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デフォールト論理と
サーカムスクリプションとの関係について

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不完全な知識を扱うときに使われる非単調推論を形式化する二種類の重要な論理的枠組み，デフォールト論理とサーカムスクリプションとの関係を調べる．そして，デフォールト論理での“信じられる”という概念を極小含意(minimal entailment)と関連づける．

まず，あるデフォールト体系(D, W)に対して，本論文で定義される$\Gamma_\infty[M_0]$を用いて制限されるWのモデル，かつ，$\Gamma_\infty[M_0]$の極小モデルM_0を(D, W)のモデルとして定義する．このモデルは(D, W)のある無矛盾な拡張(consistent extension)のモデルとなり，逆に，任意の無矛盾な拡張はこの様に制限されるモデルを持つことを示す．そして，ある*wff* ω が(D, W)で信じられるならば，ω が真となる(D, W)のモデルが存在する．次に，デフォールトの変形を通して，デフォールト論理での“信じられる”という概念と一階述語論理でのサーカムスクリプティブ推論との関わりを検討する．最後に，サーカムスクリプションの立場から，デフォールト論理の“信じられる”という概念を極小含意を用いて解釈可能であることを述べる。

On Relationship between Default Logic and Circumscription

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In this paper, we shall search out some relationship between default logic and circumscription, two kinds of important logic frameworks to formalize non-monotonic reasoning encountered in the incomplete knowledge processing, and then connect the concept of believability in default logic with minimal entailment in our way.

To begin with, we shall define a model M_0 of W restricted by $\Gamma_\infty[M_0]$ *wrt* a default theory (D, W) as the model of (D, W), in which M_0 is the minimal model of a consistent extension for (D, W); and conversely, any consistent extension for (D, W) will be the set of *wffs* satisfied by some models of (D, W). Then, it is shown that a *wff* ω believable in a default theory (D, W) is true in a model M_0 of (D, W). Next, we state that through certain transformation of defaults, it is possible to associate 'believability' in default logic with 'circumscriptive inference' in first-order logic. At last, based on all of those stated previously, our paper is ended with a conclusion that believability in default logic could be explained by minimal entailment in a different viewpoint of circumscription.

1. Introduction

In order to deal with so-called commonsense knowledge, or incomplete knowledge, default logic [9] and circumscription [3] have been proposed to formalize non-monotonic reasoning specifying how to make commonsense knowledge into use. Default logic is based on the modal symbol M (roughly meaning '*consistent*') which is beyond the framework of first-order logic and the semantics of M has not been considered. Circumscription is reasonable to be considered as some restriction posed on theories of first-order logic, in which no symbols like M in default logic are introduced, and it has a semantic counterpart, a minimal model. In this paper, we shall try to discover some relationship between two issues through transforming defaults into *wffs* in first-order language. Then the modal symbol M in default logic could be explained model-theoretically based on circumscription which is entirely corresponding to the minimal entailment.

1.1. Incomplete Knowledge and Non-Monotonic Reasoning

As we know, logical theories could been seen as collections of some represented and compact knowledge concerning certain perceived real worlds. They are assumed to be complete in classical logic, that is, no more knowledge could be relative to the real worlds except for those described by the theories. Then expanding those compact representations of knowledge causes a kind of monotonic reasoning. It suggests that those locally derivable still remain to be derivable globally when that described by theories are integrated themselves (*i.e.*, theories are consistent in logic meaning). However, it seems impossible to completely perceive real worlds and then it may be more natural to treat those theories *wrt* incompletely perceived worlds as open than to treat them as closed, such that they can be strengthened by discovering more about the real worlds. However, in this case, what received previously may be ruined later by new discoveries. Then the reasoning relative to this kind of open theories is no longer monotonic. Those locally derivable could be falsified globally. For instance, about the bird problem, we know 'A creature can fly if it is a bird, unless there is fact contrary to this.', 'Tweety is a bird' and 'Tweety is a Penguin.'. Then 'Tweety can fly.' could be conjectured locally from 'A creature can fly if it is a bird, unless there is fact contrary to this.' and 'Tweety is a bird.', while it cannot be concluded globally with all of those sentences. In our paper, two ways to specify the non-monotonic reasoning of this kind, default logic and circumscription, will be observed in same viewpoint of minimal entailment. Now we shall first recall some basic concepts concerning with default logic and circumscription.

1.2. Default Logic

Default logic is one proposal to represent incomplete knowledge with exceptions [9] and to formalize the non-monotonic reasoning on the incomplete knowledge of this kind. It is based on an extended first-order language by introducing a modal symbol M which could be roughly explained as 'consistency'. The theoretical meaning of M has not been considered in classically logical sense. Here a few terminologies and definitions relating to default logic will be recalled briefly. A *default* δ is written as $(\alpha(x): M\beta(x)/\gamma(x))$, where $\alpha(x)$, $\beta(x)$, and $\gamma(x)$ are *wffs* and x is the tuple of all variables occurring in δ. δ is said to be *closed* when every variable is bound by quantifiers. $\alpha(x)$ is called the *prerequisite*, denoted by PREREQUISITES(δ), $\beta(x)$ the *justification*, JUSTIFICATION(δ) and $\gamma(x)$ the *consequent* of δ, CONSEQUENTS(δ). For a set D of defaults, PREREQUISITES(D), JUSTIFICATION (D) and CONSEQUENTS(D) stand for sets of prerequisites, justifications and consequents respectively of all defaults in D. A default δ is roughly explained as: its consequent $\gamma(x)$ is believable if its prerequisite $\alpha(x)$ is believable unless its justification $\beta(x)$ is explicitly negated. A *default theory* is defined as a pair of sets of defaults and *wffs*, usually indicated by DT=(D, W), where D is a set of defaults and W a set of *wffs*. (D, W) is said to be *closed* when every default in D is closed and every *wff* in W is closed. (D, W) is said to be *clausal form* when every *wff* ϕ in W∪PREREQUISITES(D)∪JUSTIFICATION(D)∪CONSEQUENTS(D) is clausal form, *i.e.*, ϕ is universal quantified. The follows will give the definition of extensions for default theories.

