, e_2, \dots, e_K]^T (\hat{i}_c - c) \quad (10)$$

The positioning problem then is to find the minimum distance  $d$  between  $f_c$  and the manifold  $f(q)$ :

$$d = \min_q \| f_c - f(q) \| \quad (11)$$

If  $d$  is within some pre-determined threshold value (selected based on the noise characteristics of the image sensor), we conclude that the manipulator lies within the range of coordinates used for learning. Then, positioning is reduced to finding the coordinate  $q_c$  on the manifold corresponding to the minimum distance  $d$ . In practice, the manifold is stored in memory as a list of  $K$ -dimensional points obtained by densely re-sampling  $f(q)$ . The closest point to  $f_c$  on  $f(q)$  can be determined either by exhaustive search (if the list of manifold points is small), binary search, or indexing. In [Nene and Nayar 93] we have developed an algorithm that results in near-constant search time of approximately 20 msec on a Sun IPX workstation. Alternatively,  $q_c$  can be determined from  $f_c$  by training a regularization network of the type described in [Poggio and Girosi 90].

## Positioning Experiments

We have conducted several positioning experiments. For lack of space, we present only two of these results. All experiments were conducted using the Adept robot and hand-eye system shown in Figure 1. Figure 5(a) shows a printed circuit board. The box shown is the image area (128x128 pixels) used for learning and positioning. Note that the image is rather complex and includes a variety of subtle features. Images were acquired using an Analogics digitizer board. In this experiment, robot displacements were restricted to two dimensions ( $x$  and  $y$ ). A total of 256 images were obtained by moving the robot to 16x16 equally spaced discrete points within a 2cm x 2cm region around the desired position. A 15-dimensional eigenspace was computed using the 256 images. Each learning image was then projected to eigenspace and the 256 resulting points were interpolated to obtain a manifold with two parameters, namely,  $x$  and  $y$ . Since we are unable to display the manifold in 15-D space, we have shown it (see Figure 5(b)) in a 3-D eigenspace where the dimensions are the three most prominent eigenvectors of the eigenspace. The complete learning process including image acquisition, eigenspace computation, and manifold interpolation took approximately 11 minutes on a Sun IPX workstation. The parametric eigenspace is stored in memory as a set of  $251 \times 251 = 63001$  points obtained by resampling the continuous manifold. A robot displacement ( $x, y$ ) is stored with each manifold point.

Next, the accuracy of the positioning algorithm was tested. In these experiments, the robot was displaced by a random distance from its desired position. The random positions were uniformly distributed within the 2cm x 2cm region used for learning. Note that the random positions are generally not the same as any of the positions used while learning. The positioning algorithm was then used to estimate the robot's displacement from its desired position. This process was repeated 1000 times, each time computing the euclidean distance (error) between the robot location after positioning and the desired location. A histogram of positioning errors is shown in Figure 5(c). The average of the absolute positioning error is 0.676 mm and standard deviation is 0.693 mm. The positioning accuracy was dramatically improved by simply using a larger number of learning images. Figure 5(d) shows the error histogram for  $21 \times 21 = 441$  learning images obtained within the same 2cm x 2cm displacement region. In this case, the learning process was completed in approximately 30 minutes. The average absolute error was found to be 0.151 mm and standard deviation 0.107 mm. This reflects very high positioning accuracy, sufficient for reliable insertion of a circuit chip into its holder. This task was in fact accomplished with high repeatability using the gripper of the hand-eye system.

Similar experiments were conducted for the object shown in Figure 6(a). In this case, however, three displacement parameters were used, namely,  $x$ ,  $y$ , and  $\theta$  (rotation in the  $x$ - $y$  plane). During learning the  $x$  and  $y$  parameters were each varied within a  $\pm 1$ cm range, and  $\theta$  within a  $\pm 10$ deg range for each ( $x, y$ ) displacement. A total of  $11 \times 11 \times 11 = 1331$  learning images were obtained and a 5-D eigenspace computed. The parametric eigenspace representation in this case is a three-parameter manifold in 5-D space. In Figure 6(b) a projection of this manifold is shown as a surface ( $x$  and  $y$  are the parameters, while  $\theta = 0$ ) in 3-D eigenspace. Again, this reduced representation is used only for the purpose of display. The actual manifold is stored in memory as a set of  $65 \times 65 \times 65 = 274625$  points. In

this case, the entire learning process took approximately 5 hours. Once again, 1000 random displacements were used in the positioning experiments. The absolute euclidean positioning errors in  $x$ - $y$  space are illustrated by the histogram in Figure 6(c). An average absolute error of 0.291 mm and standard deviation of 0.119 mm were computed. The absolute errors for  $\theta$  were computed separately and found to have a mean value of 0.56 deg and deviation of 0.45 deg. These results again indicate high positioning accuracy. Figure 6(d) shows that positioning accuracy is only marginally improved for this particular object by doubling the eigenspace dimensionality. Here, 10 eigenvectors were computed to obtain a more descriptive representation of object appearance at the cost of additional memory usage. The positioning errors in this case have a mean of 0.271 mm and deviation of 0.116 mm, and the angular errors a mean of 0.44 deg and deviation of 0.33 deg. This accuracy was verified by successful insertions of a peg in the hole of the object.

