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ランダム標本化と投票によるオプティカルフローの計算

井宮 淳　岩脇圭介

千葉大学　情報画像工学科
263-8522 千葉稲毛区弥生町 1-33
Tel +81-43-290-3257 imiya@ics.tj.chiba-u.ac.jp

あらまし　本論文では, ランダム標本化と投票によるオプティカルフローの計算法を提案する. 線形オプティカルフローは注目領域の中で成立する線形連立方程式の解として与えられる. この連立方程式は, 解の次元の数よりも拘束方程式の数が多い過剰決定系である. まず, ランダム標本化と投票による方法によって過剰決定系の解が求まることを示す. すなわち, 解の次元の数と同数の線形拘束をランダムに選んで構成される正則な方程式の解を投票するハフ変換を提案する. 次いで, 画像理解における代表的な過剰決定系である線形オプティカルフローの算出にこの手法を適用する.

キーワード　ハフ変換, 最小自乗法, 模型当てはめ, ランダム算法, 線形オプティカルフロー,

Random Sampling and Voting Process for the Detection of Linear Flow Field

Atsushi IMIYA　and　Keisuke IWAWAKI

Department of Information and Image Sciences, Chiba University
1-33 Yayoi-cho, Inage-ku, 263-8522, Chiba, Japan
Tel +81-43-290-3257 imiya@ics.tj.chiba-u.ac.jp

Abstract　In this paper, we show that the random sampling and voting process detects linear flow filed. First, we summarize the theoretical aspects of randomized sampling and voting process as dynamics which solves model-fitting-problem. Second, we formalize the linear flow field detection as a model fitting problem which is solved by the least squares method. Finally, we show some numerical examples which shows the performance of our method. We introduce a new idea to solve least square model fitting problem using a mathematical fact for the construction of pseudo-inverse.

Key Words　The Hough Transform, Least-Squares Method, Model Fitting, Random Algorithms, Linear Flow Field

1 Introduction

This paper is the forth of the series of papers which deal with theoretical aspects of the Hough transform. In this paper, we deal with the random sampling and voting process as a solver of linear flow detection. In a series of papers [1, 2], the author introduced the random sampling and voting method for the problems of machine vision. The method is an extension of the randomized Hough transform which is first introduced by Finish school for planer image analysis [7]. Later they applied the method to planar motion analysis [3] and shape reconstruction from flow field [4]. These results show that the inference of parameters by voting solves the least-square problem in machine vision without assuming the predetermination of point correspondences between image frames.

The randomized Hough transform is formulated as a parallel distributed model which estimates parameters of planar lines and spatial planes, which are typical basic problems in computer vision. Furthermore, many problems in computer vision are formulated as model fitting problems in higher dimensional spaces. These problems are expressed in the framework of the least squares method (LSM) for the parameter estimation [6].

Many problems in computer vision are expressed as the minimization of the criterion

$$J(\boldsymbol{x}_\alpha) = \sum_{\alpha=1}^{N} |\boldsymbol{A}_\alpha \boldsymbol{x}_\alpha - \boldsymbol{b}_\alpha|^2, \tag{1}$$

for given $\boldsymbol{A} = \alpha$ and $\boldsymbol{b}_\alpha$, or

$$J(\xi_\alpha) = \sum_{\alpha=1}^{N} |\boldsymbol{B}_\alpha \xi_\alpha|^2, \tag{2}$$

where

$$\boldsymbol{B}_\alpha = (\boldsymbol{A}_\alpha, -\boldsymbol{b}_\alpha)\,,\ \boldsymbol{\xi} = (\boldsymbol{x}^\top, 1)^\top \tag{3}$$

These problem can be converted to the minimization of $tr(\boldsymbol{X}\boldsymbol{M})$ for matrices $\boldsymbol{X}$ and $\boldsymbol{A}$. The family of ascent equation

$$\dot{\boldsymbol{x}} = \nabla \phi,\ \phi = tr(\boldsymbol{X}\boldsymbol{A}), \tag{4}$$

provides a framework for the minimization of the least-squares method [8, 9]. Brockett introduces a dynamical system for a matching problem which is motivated by a basic problem in computer vision, matching for the motion analysis [8, 9]. He has extended and examined mathematical properties of these dynamical systems. We continue his primal motivations applying his primal idea to the Hough transform. Furthermore, the mathematical properties of of the random sampling and voting procedures for computer vision are examined as a dynamical system.