<u>*Definition1*</u>

Let DT=(D, W) be a closed default theory, and E a set of closed *wffs*. Define

(1) $E^{(0)} = W$;

(2) $E^{(i+1)} = Th(E^{(i)}) \cup \{\gamma(x) \mid (\alpha(x): M\beta(x)/\gamma(x)) \in D, \alpha(x) \in E^{(i)} \text{ and } \neg\beta(x) \notin E.\}$

E is an *extension* for DT *iff*

$$E = \bigcup_{i=0}^{\infty} E^{(i)}.$$

A *wff* ϕ is said to be *believable* in a default theory (D, W) if there is some extension for (D, W) containing ϕ. However there are not always extensions for a default theory of general form. And it is also obviously extensions are not necessary to be consistent. For a default theory, restricting the syntax of all of its defaults to be normal form, *i.e.*, the consequent of default is same with the justification of default, then it has always extensions and each of them is consistent if all of *wffs* in the default theory is consistent.

Definition2

Let DT = (D, W) be a default theory and E an extension for DT. GD(E) is introduced to denote the set of generation defaults *wrt* E, defined by:

$$GD(E) = \{ \delta \mid \delta = (\alpha(x): M\beta(x)/\gamma(x)) \in D, \alpha(x) \in E \text{ and } \neg\beta(x) \notin E \}.$$

1.3. Circumscription

Circumscription is another proposal to formalize non-monotonic reasoning without introducing any modal symbol into classical logic [3]. Commonsense knowledge or incomplete knowledge could be represented by introducing the so-called abnormal predicate and the commonsense reasoning (which is, of course, a kind of non-monotonic reasoning) can be achieved through circumscribing the abnormal predicate [4]. As an advantage of circumscription, it has a semantic counterpart, a minimal model. Now let us describe the formal definition of circumscription and declare some terminologies used in this paper.

Definition3

Let A(P) be a sentence (a formula without free occurrences of variables) with occurrences of an n-ary predicate symbol P in first-order language. The circumscription of P in A(P) is defined by the following second-order formula:

$$A(P) \wedge \forall p. [A(p) \wedge \forall x. (p(x) \supset P(x)) \supset \forall x. (P(x) \supset p(x))]$$

denoted by Circum(A(P); P), where p is an n-ary predicate variable. A(p) is the result of substituting p for each occurrence of P in A(P). x is a tuple of variables.

Let W be a set of *wffs*, S a set of predicate symbols and ϕ a closed *wff*. $W \vdash_S \phi$ stands for that ϕ is provable from the result of W circumscribing every predicate symbol occurring in S, *i.e.*, $W \vdash_S \phi$ *iff* Circum(W; S) $\vdash \phi$.

1.4. An Example And Outline

Let us intuitively observe default reasoning and commonsense reasoning relative to circumscription. In default logic, the modal symbol M is the key to understand default reasoning, how to behave and what to be derived. Informally, for a *wff* $\beta(x)$, $M\beta(x)$ represents that about $\beta(x)$ we at least know the negation of $\beta(x)$ does not hold even if $\beta(x)$ itself is not provable at present time. According to this, it seems appropriately to explain a default $(\alpha(x): M\beta(x)/\gamma(x))$ as the declaration of "when $\alpha(x)$ is derived, $\gamma(x)$ is also derived as long as the negation of $\beta(x)$ does not appear.". Then the default reasoning with such defaults seemingly encourage us to derive something as much as possible unless there are contrary facts. Such as in the bird problem, for a creature, if it is known to be a bird, the default (Bird(x): MFly(x)/Fly(x)) encourages us to believe it can fly no matter actually it can or cannot unless we are told that it is unable to fly. However, circumscription tends to suggest to minimize what could be derived and denies something unknown. For example, all we know is "A and B are two blocks.". In view of circumscription, we would like to conjecture that "Something neither A nor B is not block.". Then circumscription and default reasoning seem to stem from two entirely contrary observations of incomplete knowledge. On the one hand, *J.McCarthy* has introduced the abnormal predicate symbol ab to describe what represented by defaults, and achieved commonsense reasoning (perhaps similar to default reasoning) through circumscribing ab [4]. On the other hand, hinted by that minimal models of a sentential set of *wffs* coincide with models of extensions induced from those *wffs* by the closed world assumption, we shall try to find some deeper relationship between default logic and circumscription. However, by *Tomasz Imieliski*'s research [10], we also know by modular translation of defaults, the expressive power of circumscription is same with single seminormal default theories (*i.e.* only one extension) without prerequisites. Then we introduce a predicate symbol $^n\beta$ for every default $(\alpha(x): M\beta(x)/\gamma(x))$. $(\alpha(x): M\beta(x)/\gamma(x))$ is then transformed into $\alpha(x) \wedge \neg{}^n\beta(x) \supset \gamma(x)$ and $\neg\beta(x) \supset {}^n\beta(x)$. Based upon this transformation of defaults, we shall try to discover the relationship between default reasoning and circumscription in the following sections. In order to see the main idea intuitively, an example is illustrated as below.