## Visual Tracking

The visual processing aspects of tracking are identical to that of positioning. The primary difference is in the selection of learning parameters. In tracking applications successive images may be assumed to be close to one another since the manipulator is in the process of tracking the object and hence always close to the desired position. This implies that fewer learning samples are generally needed. For any new image acquired the positioning algorithm is used to determine the error  $q_c$  in robot coordinates. This error may be used as input to a position control system as shown in Figure 4. The control law may vary from a simple PID controller to more sophisticated adaptive controllers that incorporate the dynamics of the manipulator as well as delays introduced by the visual processing. The position controller generates a reference point  $q_r$  for the low-level robot actuator controller.

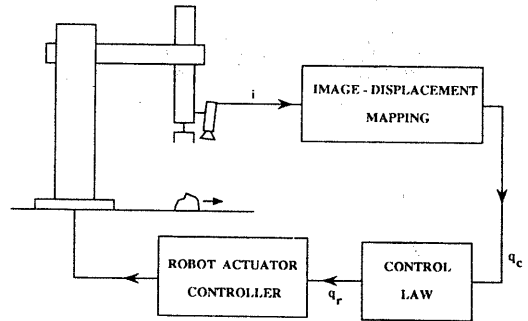


Figure 4: Schematic diagram of the visual tracking system.

## Tracking Experiments

Figure 7(a) shows an object we have used to test the tracking algorithm. The box illustrates the 96x96 pixel image region used for learning and tracking. As in the previous experiment, robot displacements were confined to three dimensions ( $x, y, \theta$ ). A total of  $13 \times 13 \times 13 = 2197$  images were acquired during the learning stage by using robot displacements within  $x = \pm 1$ cm,  $y = \pm 1$ cm,  $\theta = \pm 10$ deg. A 10-D eigenspace was used to represent the three-parameter manifold. A projection of the manifold (using  $\theta = 0$ ) in 3-D is shown in Figure 7(b).

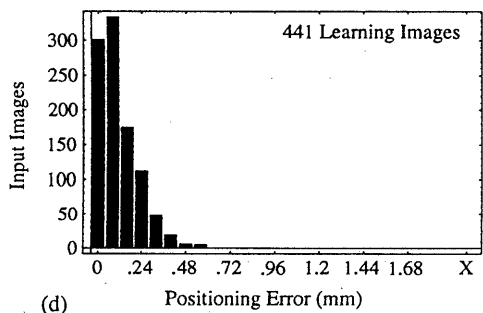
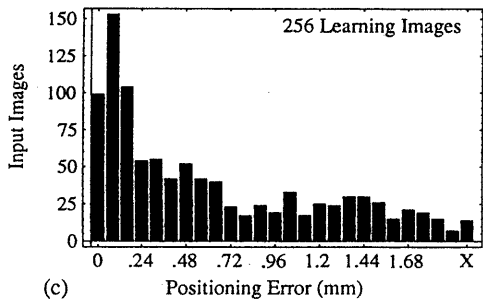
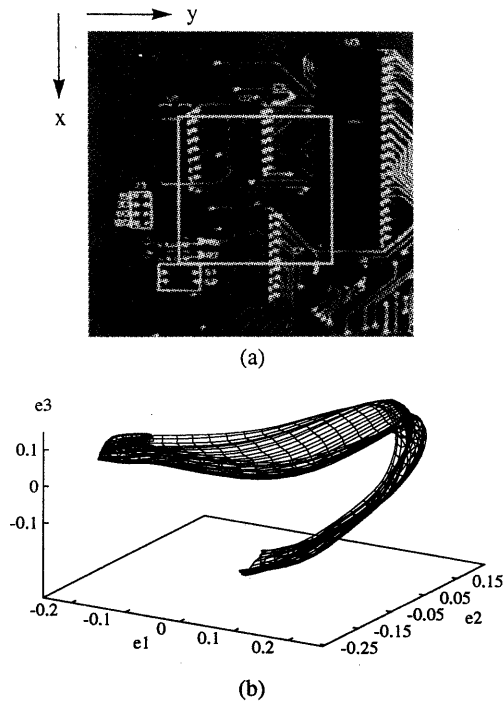


Figure 5: Visual positioning experiment: printed circuit board. (a) Image window used for learning and positioning. (b) Parametric eigenspace representation of displacement image mapping displayed in 3-D. Displacements are in two dimensions ( $x$  and  $y$ ). Histograms of absolute positioning error (in mm) for (c) 256 learning images and (d) 441 learning images.