In this paper, we show that the randomized sampling and voting process detects linear flow filed. First, we summarize the theoretical aspects of randomized sampling and voting process as dynamics which solves model-fitting-problem. Second, we formalize the linear flow field detection as a model fitting problem which is solved by LSM. Finally, we show some numerical examples which shows the performance of our method. We introduce a new idea to solve least square model fitting problem using a mathematical fact for the construction of pseudo-inverse.

2 Model Fitting and LSM

Setting $\boldsymbol{x} = (x, y)^\top$ to be a vector in two-dimensional Euclidean space $\mathbf{R}^2$, the homogeneous coordinate of vector $\boldsymbol{x}$ is expressed as $\boldsymbol{\xi} = (x, y, 1)^\top$ which expresses a point on the projective plane. For a nonzero real number λ, the the homogeneous coordinates $\boldsymbol{\xi}$ and $\lambda\boldsymbol{\xi}$ express the same point $\boldsymbol{x}$, that is, a homogeneous coordinate $\boldsymbol{\xi} = (\alpha, \beta, \gamma)^\top$ such that $\boldsymbol{\xi} \neq (0,0,0)^\top$ defines a point $(\alpha/\gamma, \beta/\gamma)^\top$. Therefore, there exist one-to-one mapping between points on $\mathbf{R}^2$ and points on the positive hemisphere S^2_+ in three-dimensional Euclidean space $\mathbf{R}^3$. We define the homogeneous coordinate $\boldsymbol{\xi}$ of a vector $\boldsymbol{x} \in \mathbf{R}^k$ for $k \geq 3$ as $\boldsymbol{\xi} = (\boldsymbol{x}^\top, 1)^\top$. We denote the Euclidean length of vector $\boldsymbol{x}$ in k-dimensional Euclidean space $\mathbf{R}^k$ for $k \geq 1$ as $|\boldsymbol{a}|$.

Let S^{n-1} be the unit sphere of $\mathbf{R}^n$ consisting of all points $\boldsymbol{x}$ with distance 1 from the origin. For $n = 1$, $S^0 = [-1, 1]$. Furthermore, the positive half-space is defined by

$$\mathbf{R}^n_+ = \{\boldsymbol{x} | x_n > 0\},\ n \geq 1. \tag{5}$$

Now, by setting

$$H^{n-1}_+ = S^{n-1} \bigcap \mathbf{R}^n_+,\ n \geq 1, \tag{6}$$

the positive unit semi-sphere is defined by

$$S^{n-1}_+ = S^{n-2}_+ \bigcup H^{n-1}_+,\ n \geq 1. \tag{7}$$

Setting $\boldsymbol{x}$ to be the valuable in n-dimensional Euclidean space, the Hough transform is a method for the estimation of parameters $\{\boldsymbol{a}_i\}_{i=1}^k$ of a collection of equations,

$$f_i(\boldsymbol{a}_i, \boldsymbol{x}) = 0,\ i = 1, 2, \cdots, k \tag{8}$$

from finite many samples $\{\boldsymbol{x}_j\}_{j=1}^m$ such that $m \gg k \geq 1$ using the voting procedure. An equation $f_i(\boldsymbol{a}_i, \boldsymbol{x}) = 0$ is called a model for the parameter estimation. The most typical and traditional models for the Hough transform are planar lines and conics if the dimension of space is two.

The model fitting for s set of sample points $\{x\}_{\alpha=1}^{n}$ is achieved by minimizing the criterion

$$J(\boldsymbol{a}) = \sum_{\alpha=1}^{n} w(\alpha)|f(\boldsymbol{a}, \boldsymbol{x}_\alpha)|^2, \quad (9)$$

with some constraines. If the model equation is expressed in the form $f(\boldsymbol{a}, \boldsymbol{x}) = \boldsymbol{a}^\top \boldsymbol{x}$, these problems are also expressed as the minimization of $tr(\boldsymbol{MA})$, where

$$\boldsymbol{M} = \sum_{\alpha=1}^{n} w(\alpha)\boldsymbol{x}_\alpha \boldsymbol{x}_\alpha^\top, \ \boldsymbol{A} = \boldsymbol{a}\boldsymbol{a}^\top. \quad (10)$$