Example

Let $DT_{Bird} = (W_{Bird}, D_{Bird})$ be the default theory describing the bird problem mentioned previously. W_{Bird} is the set of *wffs*:

Penguin(x)$\supset$Bird(x)
Penguin(x)$\supset\neg$Fly(x)
Penguin(Tweety),

and D_{Bird} is the set of a single default:

$$\frac{Bird(x): MFly(x)}{Fly(x)}.$$

The abnormal theory of DT_{Bird}, *i.e.*, the transformed form of DT, (the formal definition will be given in the coming section) is:

Bird(x)$\wedge\neg{}^n$Fly(x)$\supset$Fly(x)
$\neg$Fly(x)$\supset{}^n$Fly(x)
Penguin(x)$\supset$Bird(x)

Penguin(x)⊃¬Fly(x)
Penguin(Tweety).

Here ^{n}Fly is a new predicate symbol, and the abnormal theory of DT_{Bird} is separable *wrt* ^{n}Fly[6]. Then the circumscription of ^{n}Fly in the abnormal theory can be expressed in first-order language and is reasonably computable. The expected definition of ^{n}Fly is achieved, *i.e.*, ^{n}Fly(x) *iff* ¬Fly(x), by circumscribing ^{n}Fly in the abnormal theory. Circumscribing ^{n}Fly actually to minimize the cases that creatures being birds cannot fly.

In our opinion, a *wff* ϕ believable in DT_{Bird} is same as that ϕ is circumscriptively inferred from the abnormal theory of DT_{Bird} *wrt* ^{n}Fly.

In section2, a model M_0 of W restricted by $\Gamma_\infty[M_0]$ *wrt* s default theory (D, W) will be defined as a model of (D, W). It is also shown by theorem1 that a default theory has a consistent extension *iff* it has a model.

The relationships revealed in section3, among three concepts of the believability in default logic, the minimal entailment in first-order logic, and circumscriptive inference in circumscription are sketched in the figure.

Here we shall briefly explain it. $DT\vdash^{B}\phi$ stands for that a *wff* ϕ is believable in a default theory DT. At first, it has been known that for a theory, the circumscription of a predicate symbol is true in all of its models minimal in this predicate symbol. Next, for a default theory (D, W), through certain transformation of defaults, it will be shown that a *wff* believable in (D, W) can be circumscriptively inferred from the transformed theory of (D, W) by theorem2.

Then the commonsense reasoning and default reasoning based on circumscription and default logic respectively are connected through minimal entailment and certain transformation of defaults. The model-theoretical meaning of M in default logic could be explained by minimal entailment. It is also possible to compensate for the disadvantage of circumscription, *i.e.*, the difficulty of derivation with second-order formulas, by simulating the proof procedure on default theories.

At last, by theorem4, we come to the conclusion that: a *wff* believable in a closed default theory (D, W) of clausal form is minimally entailed by $GD^0(E)\cup W$, where E is a consistent extension for (D, W).

2. Models of Default Theories

In our opinion, models for a default theory (D, W) are models of W satisfying one extension for (D, W); and conversely any consistent extension of (D, W) is satisfied by some models of (D, W). Now we define $\Gamma_\infty[M_0]$ for any model M_0 of W. M_0 is defined as a model of (D, W) when it is the minimal model of $\Gamma_\infty[M_0]$.

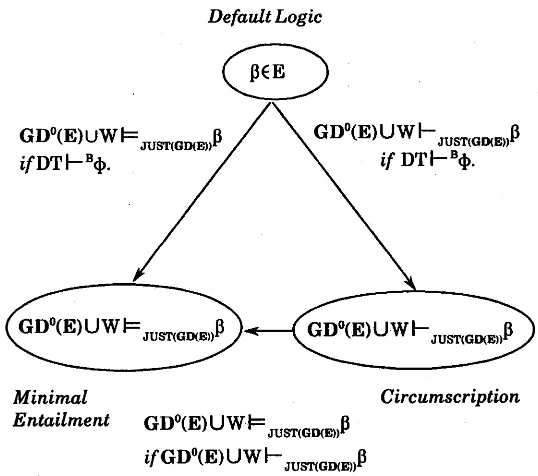

Fig. Relationship among $\models_S$, $\vdash_S$ and $\vdash^B$

Let DT=(D, W) be a closed default theory, M_0 any model of W and L the set of all formulas in first-order language.

(1) $\Gamma_0[M_0]=W$;

(2.1) $\Gamma_{i+1}[M_0]=L$
for some default $(\alpha(x): M\beta(x)/\gamma(x))$ in D,
$\alpha(x)\in\Gamma_i[M_0]$,
$M_0{}^{\sim}\models\neg\beta(x)$ and
$\neg\beta(x)\in\Gamma_i[M_0]$;

(2.2) $\Gamma_{i+1}[M_0]=Th(\Gamma_i[M_0])\cup\{\gamma(x)\mid$
$(\alpha(x): M\beta(x)/\gamma(x))\in D$,
$\alpha(x)\in\Gamma_i[M_0]$,
$M_0{}^{\sim}\models\neg\beta(x)$ and
$M_0\models\gamma(x)\}$;

(2.3) $\Gamma_{i+1}[M_0]=L$
for some default $(\alpha(x): M\beta(x)/\gamma(x))$ in D,
$\alpha(x)\in\Gamma_i[M_0]$,
$M_0{}^{\sim}\models\neg\beta(x)$ but
$M_0{}^{\sim}\models\gamma(x)$.