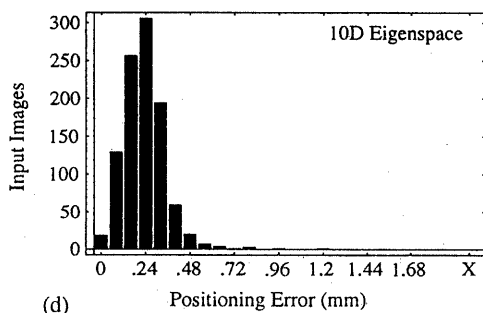
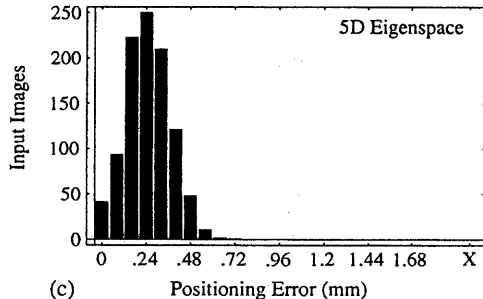
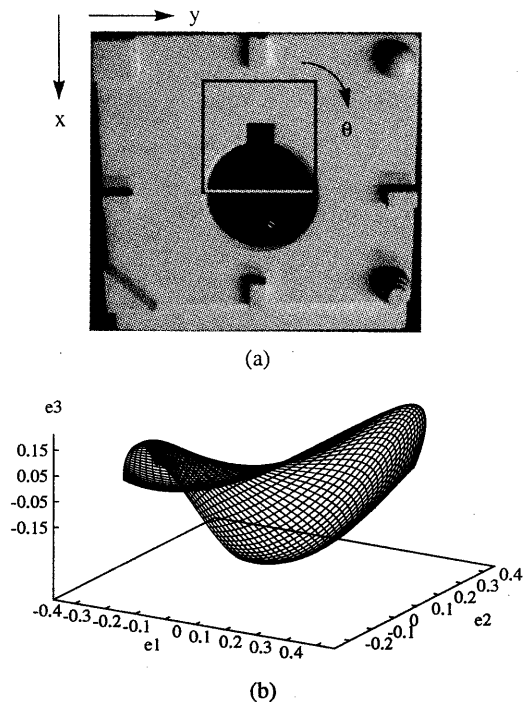
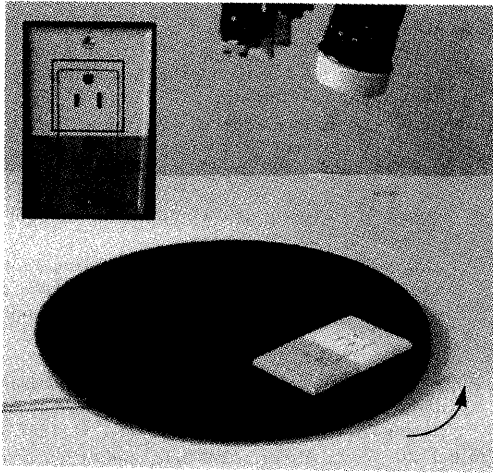
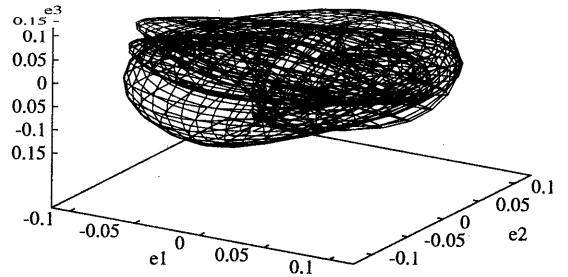