If $f(\boldsymbol{a}, \boldsymbol{x}) = \boldsymbol{a}^\top \boldsymbol{x}$, we can nomalize $|\boldsymbol{a}| = 1$. Since $\boldsymbol{A}\boldsymbol{a} = |\boldsymbol{a}|^2\boldsymbol{a}$, $rank\boldsymbol{A} = 1$, and $\boldsymbol{A}^\top = \boldsymbol{A}$, vector $\boldsymbol{a}$ is the eigenvector associated to nonzero eigenvalue of $\boldsymbol{A}$.

Our problem is equivalent to the next problem.

Problem 1 *Find $\boldsymbol{A}$ such that $\boldsymbol{A} \succeq \boldsymbol{O}$ which minimizes $tr(\boldsymbol{M}\,\boldsymbol{A})$ with the constraint $tr(\boldsymbol{IA}) = 1$, since $tr\boldsymbol{A} = |\boldsymbol{a}|^2$.*

This is the expression of the semidefinite programming problem. Therefore, the interior point method, which is a gradient decent method for convex programming problem, solves the problem.

The gradient flow

$$\frac{d\boldsymbol{A}}{dt} = -[\boldsymbol{A}, [\boldsymbol{M}, \boldsymbol{A}]] \quad (11)$$

derives the solution which minimizes the criterion

$$J(\boldsymbol{A}) = tr(\boldsymbol{AM}) \quad (12)$$

for

$$\lim_{t\to\infty} \boldsymbol{A}(t) = \boldsymbol{A}. \quad (13)$$

For a system of equations

$$\boldsymbol{\xi}_\alpha^\top \boldsymbol{a} = 0, \ \alpha = 1, 2, \cdots m, \quad (14)$$

setting

$$\Xi = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots \boldsymbol{\xi}_m)^\top \quad (15)$$

the rank of matrix Ξ is n if vector $\boldsymbol{x}_\alpha$ is an element of $\mathbf{R}^n$. Therefore, all $n \times n$ square submatrix $\boldsymbol{N}$ of Ξ is nonsingular. Setting N_{ij} to be the ij-th adjacent of matrix $\boldsymbol{N}$, we have the equality

$$f_{\alpha 1}N_{11} + f_{\alpha 2}N_{21} + \cdots + f_{\alpha n}N_{n1} = 0, \quad (16)$$

if the first column of $\boldsymbol{N}$ is $\boldsymbol{\xi} = (\boldsymbol{x}_\alpha^\top, 1)^\top$. Therefore, the solution of this system of equation is

$$\boldsymbol{a} = (n_{11}, n_{21}, \cdots, n_{n1})^\top, n_{i1} = \frac{N_{i1}}{\sqrt{\sum_{j=1}^{n} N_{j1}^2}}. \quad (17)$$

If the dimension of the parameter of model is 3, we have the solution as

$$\boldsymbol{a}_{\alpha\beta} = \frac{\boldsymbol{\xi}_\alpha \times \boldsymbol{\xi}_\beta}{|\boldsymbol{\xi}_\alpha \times \boldsymbol{\xi}_\beta|}, \quad (18)$$

for a pair of randomly selected vectors. Therefore, we can adopt the solution of a system of equations.

Since there are ${}_mC_n$ posibilities for the selection of $n\times n$ square submatrix from $\boldsymbol{M}$, we can randomly select n column vectors. Since samples are noisy, the model parameter is the vectoe which satisfies

$$|\boldsymbol{\xi}_\alpha^\top \boldsymbol{a}| = \varepsilon_\alpha, \quad (19)$$

for a small positive number ε_α. Therefore, we adopt the average of $\boldsymbol{a}$ for many combination of column vectors of Ξ. These properties lead to random sampling and voting procedure, such as

Algorithm 1

1 : Select randomly n column vectors from Ξ.

2 : Compute N_{i1} and $\boldsymbol{a}$.

3 : Vote $\boldsymbol{a}$ to accumulator space which is topologically equivalent to unit hemisphere in $\mathbf{R}^n$.

4 : After an appropriate number of iteration detect the peak in the accumulator space, and return it as the solution.

