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The Computer Solution of Jigsaw Puzzles (Part II)

— Jigsaw Piece Boundary Shape Matching, Image Merging, and Recovery of Connection Relationships —

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Abstract We proposed a method to solve the jigsaw puzzle by the computer. This method employs both the piece boundary shape information and the piece boundary image information. Firstly, the jigsaw pieces are extracted from the input image. Then, the corner points of jigsaw pieces are detected. And next, the piece classification and recognition are performed based on the jigsaw models. And then, the connection relationships are calculated according to the piece boundary shape matching and image merging. Finally, the connection relationships among the pieces are recovered. This paper, as the second part, is limited to describe the jigsaw piece boundary shape matching, the image merging, and the recovery of connection relationships among jigsaw pieces.

Keywords: jigsaw piece extraction, corner point detection, edge and piece recognition, boundary shape matching, image merging, recovery of connection relationships.

ジグソーパズルのコンピュータ解法（第2部）

—ジグソーピースの形状マッチング、画像融合、接続関係の復元—

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我々はコンピュータによるジグソーパズルを解くための一方法を提案する。この方法では、ジグソーピースの形状情報と画像情報の両方を用いる。まず、入力画像からジグソーピースを切り出して、各ピースのコーナーポイントを検出する。次に、ジグソーピースモデルに基づいてジグソーピースの分類と認識を行う。そして、ジグソーピースの形状マッチングと画像融合に基づいてジグソーピース間の接続関係を計算する。最後にジグソーピース間の接続関係を復元する。本論文は、第2部としてはジグソーピース間の形状マッチング、画像融合、接続関係の復元について述べる。

キーワード：ジグソーピース切り出し、コーナーポイント検出、エッジとジグソーピースの認識、形状マッチング、画像融合、接続関係の復元

1. Introduction

We proposed a method to solve the jigsaw puzzle by the computer. Our algorithm to solve the jigsaw puzzle can be formulated in six steps:

1) Extract the jigsaw pieces from the color input image and express the jigsaw pieces by their boundary curves.

2) Detect the dominant points of each piece, and then search the corner points from them.

3) Separate the boundary curves into four edges by taking the corner points as the separation points (note that the corner points are quartet points), and then perform the piece classification and recognition.

4) Perform the boundary shape matching to get the candidates of the neighbor pieces.

5) Perform the image merging between the present piece and all candidates.

6) Based on the result of step 4 and 5, the connection relationships among the jigsaw pieces are recovered.

Step 1), 2), and 3) are described in the previous paper[3]. The organization of the rest of this paper is as follows. Section 2 relates the Jigsaw boundary curve matching. Section 3 describes the image merging. And section 4 shows the recovery of the connection relationships. The experiment results are given in section 5. The paper ends with some concluding remarks.

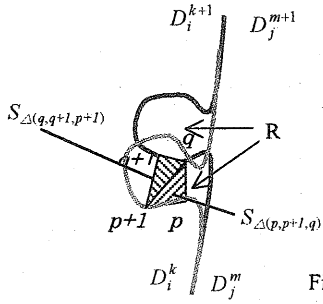


Fig.1 Edge matching.

2. Jigsaw Piece Boundary Shape Matching

Generally speaking, a human checks whether two pieces can be connected by rotating one object clockwise and the other one counterclockwise, and then bringing one piece next to the other to see the gap between them. If the gap is small enough, and the textures of images around that partial curve to be connected are also similar, two pieces are thought to be connectable at this orientation. Otherwise, the human will continue to rotate and match the two pieces to find an orientation suitable for connection. This process contains the *partial boundary shape matching* and *image merging*.

This section relates the partial boundary shape matching and the image merging will be explained in the next section.

A computer can simulate the partial boundary curve matching process by using clockwise jigsaw piece edge and counterclockwise jigsaw piece edge.

For P_i, $P_j \in S_P$ ($i \neq j$, i, j=0,1, ..., N-1), the boundary shape matching between these two pieces is shown below. Take the clockwise edge $E_i^{k,k+1}$ (modulo 4) of P_i and the counterclockwise edge $\hat{E}_j^{m,m+1}$ (modulo 4) of P_j, if they satisfy any connection relationships defined in jigsaw piece model table, the matching between them is performed. Otherwise, try to find other edge pair that can be mated. The matching between $E_i^{k,k+1}$ and $\hat{E}_j^{m,m+1}$ is described in the following. $\hat{E}_j^{m,m+1}$ is translated along the X- and Y-axis so that its initium overlaps with the initium (x_i^k, y_i^k) of $E_i^{k,k+1}$. The displacements of X- and Y-axis are given by

$$D_x(E_i^{k,k+1}, \hat{E}_j^{m,m+1}) = x_i^k - x_j^m \tag{1}$$

$$D_y(E_i^{k,k+1}, \hat{E}_j^{m,m+1}) = y_i^k - y_j^m, \tag{2}$$

where (x_i^k, y_i^k) are the coordinates of the initium of $E_i^{k,k+1}$, and (x_j^m, y_j^m) are the coordinates of the initium of $\hat{E}_j^{m,m+1}$. Then, the edge $\hat{E}_j^{m,m+1}$ is rotated counterclockwise so that the line decided by its initium and terminus overlaps with that by the initium and terminus of the clockwise edge $E_i^{k,k+1}$. The rotation angle is given by