Then $\Gamma_\infty[M_0]$ is defined as:

$$\Gamma_\infty[M_0]=\bigcup_{i=0}^{\infty}\Gamma_i[M_0]$$

Now we shall give a concept of minimal model. The model M_0 of W will be defined as the model of (D, W) if M_0 is minimal model of $\Gamma_\infty[M_0]$ *wrt* (D, W).

Definition4

The *structure* M of a sentence A of clausal form is defined as:

(i) a non-empty Herbrand universe, called the domain of M, denoted by |M|;

(ii) M[K]: $|M|^n\rightarrow|M|$, if K is an n-ary function symbol;

M[K]: $|M|^n\rightarrow$\{True, False\} if K is an n-ary predicate symbol.

$M[K^+]=\{a\in|M|^n \mid M[K](a)=True\}\subset|M|^n$;

$M[K^-]=\{a\in|M|^n \mid M[K](a)=False\}\subset|M|^n$.

Definition5

Let M and N be two structures of a sentence A. M is a *substructure* of N in a predicate symbol P, written as $M\leqq_P N$, if

(1) $|M|=|N|$;

(2) $M[Q]=N[Q]$ for any predicate symbol Q (or function symbols), $Q\neq P$;

(3) $M[P^+]\subseteq N[P^+]$.

$M[Q^+]$ is called the *extension* of predicate symbol Q in a structure M.

A model M of a sentence A is *minimal in P iff* for any model M' of A,

$M'\leqq_P M$ *only if* $M'=M$, *i.e.*, $M'[P^+]=M[P^+]$.

Let S be a set of predicate symbols. $M\leqq_S N$ means that $M\leqq_{P_i} N$ for each P_i in S.

Definition6

Let DT=(D, W) be a closed default theory. A model M_0 of W is said to be a *model* of DT *iff* $\Gamma_\infty[M_0]\neq L$, and M_0 is a model of $\Gamma_\infty[M_0]$ minimal in $[M\beta]$, the set of all predicate symbols occurring in the justifications of defaults used to construct $\Gamma_\infty[M_0]$.

ϕ is said to be *believable* in DT, written as $DT\vdash^B\phi$, *iff* there is an extension E of DT containing ϕ, *i.e.*, $\phi\in E$.

When a default theory (D, W) has a model defined in the above way, we can simply figure out:

(1) all *wffs* in W are consistent;

(2) there is a consistent extension for (D, W).

As we have known, a default theory is generally said to be *consistent* if it has at least one consistent extension. Then this suggests the default theory is consistent if it has a model.

By the definition of $\Gamma_\infty[M_0]$, there is a model for a default theory (D, W), then this model is also satisfying all of those *wffs* in W. Thus (1) is self-evident. (2) will appear in the *if-half* of the following theorem.

Theorem1

A closed default theory of clausal form DT=(D,W) has a consistent extension *iff* it has a model when when every default in D is ($\alpha(x)$: $M\beta(x)/\gamma(x)$), and $\neg\beta(x)$ contains no negative literal.

【PROOF】 I. At first, we shall prove the *if-half* of the theorem, *i.e.*, if (D, W) has a model then it has a consistent extension.

Suppose M_0 is a model for DT. By the definition6, we have $\Gamma_\infty[M_0]\neq L$. Then we can construct E in the way of definition1 based on $\Gamma_\infty[M_0]$.

(1) $E^{(0)}=W$;

(2) $E^{(i+1)}=Th(E^{(i)})\cup\{\gamma(x) \mid (\alpha(x): M\beta(x)/\gamma(x))\in D, \alpha(x)\in E^{(i)}$ and $\neg\beta(x)\notin\Gamma_\infty[M_0].\}$

$$E=\bigcup_{i=0}^{\infty}E^{(i)}.$$

In order to prove $\Gamma_\infty[M_0]$ is an extension for DT, it is sufficient to show $\Gamma_\infty[M_0]=E$

Prove $E\subseteq\Gamma_\infty[M_0]$, *i.e.*,

$$\bigcup_{i=0}^{\infty}E^{(i)}\subseteq\bigcup_{i=0}^{\infty}\Gamma_i[M_0].$$

Inductive base:

$E^{(0)}\subseteq\Gamma_0[M_0]$ by the definitions of E and $\Gamma_\infty[M_0]$;

Inductive hypothesis:

Assume $E^{(i)}\subseteq\Gamma_i[M_0]$.

Inductive step:

Prove $E^{(i+1)}\subseteq\Gamma_{i+1}[M_0]$.

For a default $\delta=(\alpha(x)$: $M\beta(x)/\gamma(x))$ in D, $\alpha(x)\in E^{(i)}$ and $\neg\beta(x)\notin\Gamma_\infty[M_0]$, to show $\alpha(x)\in\Gamma_i[M_0]$, $M_0{}^{\sim}\models\neg\beta(x)$ and $M_0\models\gamma(x)$.