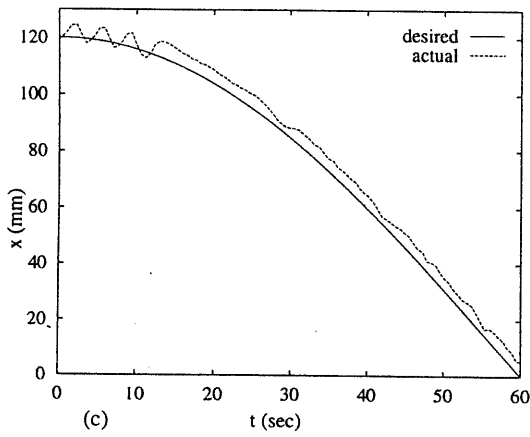
Figure 6: Visual positioning experiment: object with hole and slot. (a) Image window. (b) Parametric eigenspace representation displayed in 3-D. Displacements are in three dimensions ( $x$ ,  $y$ ,  $\theta$ ). Histograms of absolute positioning error (in mm) for (c) 5-D eigenspace and (d) 10-D eigenspace.



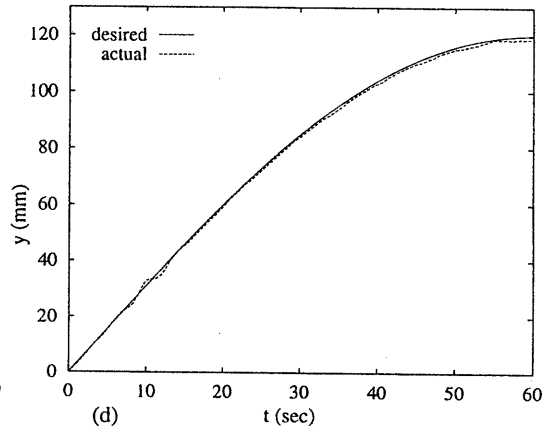
(a)



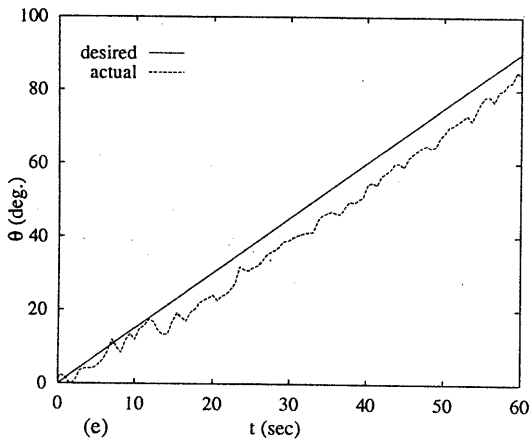
(b)



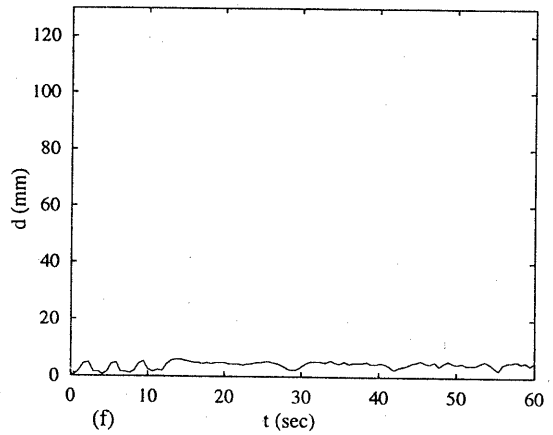
(c)



(d)



(e)



(f)

Figure 7: Visual Tracking experiment: moving electric socket. (a) Image Window. (b) Parametric eigenspace representation. Desired and actual coordinates: (c)  $x(t)$ ; (d)  $y(t)$ ; and (e)  $q(t)$ . (f) Tracking distance error  $d(t)$ .

Each cycle of the tracking algorithm involves the digitization of an input image, transfer of image data from the digitizer to the workstation, projection of the input image into eigenspace, search for the closest manifold point, computation of reference coordinates using a control law, and communication of the reference coordinates to the robot controller. In the present implementation, all computations are done on the Sun IPX workstation without the use of any customized image processing hardware. The total cycle time at present is approximately 250msec yielding a control rate of 4Hz. This restricts our present experiments to objects moving at relatively slow speeds (approximately 0.5cm/sec). It may be noted that this is merely a limitation of the current implementation. All computations involved in the image-displacement mapping are simple and can be easily done at frame-rate (30Hz) with a single frame-time delay using a more powerful workstation such as a DEC Alpha machine, or inexpensive image processing hardware such as a standard i860 board.