$$\phi(E_i^{k,k+1}, \hat{E}_j^{m,m+1}) =$$

$$\tan^{-1}\frac{y_j^{m+1} - y_j^m}{x_j^{m+1} - x_j^m} - \tan^{-1}\frac{y_i^{k+1} - y_i^k}{x_i^{k+1} - x_i^k} \tag{3}$$

where (x_i^{k+1}, y_i^{k+1}) are the coordinates of the terminus of $E_i^{k,k+1}$, and (x_j^{m+1}, y_j^{m+1}) are the coordinates of the terminus of $\hat{E}_j^{m,m+1}$. The matching error is defined by

$$\omega(E_i^{k,k+1}, \hat{E}_j^{m,m+1}) = \iint_R dxdy \tag{4}$$

where R is the region circled by the clockwise edge

$E_i^{k,k+1}$ and the counterclockwise edge $\hat{E}_j^{m,m+1}$

For the digital jigsaw boundary curve, the equation (4) can be calculated according to the following equation.

$$\omega(E_i^{k,k+1}, \hat{E}_j^{m,m+1}) =$$

$$\sum_{p=0,q=0}^{L}(S_{\Delta(p,p+1,q)} + S_{\Delta(q,q+1,p+1)}) +$$

$$W \times | L_i^{k,k+1} - L_j^{m,m+1} | \quad (5)$$

where L is the small one of the two edge length, that is, $L = \min\{ L_i^{k,k+1}, L_j^{m,m+1} \}$, W is a weighting coefficient (at present it is set at 100), $S_{\Delta(p,p+1,q)}$, $S_{\Delta(q,q+1,p+1)}$ are the areas of the triangles formed by the boundary points p, $p+1$, and q; q, $q+1$, and $p+1$; correspondingly, as shown in Fig.1. The areas of the triangles are computed as follows:

$$S_{\Delta(p,p+1,q)} = (y_j^p - y_j^{p+1})x_i^q - (x_j^p - x_j^{p+1})y_i^q +$$

$$(x_j^p - x_j^{p+1})y_j^p - (y_j^p - y_j^{p+1})x_j^p \quad (6)$$

$$S_{\Delta(q,q+1,p+1)} = (y_i^q - y_i^{q+1})x_j^{p+1} - (x_i^q - x_i^{q+1})y_j^{p+1} +$$

$$(x_i^q - x_i^{q+1})y_i^q - (y_i^q - y_i^{q+1})x_i^q \quad (7)$$

$E_i^{k,k+1} \in S_{E_j}$ of P_i (k=0, 1, 2, 3, modulo 4) is matched with $\forall\, \hat{E}_j^{m,m+1} \in S_{\hat{E}_j}$ of $P_j \in S_P$ (m=0, 1, 2, 3, modulo 4), and the matching error is calculated according to equation (5). The edge corresponding to the smallest matching error is considered as the possibly matched edge (shorted as PME) between $E_i^{k,k+1}$ of P_i and those of P_j. This is performed for $\forall P_j \in S_P$ ($j \neq i$, j=0, 1, ..., N-1), then all PMEs between $E_i^{k,k+1}$ of P_i and those of $P_j \in S_P$ ($j \neq i$) are obtained. All PMEs of $E_i^{k,k+1}$ of P_i are sorted according to the matching error, from small to big. The top M of the sorted PMEs are thought of as the truly matched edge (abbreviated as TME) candidates.

The above processing is applied for $\forall P_i \in S_P$ (i=0, 1, ..., N-1), the top M TME candidates of the four edges of P_i are obtained. One of TME candidates of the each edge will be determined as the TME by employing the image merging which will be described in the next section. At present, M is set at 6.

It is necessary to notice that "L" type edge does not have TME. The edge type recognition is given in the literature [3].

3. Image Merging

The purpose of image merging is to determine the TME from TME candidates. This is based on the integration of image features.

3.1 Definition of the Integration Degree of Image Features

The integration degree of image features is defined from the separability of image features. Suppose a region R of an image can be classified into sub-region R_1 and R_2, the separability of image features is defined as follows [2]

$$\eta = \frac{\sigma_b^2}{\sigma_T^2} \quad (8)$$

$$\sigma_b^2 = n_1(\overline{P}_1 - \overline{P}_m)^2 + n_2(\overline{P}_2 - \overline{P}_m)^2 \quad (9)$$

$$\sigma_T^2 = \sum_{i=1}^{N}(P_i - \overline{P}_m)^2 \quad (10)$$

where n_1 and n_2 are the number of image pixels in R_1 and

R_2, respectively, N = $n_1 + n_2$, $\overline{P_1}$, $\overline{P_2}$, and $\overline{P_m}$ are the average of the luminance in R_1, R_2, and R correspondingly, P_i is the luminance at i-th pixel, and σ_T is the variance in R. η lies in the range $0< \eta \leq 1.0$. If R_1 and R_2 can be separated completely, η =1.0. If R_1 and R_2, cannot be separated, η is near to 0. P_i can be the saturation, hue, texture, and so on, instead of the luminance, of at i-th pixel.

Generally, for the L standardized image features, the separability is defined below [1]

$$\eta = \sum_{f=1}^{L} \sigma_b^2 \Big/ \sum_{f=1}^{L} \sigma_{Tf}^2 \tag{11}$$

$$\sigma_f^2 = n_1(\overline{P_{1f}} - \overline{P_{mf}})^2 + n_2(\overline{P_{2f}} - \overline{P_{mf}})^2 \tag{12}$$

$$\sigma_{Tf}^2 = \sum_{i=1}^{N} (P_{if} - \overline{P_{mf}})^2 \tag{13}$$

where σ_{Tf} is the variance of image feature f (f=1, ..., L) in R, $\overline{P_{1f}}$, $\overline{P_{2f}}$, and $\overline{P_{mf}}$ are the average of the feature f in R_1, R_2, and R correspondingly, and η also lies in the range $0< \eta \leq 1.0$.