(i) Prove $\alpha(x)\in\Gamma_i[M_0]$.

By the inductive hypothesis $E^{(i)}\subseteq\Gamma_i[M_0]$ and $\alpha(x)\in E^{(i)}$, we have $\alpha(x)\in\Gamma_i[M_0]$;

(ii) Prove $M_0{}^{\sim}\models\neg\beta(x)$.

(a) Let $\neg\beta(x)$ be an atom P(t). Obviously we have $P(t)\in[M\beta]$, where $[M\beta]$ is the set of predicate symbols occurring in the justifications of defaults used to construct $\Gamma_\infty[M_0]$.

Suppose $M_0\models P(t)$. Then we can construct a proper substructure M of M_0 in the way:

$M[K]=M_0[K]$ for every $K\neq P$ and

$M[P]=M_0[P]-(t)$.

Because of $P(t)\notin\Gamma_\infty[M_0]$ and $M_0\models\Gamma_\infty[M_0]$. Then M is a model of $\Gamma_\infty[M_0]$. This contradicts the minimality of M_0. Thus $M_0{}^{\sim}\models\neg\beta(x)$;

(b) Let $\neg\beta(x)$ be $P(t_1)\vee Q(t_2)$ or $P(t_1)\wedge Q(t_2)$. The proof of $M_0{}^{\sim}\models\neg\beta(x)$, similar with (a), will be omitted here.

Thus we get $M_0{}^{\sim}\models\neg\beta(x)$;

(iii) To prove $M_0\models\gamma(x)$.

By the definition of $\Gamma_\infty[M_0]$, we know that when $\alpha(x)\in\Gamma_i[M_0]$, $M_0{}^{\sim}\models\neg\beta(x)$, $\gamma(x)$ is satisfied by M_0 if $\Gamma_\infty[M_0]\neq L$; otherwise, $\Gamma_\infty[M_0]=L$.

Thus we get $M_0\models\gamma(x)$.

By (i), (ii) and (iii), we have $E^{(i+1)}\subseteq\Gamma_{i+1}[M_0]$. Therefore $E\subseteq\Gamma_\infty[M_0]$.

Prove $\Gamma_\infty[M_0]\subseteq E$, *i.e*,

$$\bigcup_{i=0}^{\infty}\Gamma_i[M_0]\subseteq\bigcup_{i=0}^{\infty}E^{(i)}.$$

Inductive base:

$\Gamma_0[M_0]\subseteq E^{(0)}$ by the definitions of E and $\Gamma_\infty[M_0]$;

Inductive hypothesis:

Assume $\Gamma_i[M_0]\subseteq E^{(i)}$;

Inductive step:

Prove $\Gamma_{i+1}[M_0] \subseteq E^{(i+1)}$.

For a default $\delta = (\alpha(x) \colon M\beta(x)/\gamma(x))$ in D, $\alpha(x) \in \Gamma_i[M_0]$, $M_0{}^{\sim} \models \neg\beta(x)$ and $M_0 \models \gamma(x)$, to show $\alpha(x) \in E^{(i)}$ and $\neg\beta(x) \notin \Gamma_\infty[M_0]$.

(i) Prove $\alpha(x) \in E^{(i)}$.

By the inductive hypothesis $\Gamma_i[M_0] \subseteq E^{(i)}$ and $\alpha(x) \in \Gamma_i[M_0]$, we have $\alpha(x) \in E^{(i)}$;

(ii) Prove $\neg\beta(x) \notin \Gamma_\infty[M_0]$.

Suppose $\neg\beta(x) \in \Gamma_\infty[M_0]$. Then it is reasonable to assume there is an j, $j \geqq i$, $\neg\beta(x) \in \Gamma_j[M_0]$. Together with $\alpha(x) \in \Gamma_i[M_0]$, $M_0{}^{\sim} \models \neg\beta(x)$ and $j \geqq i$, we have $\alpha(x) \in \Gamma_j[M_0]$, $M_0{}^{\sim} \models \neg\beta(x)$ and $\neg\beta(x) \notin \Gamma_j[M_0]$. Then according to (2.1) in the definition of $\Gamma_\infty[M_0]$, $\Gamma_\infty[M_0] = L$ is obtained. This contradicts that M_0 is a model of $\Gamma_\infty[M_0]$. Hence $\neg\beta(x) \notin \Gamma_\infty[M_0]$.

By (i) and (ii), we have $\Gamma_{i+1}[M_0] \subseteq E^{(i+1)}$. Therefore $\Gamma_\infty[M_0] \subseteq E$.

Together with $E \subseteq \Gamma_\infty[M_0]$, $\Gamma_\infty[M_0] = E$ has been proven.

II. Next, we shall prove the *only-if-half* of the theorem, *i.e.*, if (D, W) has a consistent extension, then it has a model.

In order to show (D, W) has a model, it is sufficient to construct an $\Gamma_\infty[M_0]$ based on a model M_0 of W *wrt* (D, W) and prove that M_0 is a model of $\Gamma_\infty[M_0]$ minimal in $[M\beta]$.

Let E be an extension for (D, W). Because (D, W) is clausal form, then E is also a theory of clausal form. At the same time, a clausal theory has always a minimal model. Thus it is reasonable to assume M_0 be a model of E minimal in $[M\beta]$, the set of predicate symbols occurring in the justifications of defaults in GD(E).