A generalization of this property is based on the following proposition.

Proposition 1 *[5] Assuming that matrices $\boldsymbol{P}_k$ and $\boldsymbol{O}$ are a $k \times k$ permutation matrix and the $(m-n)\times n$ null matrix, respectively, for a $m \times n$ matrix $\boldsymbol{A}$ such that $m > n$ and $rank\boldsymbol{A} = n$, vector $\boldsymbol{a}$ which is defined as*

$$\boldsymbol{a} = \left(\boldsymbol{A}_n^{-1} \ \boldsymbol{O}\right) \boldsymbol{y} \quad (20)$$

for

$$\boldsymbol{A}_n = (\boldsymbol{P}_n \ \boldsymbol{O})\, \boldsymbol{P}_m \boldsymbol{A} \quad (21)$$

minimizes a criterion $|\boldsymbol{y} - \boldsymbol{Aa}|^2$.

There are ambiguities for the selection of $\boldsymbol{P}_n$ and $\boldsymbol{P}_m$. The proposition implies that if the $m \times n$ system matrix is column full-rank,

1. selecting n equations from the system of equations, and

2. solving this nonsingular equation,

we obtain a solution of the least square optimization. If we randomly select column vectors, this method also derives an extension of the randomize Hough transform. We show a simple example for the application of this proposition.

Example 1 *Let a system of equation be*

$$\begin{pmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \\ 1/4 & 3/4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{22}$$

Since the rank of the system matrix is 2, we have the following three systems of equations,

$$\begin{pmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{23}$$

$$\begin{pmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{24}$$

$$\begin{pmatrix} 2/3 & 1/3 \\ 1/4 & 3/4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{25}$$

The solutions of these equations are all $(a,b)^\top = (1,1)^\top$.

3 Flow Field Detection

3.1 The Hough Transform

A line on two-dimensional Euclidean plane $\mathbf{R}^2$ is expressed as

$$\boldsymbol{\xi}^\top \boldsymbol{a} = 0 \tag{26}$$

where the parameter vector $\boldsymbol{a} = (a,b,c)$ is normalized to the unit length, that is, $|\boldsymbol{a}| = 1$.

Let m lines exist on $\mathbf{R}^2$ and sample-points P be separated to clusters of points such as

$$P = \bigcup_{i=1}^{m} P_i, \text{ s.t.} P_i = \{\boldsymbol{x}_{ij}\}_{j=1}^{n(i)}. \tag{27}$$

Furthermore, we assume that in each cluster there exists k points, that, is $k = n(i)$ and $k \times m = n$. The Hough transform for the line detection achieves both the classification of sample points and the model fitting concurrently. Therefore, in this section, we formulate the randomized Hough transform as the LSM for the model fitting problem. After clustering sample-points, we have the equation,

$$\boldsymbol{\xi}_{ij}^\top \boldsymbol{a}_i = \boldsymbol{0}, i = 1,2,\cdots m. \tag{28}$$

For a collection of sample points, we call matrix $|\boldsymbol{Xi}$,

$$\Xi = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_n) \tag{29}$$

the data matrix. If there is no error in sample points, the parameters of a line satisfies the equation

$$\Xi^\top \boldsymbol{a} = \boldsymbol{0}. \tag{30}$$

Let $\boldsymbol{Q}$ be an appropriate permutation matrix. Setting

$$\Xi \boldsymbol{Q} = (\boldsymbol{\xi}_{\sigma(1)}, \boldsymbol{\xi}_{\sigma(2)}, \cdots, \boldsymbol{\xi}_{\sigma(n)})^\top \tag{31}$$

where $\sigma(\cdot)$ is an appropriate permutation associated with $\boldsymbol{Q}$, we have

$$\Xi \boldsymbol{Q} = (\Xi_1, \Xi_2, \cdots, \Xi_n) \tag{32}$$

where each $\boldsymbol{Xi}_i$ is $3 \times k$ matrix. If each Ξ_i forms cluster which determines a line, matrix Ξ_i and parameter $\boldsymbol{a}_i$ satisfy the relation

$$\Xi_i \boldsymbol{a}_i = 0, \ i = 1,2,\cdots,m. \tag{33}$$

Therefore, setting

$$^\top(\Xi_1, \Xi_2, \cdots, \Xi_m) = (\Xi_1^\top, \Xi_2^\top, \cdots, \Xi_m^\top)^\top \tag{34}$$