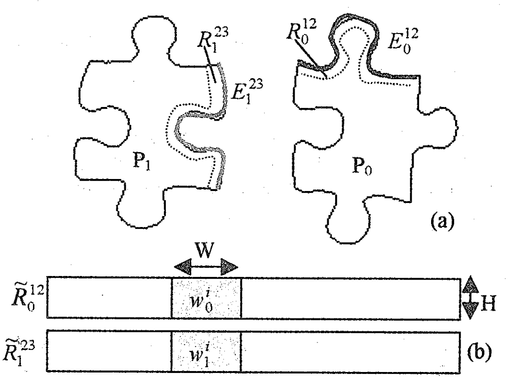


Fig.2 Integration degree of image.

From equation (11), the integration degree of image features is defined as

$$\rho = 1 - \eta. \tag{14}$$

If R_1 and R_2 can be merged completely, ρ takes the value near to 1.0. If R_1 and R_2, cannot be merged, ρ is equal to 0.

ρ is used to determine the TME from TME candidates. Details are given in the next section.

3.2 Image Merging

Let employ the Fig.2 to explain the image merging between jigsaw pieces. According to the boundary shape matching as described in section 2, one TME candidate of E_0^{12} of P_0 is E_1^{23} of P_1 as shown in Fig.2 (a). However, we cannot say definitely say that E_0^{12} of P_0 can be connected to E_1^{23} of P_1 because the shape information is not enough. We must also check the images in regions next to the edge E_0^{12} and E_1^{23}, that is, the regions R_0^{12} and R_1^{23}, can be merged or not according the integration of image features defined in equation (14). Details are given below.

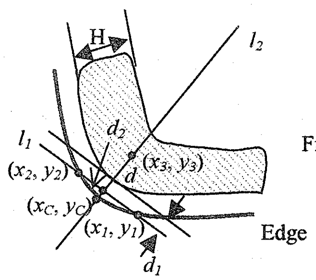


Fig.3 Image region used to calculate the integration degree.

For the convenience to calculate the integration of image features between the image region nearby the edge E_0^{12} and that nearby E_1^{23}, R_0^{12} and R_1^{23} are spread into rectangle regions $\tilde{R}_0^{12}$ and $\tilde{R}_1^{23}$ as shown in Fig.2 (b), respectively. $\tilde{R}_0^{12}$ and $\tilde{R}_1^{23}$ are separated into small regions with same width and height. The integration degree between $\tilde{R}_0^{12}$ and $\tilde{R}_1^{23}$ is defined by

$$\overline{\rho} = \frac{1}{M} \sum_{i=0}^{M-1} \rho_i \tag{15}$$

$$M = \min\left\{ \left\lfloor \frac{L_0^{12}}{W} \right\rfloor, \left\lfloor \frac{L_1^{23}}{W} \right\rfloor \right\} \tag{16}$$

where ρ_i is the integration degree between i-th small region w_0^i of $\tilde{R}_0^{12}$ and w_1^i of $\tilde{R}_1^{23}$, which is given in equation (14), and M is the minimal number of small regions in $\tilde{R}_0^{12}$ and $\tilde{R}_1^{23}$, W is the width of the small region. And the image features used in equation (14) are R-, G-, and B-components.

It is necessary to notice that when spreading the image data in R_0^{12} to the rectangle region $\tilde{R}_0^{12}$, the image data is copied along the normal line, and the start point is not from the point on edge but the point that leaves d_1 dots apart from the edge. This is shown in Fig.3. The points P_1 (x_1, y_1) and P_2 (x_2, y_2) are the points which leave d_2 dots apart from the point in question to the left and right along the edge, and P_c (x_c, y_c) is the mediate point of P_1 and P_2. The coordinates of the point P_3 (x_3, y_3) on the normal line l_2 are given by