At first we can construct $\Gamma_\infty[M_0]$ in the way:

(1) $\Gamma_0[M_0] = W$;

(2) $\Gamma_{i+1}[M_0] = Th(\Gamma_i[M_0]) \cup \{\gamma(x) \mid (\alpha(x) \colon M\beta(x)/\gamma(x)) \in D,\ \alpha(x) \in \Gamma_i[M_0],\ M_0{}^{\sim} \models \neg\beta(x) \text{ and } M_0 \models \gamma(x)\}$.

$$\Gamma_\infty[M_0] = \bigcup_{i=0}^{\infty} \Gamma_i[M_0]$$

Prove $\Gamma_\infty[M_0] \subseteq E$, *i.e.*,

$$\bigcup_{i=0}^{\infty} \Gamma_i[M_0] \subseteq \bigcup_{i=0}^{\infty} E^{(i)}.$$

Inductive base:

$\Gamma_0[M_0] \subseteq E^{(0)}$ because of the definitions of $\Gamma_\infty[M_0]$ and E;

Inductive hypothesis:

Assume $\Gamma_i[M_0] \subseteq E^{(i)}$;

Inductive step:

Prove $\Gamma_{i+1}[M_0] \subseteq E^{(i+1)}$.

For a default $(\alpha(x) \colon M\beta(x)/\gamma(x))$ in D, $\alpha(x) \in \Gamma_i[M_0]$, $M_0{}^{\sim} \models \neg\beta(x)$ and $M_0 \models \gamma(x)$, to show $\alpha(x) \in E^{(i)}$ and $\neg\beta(x) \notin E$.

(i) Prove $\alpha(x) \in E^{(i)}$.

By the inductive hypothesis $\Gamma_i[M_0] \subseteq E^{(i)}$ and $\alpha(x) \in \Gamma_i[M_0]$, we get $\alpha(x) \in E^{(i)}$;

(ii) Prove $\neg\beta(x) \notin E$.

Suppose $\neg\beta(x) \in E$. Then it is true in every model of E. This contradicts $M_0{}^{\sim} \models \neg\beta(x)$ and $M_0 \models E$.

By (i) and (ii), we have $\Gamma_{i+1}[M_0] \subseteq E^{(i+1)}$. Thus $\Gamma_\infty[M_0] \subseteq E$.

Prove $E \subseteq \Gamma_\infty[M_0]$, *i.e.*,

$$\bigcup_{i=0}^{\infty} E^{(i)} \subseteq \bigcup_{i=0}^{\infty} \Gamma_i[M_0].$$

Inductive base:

$E^{(0)} \subseteq \Gamma_0[M_0]$ by the definitions of E and $\Gamma_\infty[M_0]$;

Inductive hypothesis:

Assume $E^{(i)} \subseteq \Gamma_i[M_0]$;

Inductive step:

Prove $E^{(i+1)} \subseteq \Gamma_{i+1}[M_0]$.

For a default $(\alpha(x) \colon M\beta(x)/\gamma(x))$ in D, $\alpha(x) \in E^{(i)}$ and $\neg\beta(x) \notin E$, to show $\alpha(x) \in \Gamma_i[M_0]$, $M_0{}^{\sim} \models \neg\beta(x)$ and $M_0 \models \gamma(x)$.

(i) Prove $\alpha(x) \in \Gamma_i[M_0]$.

By the inductive hypothesis $E^{(i)} \subseteq \Gamma_i[M_0]$ and $\alpha(x) \in E^{(i)}$, we get $\alpha(x) \in \Gamma_i[M_0]$;

(ii) Prove $M_0{}^{\sim} \models \neg\beta(x)$.

According to $M_0 \models E$, $\neg\beta(x) \notin E$ and the minimality of M_0, we can reason out $M_0 \models \neg\beta(x)$. The details of this proof, similar with (ii) to show $\Gamma_\infty[M_0] \subseteq E$ in the proof of *if-half*, will be omitted here;

(iii) Prove $M_0 \models \gamma(x)$.

Because of $\alpha(x) \in E^{(i)}$ and $\neg\beta(x) \notin E$, then we have $\gamma(x) \in E$ by the property of extension. Thus $M_0 \models \gamma(x)$ follows $M_0 \models E$.

By (i), (ii) and (iii), we have $E^{(i+1)} \subseteq \Gamma_{i+1}[M_0]$. Thus $E \subseteq \Gamma_\infty[M_0]$.

Together with $\Gamma_\infty[M_0] \subseteq E$, there is $\Gamma_\infty[M_0] = E$. Therefore M_0 is a model of $\Gamma_\infty[M_0]$ in $[M\beta]$ because M_0 is a model of E minimal in $[M\beta]$.

Then theorem has been proven. **QED**

Generally, a default $(\alpha(x) \colon M\beta(x)/\gamma(x))$, in which $\neg\beta(x)$ contains negative literals, can be changed into default $(\alpha(x) \colon M\beta'(x)/\gamma(x))$, in which $\neg\beta'(x)$ contains no negative literal. For instance, (Bird(x): MFly(x)/Fly(x)) can be represented by (Bird(x): $M\neg{}^n$Fly(x)/Fly(x)) and $\neg$Fly(x)$\supset{}^n$Fly(x). Therefore, theorem1 may be appropriate for any default theories if we slightly change the form of defaults appearing in those default theories.