The present control law is based on a simple interpolation/prediction scheme to facilitate smooth manipulator motion. The tracking accuracy was determined by moving the object at known velocity along a circle using a motorized turntable (Figure7(a)). The turntable was rotated through 90 deg, moving the object through a total distance of 19 cm. In Figures7(c)-(e) the desired and actual coordinates of the robot are plotted as a function of time. The deviations and lags that result while tracking are attributed mostly to delays introduced by the vision computations and the simple control scheme used. Our current work is geared towards overcoming these limitations to achieve higher tracking speeds. Also, the experiments reported here were confined to three end-effector parameters  $(x, y, \theta)$ . We are currently exploring extensions to positioning/tracking problems that involve more than three degrees of freedom.

### Acknowledgements

The authors would like to thank Nikolaos Papanikolopoulos and Peter Allen for their comments on this paper.

### References

- [Weiss et al. 87] L. Weiss, A. Sanderson, and C. Neuman, *Dynamic sensor-based control of robots with visual feedback*, IEEE Journal of Robotics and Automation, Vol. RA-3, No. 5, pp. 404-417, Oct. 1987.
- [Feddema et al. 91] J. Feddema, C.S.G. Lee, and O. Mitchell, *Weighted selection of image features for resolved rate visual feedback control*, IEEE Transactions on Robotics and Automation, Vol. 7, No. 1, pp. 31-47, Feb. 1991.
- [Koivo and Houshangi 91] A. Koivo and N. Houshangi, *Real-time vision feedback for servoing robotics manipulator with self-tuning controller*, IEEE Transactions on Systems, Man, and Cybernetics, Vol. 21, No. 1, pp. 134-142, Feb. 1991.
- [Hashimoto et al. 91] K. Hashimoto, T. Kimoto, and H. Kimura, *Manipulator Control with Image-Based Visual Servo*, Proceedings of IEEE International Conference on Robotics and Automation, 1991, pp. 2267-2271.
- [Papanikolopoulos et al. 91] N. Papanikolopoulos, P. Khosla, and T. Kanade, *Adaptive robotic visual tracking*, Proceedings of Automatic Control Conference, 1991.
- [Luo et al. 88] R. Luo, R. Mullen, and D. Wessel, *An Adaptive Robotic Tracking System using Optical Flow*, Proceedings of IEEE International Conference on Robotics and Automation, 1988, pp. 568-573.
- [Allen et al. 92] P. K. Allen, A. Timcenko, B. Yoshimi, and P. Michelman, *Trajectory Filtering and Prediction for Automated Tracking and Grasping of a Moving Object*, Proceedings of IEEE International Conference on Robotics and Automation, Nice, 1992, pp. 1850-1856.
- [Kuperstien 87] M. Kuperstien, *Adaptive visual-motor coordination in multi-joint robots using parallel architecture*, Proceedings of IEEE International Conference on Robotics and Automation, Raleigh, N.C., 1987, pp. 1595-1602.
- [Mel 87] B. W. Mel, *MURPHY: A robot that learns by doing*, AIP Proceedings of Neural Information Processing System Conference, Denver, CO, 1987.
- [Miller 89] W. T. Miller, *Real-time application of neural networks for sensor-based control of robots with vision*, IEEE Transactions on Systems, Man, and Cybernetics, Vol. 19, No. 4, pp. 825-831, July/August, 1989.
- [Walter et al. 90] J. Walter, T. Martinez, and K. Schulten, *Industrial robot learns visuo-motor coordination by means of neural-gas network*, Proceedings of International Joint Conference on Neural Networks, June, 1990.
- [Albus 75] J. S. Albus, *A new approach to manipulator control: The cerebellar model*, Transactions of ASME, Journal of Dynamic Systems Measurement and Control, Vol. 97, pp. 220-227, Sept. 1975.
- [Miller 87] W. T. Miller, *Sensor-based control of robotic manipulators using a general learning algorithm*, IEEE Journal of Robotics and Automation, Vol. RA-3, No. 2, pp. 157-165, April, 1987.
- [Oja 83] E. Oja, *Subspace methods of Pattern Recognition*, Research Studies Press, Hertfordshire, 1983.
- [Murase and Nayar 93] H. Murase and S. K. Nayar, "Learning and Recognition of 3D Objects from Appearance," *Proc. of IEEE Workshop on Qualitative Vision*, pp. 39-50, June 1993.
- [Rogers 90] D. F. Rogers, *Mathematical Elements for Computer Graphics*, 2nd ed., McGraw-Hill, New York, 1990.
- [Nene and Nayar 93] S. A. Nene and S. K. Nayar, "Binary Search Through Multiple Dimensions," Technical Report CUCS-26-93, Department of Computer Science, Columbia University, August, 1993.
- [Poggio and Girosi 90] T. Poggio and F. Girosi, "Networks for Approximation and Learning," *Proceedings of the IEEE*, Vol. 78, No. 9, pp. 1481-1497, September 1990.