$$x_3 = x_c + \frac{d}{\|P_2 - P_1\|}(y_2 - y_1) \tag{17}$$

$$y_3 = y_c - \frac{d}{\|P_2 - P_1\|}(x_2 - x_1) \tag{18}$$

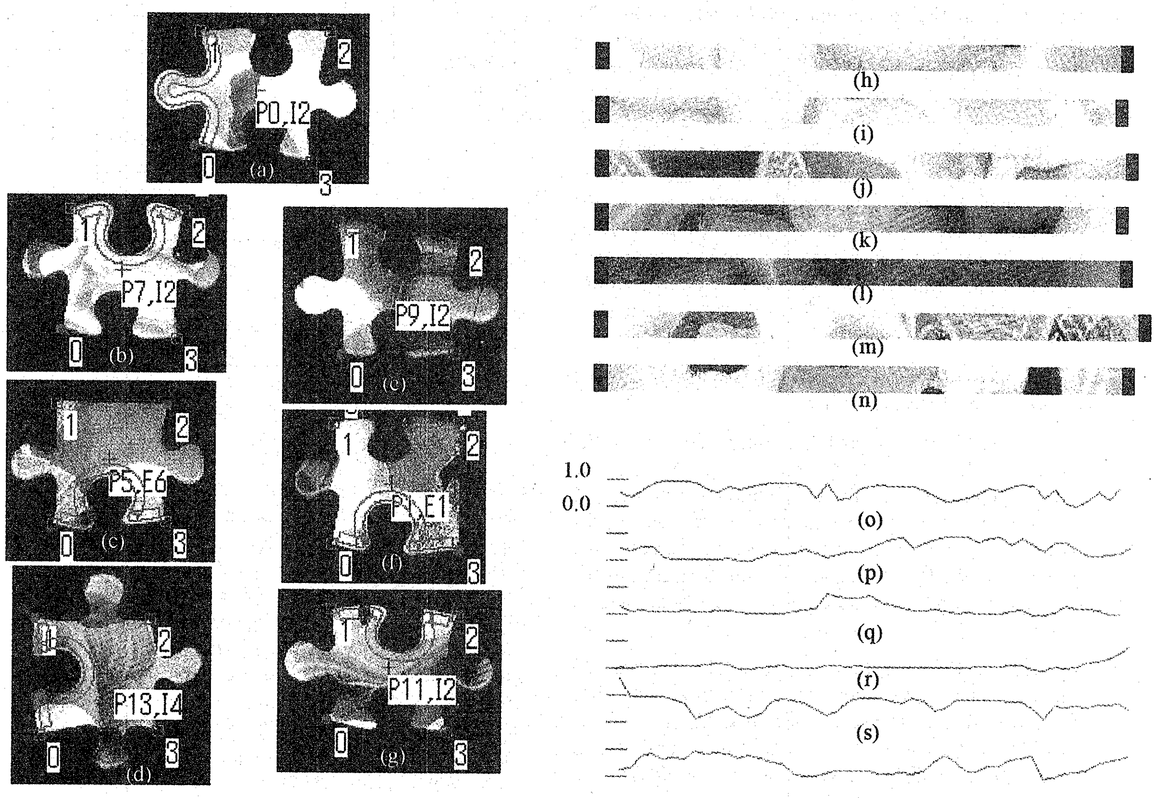


Fig.4 Edges in (b) ~ (g) marked by double red curves are six TME candidates of the edge also marked by double red curves in (a). Image regions nearby the edges in (a) ~ (g) are converted to the rectangle regions in (h) ~ (n). Image integration degrees between (h) and (i) ~ (n) are shown in (o) ~ (t).

where d is the distance from P_c along the normal line l_2. At present, W, d_1, and d_2 are set at 10 dots, H 20 dots.

The edges in Fig.4 (b) to (g) marked by two red lines are the TME candidates of the edge in Fig.4 (a) also marked by two red lines. These are obtained by the piece boundary shape matching. The images nearby these edges in (a) to (g) are spread into the rectangle regions as shown in (h) to (n), respectively. The integration degrees between the rectangle region in (h) and those in (i) to (n) are shown in (o) to (t), correspondingly.

4. Recovery of Connection Relationships

TME candidates obtained in last section are put into the jigsaw piece connectedness table. The connection relationships are recovered according to this table. The format of the jigsaw piece interconnected table is given in Table 1. The first column, C, shows the candidate number which is in repetition with length M, that is, the first M shows candidate number of the first edge of P_0, the second of the second edge of P_0, and so on. The second and the third columns, X_1 and Y_1, are the piece number and edge number of the piece in question, respectively, and the fourth and fifth columns, X_2 and Y_2, are the piece number and edge number of the neighbor of the piece in question. The sixth column, ω_k, is the jigsaw piece boundary shape matching error, which is sorted from small to big. The seventh column, ρ_k, is the integration degree of the images between the regions nearby the edge X_1 of the piece Y_1 and X_2 of Y_2. The eighth column, τ_k, is the ratio between the matching error of k-th candidate and that of the first, that is, $\tau_k = \omega_k / \omega_0$, where $k = 0, 1, \ldots, M\text{-}1$.

The recovery of the connection relationships is based on the following *decision rules*.

Rule 1: If the k-th candidate ranks first both in the boundary shape matching error field and in the integration degree field, the k-th candidate is considered as TME. It is necessary to notice that first rank in the matching error field means its matching error is the smallest among the candidates, and the first rank in the integration degree field means that its integration degree is largest.

Rule 2: If the m-th candidate ranks first in the boundary shape matching error field, n-th candidate ranks first in the integration degree field, and if τ_n is bigger than the threshold τ_{thres}, the m-th candidate is thought of as TME. At present the τ_{thres} is determined experimentally and set at 1.5.

Rule 3: If the s-th candidate ranks first in the boundary shape matching error field, t-th candidate ranks first in the integration degree field, and if τ_t is smaller than the threshold τ_{thres}, then the t-th candidate is thought of as TME.

Rule 4: In other cases, TME candidates are left unsolved.

Moreover, because the jigsaw pieces are interconnected, the contents of jigsaw piece connectedness table can be corrected according to the following *correction rule*.

Correction Rule: The jigsaw piece connectedness table is searched from top to bottom. For P_m, if its k-th edge is determined to connect the s-th edge of P_n according to the decision rule 1, 2, or 3, then it is able to definitely say that the s-th edge of P_n is connected to the k-th edge of P_m. Therefore, for the first TME candidate of the s-th edge of P_n, its neighbor piece number and the edge number of the neighbor piece, that is, X_2 and P_2, can be can be determined as P_m and k, respectively. Its integration degree, that is, ρ_0 can be set to 1.0, and the matching error, i.e., ω_0, can be set to a positive non-zero value as small as possible, e.g., 1.0. Next, the jigsaw piece connectedness table is searched from bottom to top, and above processing is repeated again. In this way, TME can be rescued from the unsolved TME candidates.