According to this, a default theory has extensions is identified with that it has models, which are actually the models for all *wffs* and satisfy those restrictions represented by some defaults in the default theory. Let us consider a model M_0 for a default theory (D, W) and an $\Gamma_\infty[M_0]$ *wrt* (D, W). Shown by theorem1, $\Gamma_\infty[M_0]$ is an extension for (D, W).

<u>*Corollary1.1*</u>

Let DT=(D, W) be a closed default theory of clausal form. A *wff* ϕ believable in DT is true in a model M_0 of DT, *i.e.*, $M_0 \models \phi$ if $DT \vdash^B \phi$.

3. Transformation of Default Theories

As shown in [10], *Tomasz Imielinski* defines an abstract ordering among models of a theory *wrt* a predicate symbol and explains the circumscription of this predicate symbol in the theory by the smallest models *wrt* defined ordering. He tries to represent defaults by this semantic ordering. However, he achieved a very surprising result that it is incapable to represent an important class of defaults, normal defaults with prerequisites, and it is only capable to represent a simple (*i.e.*, with only one extension) default theory without prerequisites by semantic ordering. Then in order to connect default reasoning with circumscription, it seems not sufficient to introduce only an abstract ordering among models. In our case, for every default in a default theory, a predicate symbol *wrt* its justification is defined. The reasoning upon this default theory will be described through circumscribing those additional predicate symbols in the transformed theory. Now the relative definitions are stated formally as follows.

<u>*Definition7*</u>

Let D be a set of defaults and δ a default.

δ^0 and D^0 are defined as:

$\delta^0 = (\alpha(x) \wedge \neg {}^n\beta(x) \supset \gamma(x)) \wedge (\neg \beta(x) \supset {}^n\beta(x))$ if $\delta = (\alpha(x): M\beta(x)/\gamma(x))$;

$D^0 = \{ \delta^0 \mid \delta \in D \}$;

JUST(D) is a set of predicate symbols, defined as:

$JUST(D) = \{ {}^n\beta \mid (\alpha(x): M\beta(x)/\gamma(x)) \in D. \}$

$W \cup D^0$ is called the *abnormal theory* of a default theory (D, W).

It has been known that if a *wff* ϕ is believable in (D, W) then there is a model M_0 of (D, W) satisfying ϕ by corollary1.1. Observing $\Gamma_\infty[M_0]$ and the definition of circumscriptive inference, we could figure out a relationship established between two concepts, relative to non-monotonic reasoning, of believability in default logic and circumscriptive inference in first-order logic.

Now let us observe the relationship between believability and circumscription. Suppose E is an extension of a default theory DT=(D, W) and GD(E) the set of generation defaults *wrt* E. $GD^0(E) \cup W \vdash_{JUST(GD(E))} \phi$ means that ϕ is derived from $GD^0(E) \cup W$ by circumscribing every predicate symbol ${}^n\beta$ in JUST(GD(E)), appearing in definition7. For a default theory, deriving $M\beta(x)$, which suggests no $\neg\beta(x)$ explicitly exists, could be achieved by minimizing ${}^n\beta$ in its abnormal theory.

Preparatory to presenting next one of our main results on default reasoning and circumscription, we shall give several concepts useful to prove theorem.

<u>*Definition8*</u>

Let T be a *first*-order theory consisting of sentences and P a predicate symbol occurring in T. Circum(T; P) is *reducible iff* there is an *first*-order theory, written as $T_{Circum}(T; P)$, model-theoretically equivalent to Circum(T; P). That is, for any sentence β in the first-order language,

$Circum(T; P) \models \beta$ *iff* $T_{Circum}(T; P) \models \beta$.

T is said to be *circumscriptively reducible* on P when Circum(T; P) is reducible.

<u>*Definition9*</u>

A formula F is called *solitary wrt* a predicate symbol P if it is the conjunction of:

(i) formulas without positive occurrences of P;

(ii) formulas of the form $\forall x\ (U(x) \supset P(x))$, where U(x) does not contain P.

A formula F is called *separable* with respect to P if it is a disjunction of solitary formulas.

Suppose that

$$L_1 \wedge \ldots \wedge L_m \supset P(t_1, \ldots, t_n) \qquad (1)$$

is a clause about a predicate symbol P. Let = be the equality relation, and $x_1, \ldots, x_n$ be variables not appearing in the clause. (1) is equivalent to the clause

$$x_1 = t_1 \wedge \ldots \wedge x_n = t_n \wedge L_1 \wedge \ldots \wedge L_m \supset P(x_1, \ldots, x_n)$$

Finally, if $y_1, \ldots, y_r$ are the variables in (1), it is itself equivalent to

$$(\exists y_1, \ldots, y_r)[x_1 = t_1 \wedge \ldots \wedge x_n = t_n \wedge L_1 \wedge \ldots \wedge L_m] \supset P(x_1, \ldots, x_n) \qquad (2)$$

we call this the *general form* of the clause.

Suppose there are exactly *k* clauses, $k > 0$, about the predicate symbol P. Let

$$\begin{array}{l} E_1 \supset P(x_1, \ldots, x_n) \\ \ldots\ldots \\ E_k \supset P(x_1, \ldots, x_n) \end{array} \qquad (3)$$

be *k* general forms of these clauses. Each of E_i will be an existentially quantified conjunction of literals as in (2). The *definition of P*, implicitly given by all of those *k* clauses, is

$$(\forall x_1, \ldots, x_n)[E_1 \vee E_2 \vee \ldots \vee E_k \equiv P(x_1, \ldots, x_n)] \qquad (4)$$

The *if-half* of this definition is just the *k* general form clauses (3) grouped as a single implication. The *only-if-half* is the *completion axiom* for P.