and

$$\boldsymbol{a} = \boldsymbol{a}_1, \boldsymbol{a}_2 \cdots, \boldsymbol{a}_m \tag{35}$$

the Hough transform achieves the minimization of

$$J(\boldsymbol{a}, \boldsymbol{Q}) = \left| \left\{ ^\top(\Xi \boldsymbol{Q}) \right\}^\top \boldsymbol{a} \right|^2 \tag{36}$$

with constrains

$$|\boldsymbol{a}_i|^2 = 1, \ i = 1,2,\cdots,m. \tag{37}$$

This property implies that the classification of sample data is equivalent to the permutation of elements of data matrix Ξ. There exist many possibilities for the selection of a permutation $\boldsymbol{Q}$, if we do not know the estimate of $\{\boldsymbol{a}_i\}_{i=1}^m$. These expressions of the line fitting problem conclude that the Hough transform achieves both the permutation of data matrix Ξ and the computation of the solution which satisfies eq. (??) concurrently.

Setting

$$\boldsymbol{a}_{ij} = \frac{\boldsymbol{\xi}_i \times \boldsymbol{\xi}_j}{|\boldsymbol{\xi}_i \times \boldsymbol{\xi}_j|} \tag{38}$$

for all $\boldsymbol{\xi}_i$ and $\boldsymbol{\xi}_j$ such that $i \neq j$. For noisy data, the average of $\boldsymbol{a}_{ij}$

$$\boldsymbol{a} = \frac{1}{{}_nC_2} \sum_{i<j} \boldsymbol{a}_{ij} \tag{39}$$

might provide a good estimate of the parameter of a line. This property implies the following algorithm for the estimate of parameter vector $\boldsymbol{a}$ from noisy data

Next, we calculate the accuracy of solutions which Algorithm 1 yields. Assuming that $|P_i| = p$ and the

```
procedure   : Random Algorithm 2
 begin
   select Tmax and p
   set scor(a) := 0
     while  t ≤ Tmax   do
       begin
         select ξ_i and ξ_j, randomly
         compute a = ξ_i×ξ_j / |ξ_i×ξ_j|
         set scor(a) := scor(a) + 1
         if  scor(a) > p  then  retern a
              else  t := t + 1
       end
 end
```

Figure 1: An Algorithm for Line Detection

dimension of the parameter is q for $p \gg q$, the number of the combinations of the equations which yield the valid solution is ${}_pC_q$ for each parameter. Furthermore, the number of the combination of q equations from n samples is ${}_nC_q$. Therefore, the probability for obtaining valid solutions is ${}_pC_q/{}_nC_q$, since we assume that there exist m models and $p \times m(= n)$ sample points. This property implies that the probability of a valid solution is

$$P_t =_p C_q/(m \times_n C_q), \tag{40}$$

and the probability of an invalid solution is

$$P_f = \frac{1 -_p C_q/_nC_q}{_nC_q -_p C_q}. \tag{41}$$

Henceforward, we have the inequality $P_t \gg P_f$ if $n \gg m$. This property implies that, after the sufficient number of iterations, Algorithm 1 achieves both the classification of data and the estimation of parameters concurrently.

3.2 Linear Flow Field Detection

Setting $f(x,y,t)$ to be a time dependent gray-scale image, the linear optical flow $\boldsymbol{u} = (u,v,1)^\top$ is the solution of the linear equation

$$\boldsymbol{f}^\top \boldsymbol{u} = 0, \tag{42}$$

for

$$\frac{df(x,y,t)}{dt} = \boldsymbol{f}^\top \boldsymbol{u}, \tag{43}$$

where

$$\boldsymbol{f} = \left(\frac{\partial f(x,y,t)}{\partial x}, \frac{\partial f(x,y,t)}{\partial y}, \frac{\partial f(x,y,t)}{\partial t}\right)^\top \tag{44}$$

Assuming that the flow vector $\boldsymbol{u}$ is constant in an area S, the flow vector in an area is the solution of the minimization problem

$$J(\boldsymbol{u}) = \sum_{\alpha=1}^{n} (\boldsymbol{f}_\alpha^\top \boldsymbol{u})^2, \tag{45}$$

where

$$\boldsymbol{f}_\alpha = \left(\frac{\partial f(x,y,t)}{\partial x}, \frac{\partial f(x,y,t)}{\partial y}, \frac{\partial f(x,y,t)}{\partial t}\right)^\top \Bigg|_{x=x_\alpha, y=y_\alpha} \tag{46}$$

for a sample point $(x_\alpha, y_\alpha)^\top$ in an area S.