Table 1 Format of jigsaw piece connectedness table.

	C	X_1	Y_1	X_2	Y_2	ω_k	ρ_k	τ_k
TME candidates of 1st edge of P_0	0							
	1							
	· · ·							
	M-1							
TME candidates of 2nd edge of P_0	0							
	1							
	· · ·							
	M-1							
· · ·	0							
	1							
	· · ·							
	M-1							
	· · ·							

After TMEs are determined, the connection relationships among the jigsaw pieces can be recovered. Details are given in the following.

(1) Search the jigsaw piece connectedness table and find the corner piece. Put this piece into the *buffer*, and set the index of the buffer i to 0, number of jigsaw pieces in the buffer n to 1. And then make the output image I_{out}, and initialize I_{out} with 0. Copy the image of the corner piece to I_{out}.

(2) For the k-th edge of i-th piece P_i in the *buffer*, find its neighbor piece from the jigsaw piece connectedness table. If its neighbor piece P_j is found,

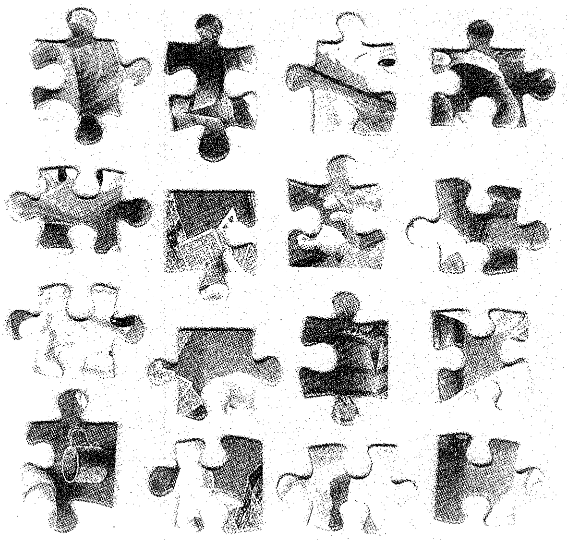
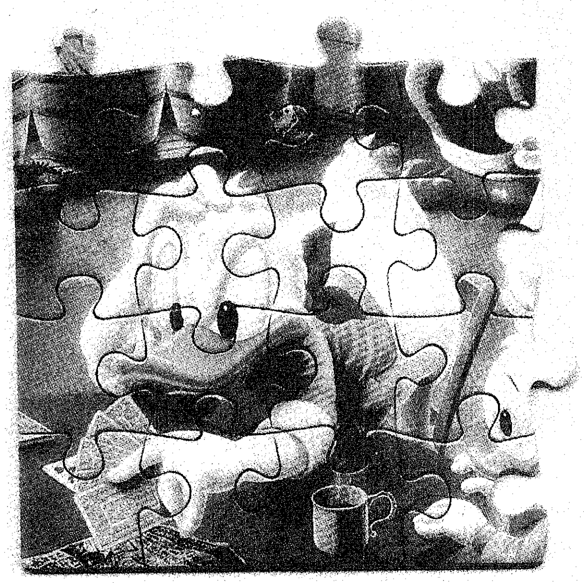


Fig.5 One of the input image (white background).



(a)

I_4	I_3	I_2	I_1
I_8	I_7	I_6	I_5
I_{12}	I_{11}	I_{10}	I_9
I_{16}	I_{15}	I_{14}	I_{13}

(b)

Fig.6 (a) A portion of a MPMCA jigsaw puzzle. (b) Assigned number to the jigsaw piece in (a).

then after being translated and rotated, the image of P_j is copied to I_{out}, and n is increased by 1. The displacements and rotation angle can be calculated according to equation (1) to (3). The edge index, k, changes from 0 to 3.

(3) The operation in (2) is repeated for i from 0 to n.

In this way, the connection relationships among the jigsaw pieces can be recovered.

5. Experiment Results

We employ the real-world images to test the algorithm related above. The algorithms are implemented on Windows platform, and the programming language C++.

Fig.5 shows one of the input image which is the input image with the white background. This image includes 16 MPMCA jigsaw pieces that consist of a portion of a MPMCA jigsaw puzzle as shown in Fig.6 (a). To mark the piece correspondence in Fig.5 and Fig.6 (a), each jigsaw piece in Fig.6 (a) is assigned a number from right to left for rows, and from top to bottom for column, as shown in Fig.6 (b). The correspondence between the Jigsaw piece number in the order of extraction from the input image and that assigned in Fig.6 (b) is given in Table 2.

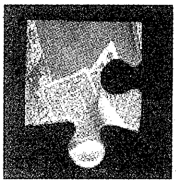
Table 2 Correspondence between the Jigsaw piece number in the order of extraction from the input image and that assigned in Fig.6 (b).

P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
I_6	I_{15}	I_{14}	I_8	I_4	I_{12}	I_5	I_7
P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
I_{16}	I_2	I_{13}	I_{11}	I_3	I_{10}	I_9	I_1

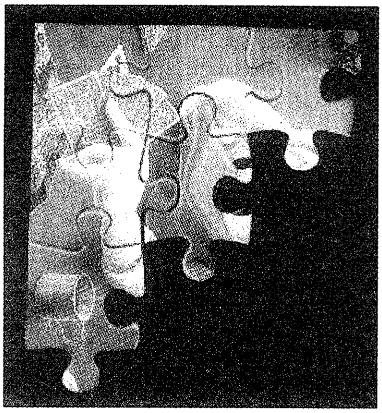
Table 3 Constraint conditions for the puzzle in Fig.5.

Piece No.	Edge No	Constraint Condition
$P_4 = I_4$	3	No neighbor
$P_{12} = I_3$	0	No neighbor
$P_9 = I_2$	2	No neighbor
$P_{15} = I_1$	0	No neighbor
	3	No neighbor
$P_6 = I_5$	1	No neighbor
$P_{14} = I_9$	1	No neighbor
$P_{10} = I_{13}$	2	No neighbor

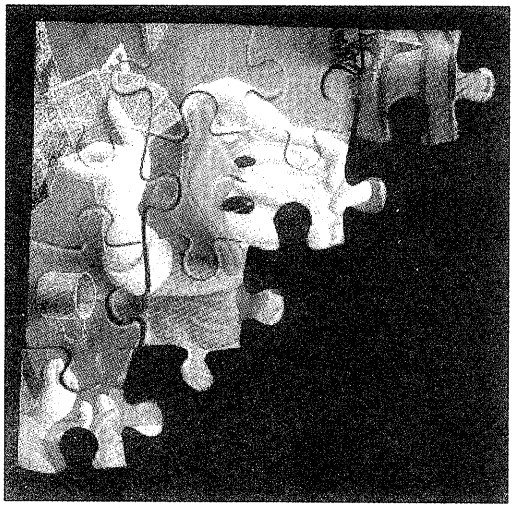
Because the jigsaw pieces in Fig.5 consists of a portion of MPMCA jigsaw puzzle as shown in Fig.6, it is necessary to add *constraint conditions* that the jigsaw pieces at the top and right edges in Fig.6 do not have the neighbor pieces. These constraint conditions are given in Table 3.



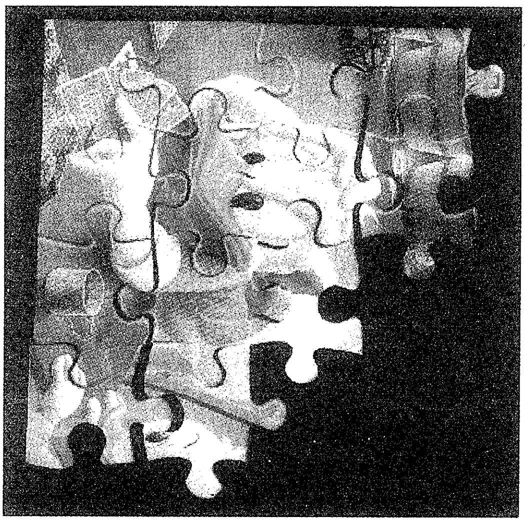
(a) Repetition times $i = 0$.



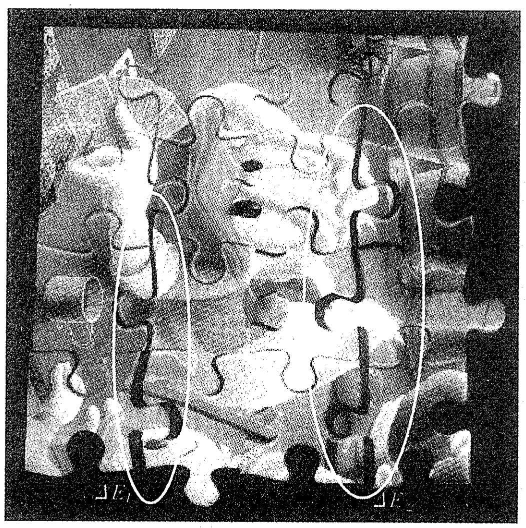
(b) Repetition times $i = 3$.



(c) Repetition times $i = 6$.



(d) Repetition times $i = 9$.



(e) Repetition times $i = 15$.

Fi.g.7 Recovery process of the connection relationships in the input image in Fig.5.

By applying the jigsaw piece boundary shape matching and image merging as described in the section 2 and 3, the connectedness table is obtained. A portion of the connectedness table for the input image in Fig. 5 is given in Table 4. The ninth column of Table 4 shows the applied decision rule. There are 64 interconnected relationships in this table, among them there are eight constraint conditions as shown in Table 3, and there are eight "L" edges that do not have neighbor pieces. For the left 48 interconnected relationships, by applying the decision rules, 46 of them can be determined, and two are left unsolved. Interconnected relationships determined by rule 1, 2, and 3 are 27, 10, and 9, respectively. By applying the correction rule, two unsolved interconnected relations are rescued as shown by the arrow-line in Table 4. Therefore, for the input image in Fig.5, the connection relationships can be recovered completely. The recovery process at step 0 , 3, 6, 9, and 15 are in Fig.7.