Therefore, methods described in the previous sections are valid for the detection of the flow filed. Setting the eigenvector associated to the minimum eigenvalue of

$$M = \sum_{\alpha=1}^{N} \boldsymbol{f}_\alpha \boldsymbol{f}_\alpha^\top \tag{47}$$

to be

$$\boldsymbol{a} = (A,B,C)^\top, \tag{48}$$

our solution is

$$\boldsymbol{u} = \left(\frac{A}{C}, \frac{B}{C}, 1\right)^\top. \tag{49}$$

For a system of equations

$$\boldsymbol{f}_\alpha^\top \boldsymbol{a} = 0,\ \alpha = 1,2,\cdots,N, \tag{50}$$

in a windowed area, we have

$$\boldsymbol{a} = \frac{\boldsymbol{f}_\alpha \times \boldsymbol{f}_\beta}{|\boldsymbol{f}_\alpha \times \boldsymbol{f}_\beta|},\ \boldsymbol{a} = (A,B,C)^\top, \tag{51}$$

if we assume that $|\boldsymbol{a}| = 1$. Furthermore, since, $\boldsymbol{u} = (u,v,1)^\top$, setting

$$\boldsymbol{u} = \lambda(\boldsymbol{f}_\alpha \times \boldsymbol{f}_\beta),\ \boldsymbol{\alpha} = (\alpha,\beta,\gamma)^\top \tag{52}$$

for a nonzero real constant λ, we have also

$$\boldsymbol{u} = (\frac{\alpha}{\gamma}, \frac{\beta}{\gamma}, 1)^\top. \tag{53}$$

Moreover, setting

$$f_{x\alpha} = \frac{\partial f(x,y,t)}{\partial x}\bigg|_{x=x_\alpha, y=y_\alpha, t=\tau}, \tag{54}$$

$$f_{y\alpha} = \frac{\partial f(x,y,t)}{\partial y}\bigg|_{x=x_\alpha, y=y_\alpha, t=\tau}, \tag{55}$$

$$f_{\tau\alpha} = \frac{\partial f(x,y,t)}{\partial t}\bigg|_{x=x_\alpha, y=y_\alpha, t=\tau}, \tag{56}$$

the linear constrain for the linear optical flow is

$$f_{x\alpha}u + f_{y\alpha}v + f_{\tau\alpha} = 0 \tag{57}$$

Setting

$$a_\alpha = -\frac{f_{x\alpha}}{f_{\tau\alpha}},\ b_\alpha = -\frac{f_{y\alpha}}{f_{\tau\alpha}}, \tag{58}$$

for $f_{\tau\alpha} \neq 0$, eq. (57) becomes

$$\frac{u}{a_\alpha} + \frac{v}{b_\alpha} = 1. \tag{59}$$

Therefore, $(u, v)^\top$ is the common point of lines which connect $(a_\alpha, 0)^\top$ and $(0, b_\alpha)^\top$, and $(a_\beta, 0)^\top$ and $(0, b_\beta)^\top$. This property implies that the clasical Hough transform achieves the linear flow-filed detection voting lines onto the accumulator space.

4 Numerical Examples

Since our images are sampled and expressed in the pixel form, we approximate,

$$\begin{aligned} f_x &= \frac{1}{4}\{f(x+1,y,t) + f(x+1,y+1,t) \\ &\quad + f(x+1,y,t+1) + f(x+1,y+1,t+1)\} \\ &\quad - \frac{1}{4}\{f(x,y,t) + f(x,y+1,t) \\ &\quad + f(x,y,t+1) + f(x,y+1,t+1)\}, \end{aligned} \tag{60}$$