Table 4 Connectedness table for input image in Fig.5

C	X_1	Y_1	X_2	Y_2	ω_k	ρ_k	τ_k	R
0	0	0	7	1	1989.3	0.66890	1.00	R2
1	0	0	5	3	2026.4	0.47689	1.02	
⋮								
0	0	3	9	2	1953.1	0.42560	1.00	R4
1	0	3	13	1	2180.7	0.44246	1.12	
2	0	3	1	0	4305.5	0.40391	2.20	
3	0	3	14	0	4310.3	0.46123	2.21	
4	0	3	7	0	4584.3	0.44262	2.35	
5	0	3	5	0	4999.5	0.39152	2.56	
⋮						Correction rule		
0	11	2	1	3	2139.2	0.33707	1.00	R3
1	11	2	13	0	2367.4	0.53802	1.11	
2	11	2	7	1	2558.9	0.21289	1.20	
3	11	2	0	1	2734.5	0.23092	1.28	
4	11	2	9	1	3137.8	0.47419	1.47	
5	11	2	5	3	3433.9	0.38821	1.61	
						Correction rule		
0	13	0	12	1	1257.5	0.43268	1.00	
1	13	0	5	0	2052.7	0.54184	1.63	
2	13	0	4	3	2211.6	0.49541	1.76	
3	13	0	15	2	2304.1	0.54444	1.83	
4	13	0	0	0	2468.4	0.25630	1.96	
5	13	0	11	2	3065.8	0.57924	2.44	R4
0	13	1	0	3	1617.9	0.43169	1.00	R2
1	13	1	11	3	4238.5	0.45921	2.62	
2	13	1	10	2	4726.1	0.47250	2.92	
3	13	1	8	2	5248.4	0.56958	3.24	
4	13	1	6	0	5736.8	0.42839	3.55	
5	13	1	3	3	5912.1	0.37956	3.65	
3	15	3	10	1	6036.5	0.30126	1.10	
4	15	3	9	0	6140.1	0.35374	1.12	
5	15	3	8	3	6567.3	0.48443	1.20	

Another experiment uses the input image as shown in Fig.8, which is one of the five input images. This

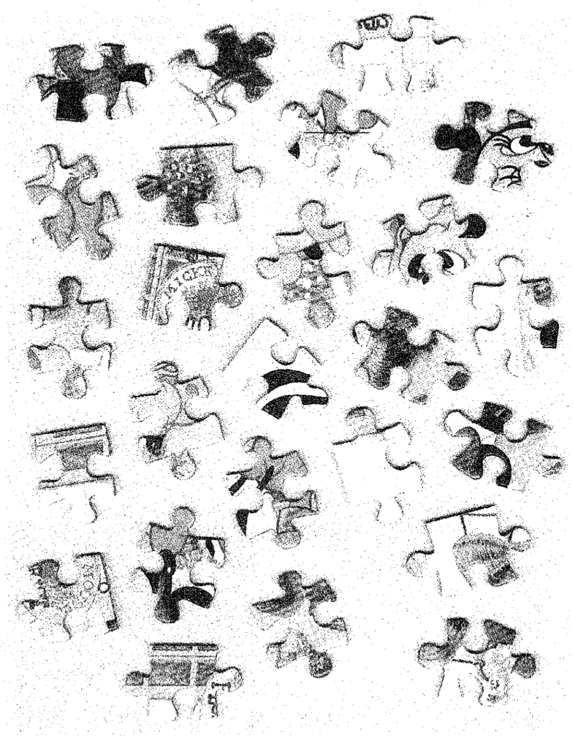


Fig.8 One of the input image (white background).

image includes 25 MPMCA jigsaw pieces. The recovery process at step 0 , 5, 10, 15, and 24 are in Fig.9.