$$\begin{aligned} f_y &= \frac{1}{4}\{f(x,y+1,t) + f(x+1,y+1,t) \\ &\quad + f(x,y+1,t+1) + f(x+1,y+1,t+1)\} \\ &\quad - \frac{1}{4}\{f(x,y,t) + f(x+1,y,t) \\ &\quad + f(x,y,t+1) + f(x+1,y,t+1)\}, \end{aligned} \tag{61}$$

$$\begin{aligned} f_t &= \frac{1}{4}\{f(x,y,t+1) + f(x+1,y,t+1) \\ &\quad + f(x,y+1,t+1) + f(x+1,y+1,t+1)\} \\ &\quad - \frac{1}{4}\{f(x,y,t) + f(x+1,y,t) \\ &\quad + f(x,y+1,t) + f(x+1,y+1,t)\}. \end{aligned} \tag{62}$$

Furthermore, we assume that our window is 5×5, pixel. Therefore, we have the ${}_{25}C_2$ combinations for the selection of a pair of linear constraints from 25 constrains.

For the computation of flow vector $\boldsymbol{u}$, we adopt two methods which compute eq. (51) and the common point of two lines which are defined by eq. (57). For the case of the common point of a pair of lines, we use two accumulator space. First as the classical Hough transform, we first vote a pair of lines in the usual accumulator space. Second we vote this common point to the second accumulator. After voting this point to the second accumulator space, for the clearance of the first accumulator space, we delate lines from the first accumulator space. This procedure is introduced to save the amount of memory for the voting.

The linear constrains of for the linear optical flow field are derived as the first order approximation of intensity constrain. Furthermore, because of digitization, f_x, f_y, and f_t contain numerical errors. These errors distribute the solutions in the accumulator space. Therefore, for both cases, we adopt the median of peaks in the accumulator space. This is the significant difference from the usual Hough transform for the detection of lines and conics on a plane.

Figures 2 and 3 show the results for eq. (51) and the common point of a pair of lines for "Hamburg Taxi. " Our method detects motion of all cars. The results show the average filed from frame 1 to frame 21. The solutions are detected from 100 samples. These results show that the random sampling process speed up the computational time.

5 Conclusions

In this paper, we showed that the random sampling and voting process detects linear flow filed. First, we summarized the theoretical aspects of randomized sampling and voting process as dynamics which solves model-fitting-problem. Second, we formalized the linear flow field detection as a model fitting problem which is solved by LSM. Finally, we show some numerical examples which shows the performance of our method. We introduced a new idea to solve least square model fitting problem using a mathematical fact for the construction of pseudo-inverse. The most advantage of the proposed method is simple because we used the same engine with the Hough transform for the planar line detection.

Setting k to be the number of windowed regions in a frame, a system of equations which yields the flow field is described as

$$\boldsymbol{A}_i\boldsymbol{x}_i = \boldsymbol{b}_i,\ i = 1, 2, \cdots, k. \tag{63}$$

This system is expressed as

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \tag{64}$$

where

$$\begin{aligned} \boldsymbol{A} &= \mathrm{Diag}(\boldsymbol{A}_1, \boldsymbol{A}_2, \cdots, \boldsymbol{A}_k) \\ \boldsymbol{x} &= \mathrm{diag}(\boldsymbol{x}_1^\top, \boldsymbol{x}_2^\top, \cdots, \boldsymbol{x}_k^\top)^\top \\ \boldsymbol{b} &= \mathrm{diag}(\boldsymbol{b}_1^\top, \boldsymbol{b}_2^\top, \cdots, \boldsymbol{b}_k^\top)^\top. \end{aligned} \tag{65}$$

The LSM solution of eq. (64) is

$$\boldsymbol{x} = (\boldsymbol{A}^\top\boldsymbol{A})^{-1}\boldsymbol{A}^\top\boldsymbol{b}, \tag{66}$$

since matrix $\boldsymbol{A}_i$ is column full rank for $i = 1, 2, \cdots, k$.

It is possible to solve eq. (64) using dynamical system. However, each subproblem of eq. (64) as the same mathematical structure. Therefore, if we have the same number of machines with the windowed region, we have the flow vector at each reagin concurrently. This mathematical structure means that the parallel distributed framework is suitable for the detection of flow field. This system is similar to the visual system of insects which has many small eyes for the detection of motions of objects.