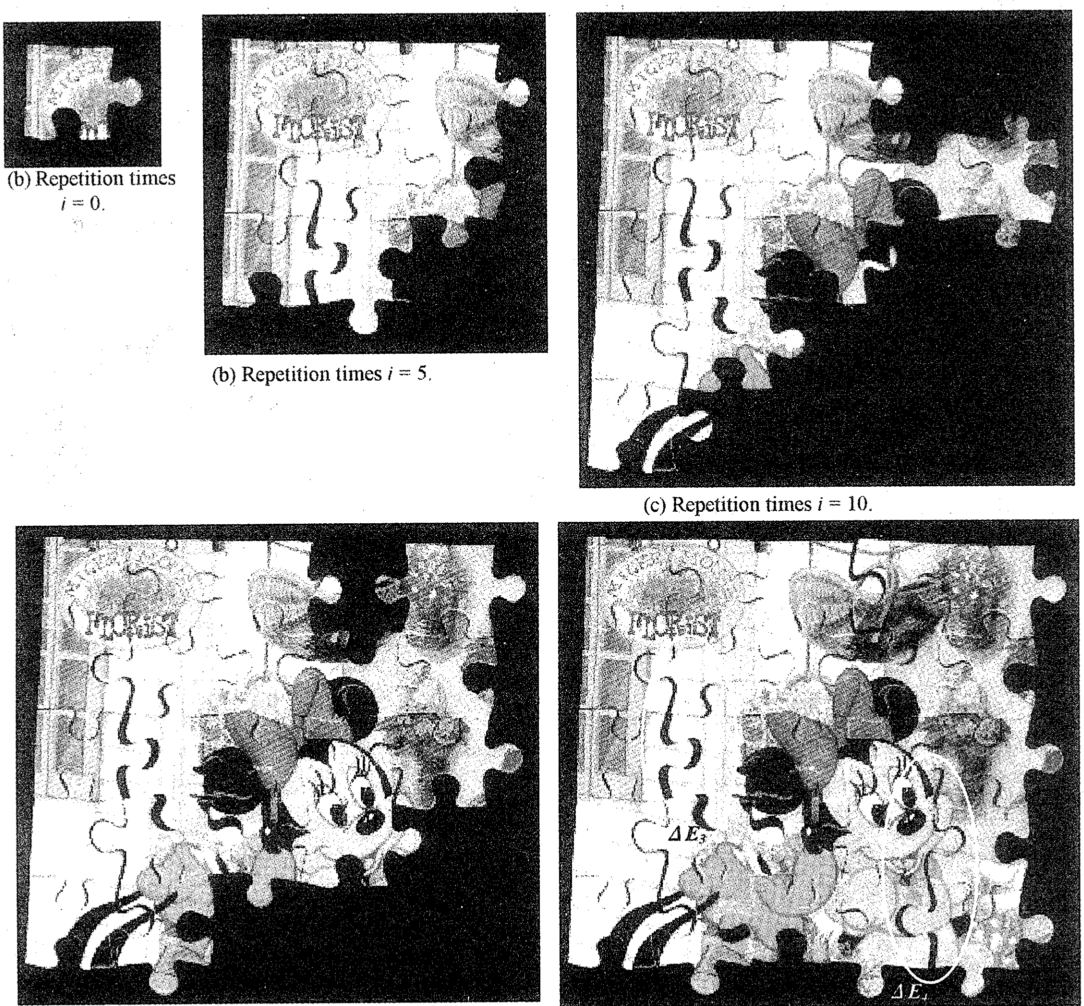
6. Conclusions and Discussions

This paper discussed the computer solution of jigsaw puzzles. The experiments are performed with the real world images. The experiment results show that our method is of validity. This method to solve the jigsaw puzzle can be applied in the intelligent robot assembly system, and in map matching.

In this method, we categorized the MPMCA jigsaw pieces into 18 patterns. This greatly reduced the computation time and simplified the question. This method includes the jigsaw piece extraction, the corner point detection, and piece type recognition in the first phase, and the boundary shape matching, the image merging, and the connected ness recovery in the second phase.

In the first phase, we used multiple input images with different background colors. From the experiment results, it is clear that this method is successful. The computation time in first phase is mainly occupied by the jigsaw piece extraction. It can be given by

$$T_{extract\text{-}piece} = t_{con} \times O(P \times Q) \quad (19)$$



(b) Repetition times $i = 0$.

(b) Repetition times $i = 5$.

(c) Repetition times $i = 10$.

(d) Repetition times $i = 15$.

(e) Repetition times $i = 24$.

Fi.g.9 Recovery process of the connection relationships in the input image in Fig.8.

where P and Q are width and height of the input images, respectively, t_{con} is the conjugation time for one corresponding pixel in five input images.

The corner point detection is of knowledge reasoning based on its five properties. The jigsaw piece recognition consists of the edge recognition and the model pattern matching. From the experiment result by using the real-world images, we can say these methods are efficient.

In the second phase, the computation time of boundary shape matching is given by

$$T_{shape\text{-}match} = t_{match} \times O(16 \times N \times (N\text{-}1)) \qquad (20)$$

where N is the number of jigsaw pieces in the input image, and t_{match} is the matching time between two edges of two different pieces. t_{match} contains the time for copying the coordinates of the points on edges, for translating and rotating the coordinates of an edge in matching, and for calculating the matching error. It is directly proportional to the edge length that depends on the size of the input image. All edges are matched each other to get the TME candidates. The matching results are sorted by the matching error from small to big, and the top M (=6) of them are considered as the TME candidates. If the N becomes large, the value of M should be also set larger.

The image merging is to calculate the integration degree of image in the region nearby the edge in question and those of TME candidate. The computation time is given by

$$T_{image\text{-}merge} = t_{merge} \times O(M) \quad (21)$$

where t_{merge} is the time for calculating the integration degree of the images in the regions nearby the edge in question and those nearby its TME candidates. It contains the time for spreading the image in the region along the edge into rectangle region, for calculating the integration degree between the corresponding small regions of two rectangle regions. The width and height of the small region are 10 dots and 20 dots. The experiment results show that they are appropriate. It may be changed into other values for the different jigsaw puzzles.

We made three decision rules and one correction rule to determine the TME from its candidates. For the large size jigsaw puzzles, these rules need to be improved. The computation time for recovering the connection relationships among jigsaw puzzles is given by

$$T_{recovery} = t_{recovery} \times O(N) \quad (22)$$

where $t_{recovery}$ is the time for copying the image data of a jigsaw piece from the input image, rotating these image data and putting them into the output image.

Although the TMEs cannot be determined at a hundred percent, the jigsaw puzzle may be solved completely. For example, in Fig.9 (d), the neighbor piece of the third edge of P_5 cannot be determined from its TME candidates, however, it can be determined from the TME candidates of the first edge of P_{19}, as shown in Fig.9 (e).

The gaps marked by the white circles in Fig.7 (represented by ΔE_1 and ΔE_2) and in Fig.9 (shown by ΔE_3 and ΔE_4) are the errors in the recovery of the connection relationships among jigsaw pieces. There are two factors to cause these errors. Firstly, because the image is scanned from top to bottom, the brightness of the edges on the top-side and bottom-side are different, and there exist shades on the bottom-sides (see Fig.5 and Fig.8). This gives rise to the deformation of boundary curves. Secondly, the minimal the rotation angle of the edges and image data is 1 degree. This may cause the error in piece orientation. The gaps become larger and larger in accompanying the continuation of the recovery process. This is because the errors are integrated.

At present, we tested this method by using the image containing tens of pieces. It is also necessary to test it by using image containing more pieces. As an application of the method to solve the jigsaw puzzles, it can be directly applied to intelligent robot assembly system. These are left to do in the future.

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