As we mentioned in section 3.1, the Hough transform achieves grouping of sample points applying the permutation of data matrix. However, for the detection o linear flow field, we need not to achieve grouping of sample points, because we solves the model-fitting problem using the random sampling and voting. This is the fundamental difference between the Hough transform and the flow field detection.

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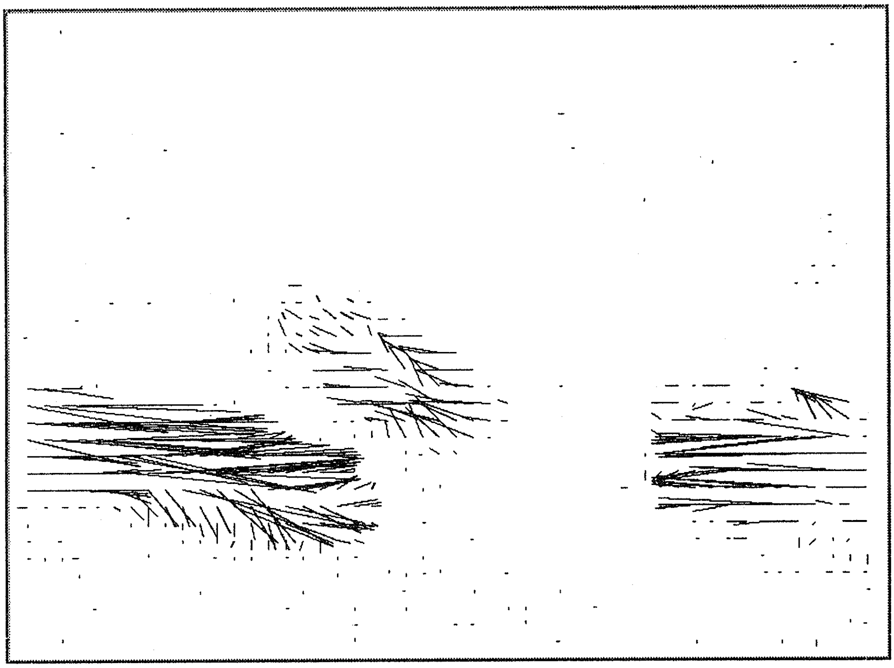

Figure 2: Detected Flow Field by the Line Voting Method

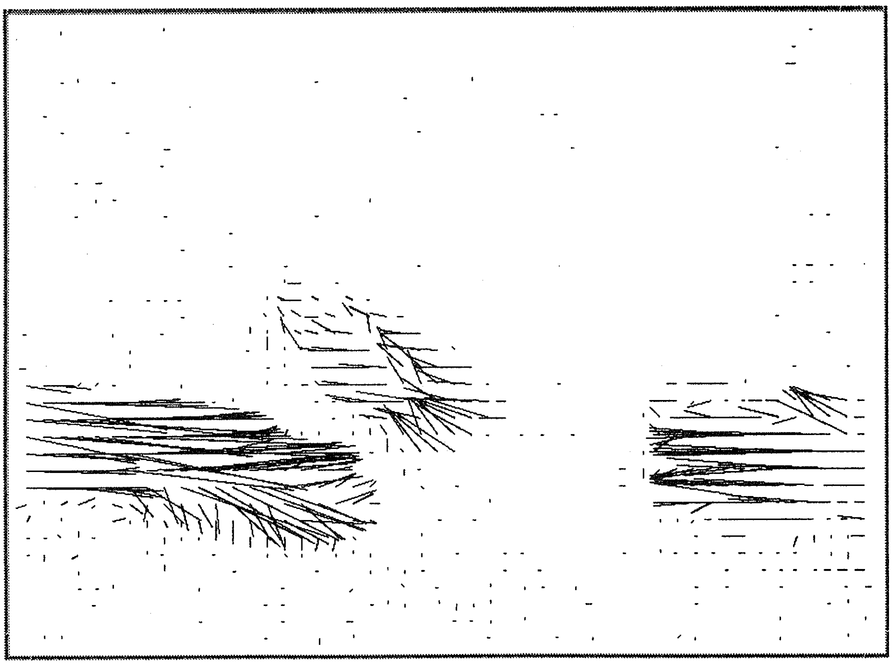

Figure 3: Detected Flow Field by the Point Voting Method