For a theory T of clausal form, it may be partitioned into two disjoint sets:

T_P: those clauses in T containing exactly one positive literal in P, and

$T - T_P$: those clauses of T containing no positive (but possibly negative) literals on P.

The *completion* of T *wrt* P, Comp(T; P), is a theory of T with T_P replaced by the definition of P.

Lemma2.1

Let DT = (D, W) be a clausal default theory. Then the abnormal theory of DT is separable *wrt* JUST(D).

Lemma2.2

Let T be a theory of *wffs* of clausal form. Then T is circumscriptively reducible *wrt* a predicate symbol P if T is separable *wrt* P and the circumscriptively reduced theory T_{Circum}(T; P) of T is Comp(T; P).

The details about those issues will be omitted in this paper. Who interested in them are suggested to refer [1], [2] and [6].

Then, we define a concept of default proof for a *wff* in a default theory and show that for a closed consistent default theory DT, if it has a consistent extension containing a *wff* ϕ, then ϕ has a default proof in DT. Because default theories of general form have no semi-monotonicity, it is certainly we cannot expect a *wff*, having default proof, could be definitely believable.

Definition10

Let DT = (D, W) be a closed default theory of general form and ϕ a *wff*. The finite sequence D_0, D_1,...,D_n of finite subsets of D is said to be *default proof* of ϕ *iff*

(1) $W \cup CONSEQUENTS(D_0) \vdash \phi$;

(2) for any i, $1 \leqq i \leqq n$, $W \cup CONSEQUENTS(D_i) \vdash PREREQUISITES(D_{i-1})$;

(3) $D_n = \varnothing$;

(4) $W \bigcup_{i=0}^{n} JUSTIFICATION(D_i) \bigcup_{i=0}^{n} CONSEQUENTS(D_i)$ is satisfiable.

As we have known, the concept of a default proof of a *wff* has also been defined by *R.Reiter* in [9]. Our definition of default proof is slightly different. Firstly, he defines a default proof on a closed normal default theory because it seems that he expects to establish a complete relation between the believability and default proof of a *wff* in normal default theories. Actually, he has successfully shown that a consistent closed normal default theory has an extension containing a *wff* ϕ *iff* there is a default proof of ϕ *wrt* the default theory. Secondly, it follows normality of default theories that in (4) the satisfication of justifications is not necessary to be considered for it is automatically met by the satisfication of all of those consequents of defaults involved in the default proof. In our definition, the underlying default theory is general form, then in property (4) the satisfication of justifications of D_i is no more trivial. All of our intensions to modify the concept of default proof in this way is to show following lemma for the coming theorem. Because general default theories lack semi-monotonicity possessed by normal default theories, *i.e.*, for a default theory (D, W), its extension is not always a super set of sub-default theories consisting of W and subsets of D, the lemma can obviously not be conversed. That is, for a *wff*, the existence of its default proof does not determine its believability in default theories of general form.

Lemma2.3

Let DT = (D, W) be a closed default theory. A closed *wff* ϕ has a default proof in DT if there is a consistent extension E of DT containing ϕ.

【PROOF】 In order to prove this, we introduce $GD(E^{(i)})$ to stand for the defaults relative to $E^{(i)}$ in the definition of extension.

If $\phi \in E$, then there is at least an $E^{(i)}$, such that $\phi \in E^{(i)}$. Construct D_0, D_1, . . ., D_i in following way:

$$\text{for any } j,\ 0 \leqq j \leqq i,\ \text{take } D_j = \bigcup_{k=0}^{i-j} GD(E^{(k)}).$$

Obviously D_0, D_1, . . ., D_i satisfy the properties (1)~(4) in definition10.

(1) $W \cup CONSEQUENTS(D_0) \vdash \phi$ is caused by $\phi \in E^{(i)}$;

(2) For any v, $1 \leqq v \leqq i$, $W \cup CONSEQUENTS(D_v) \vdash PREREQUISITES(D_{v-1})$. We can say this by the definition of $E^{(i-v+1)}$, $PREREQUISITES(D_{v-1}) \in W \cup CONSEQUENTS(D_v)$;

(3) $D_i = \varnothing$ because there is $E^{(0)} = W$ then $GD(E^{(0)}) = \varnothing$;

(4) Because E is consistent and for any β in

$$\bigcup_{j=0}^{i} JUSTIFICATION(D_j),$$

there is $\neg\beta \notin E$; Then

$$W \bigcup_{j=0}^{i} JUSTIFICATION(D_j) \bigcup_{j=0}^{i} CONSEQUENTS(D_j)$$

is satisfiable.

Thus the sequence of D_0, D_1, . . . , D_i is a default proof of ϕ. **QED**

According to lemma2.3, a closed *wff* ϕ believable in a closed default theory has a default proof. Transforming those defaults involved in the default proof, then ϕ can be circumscriptively inferred from this transformed abnormal theory.

Theorem2

Let DT=(D, W) be a closed consistent default theory of clausal form and E a consistent extension of DT. A closed *wff* ϕ believable in (D, W) is circumscriptively inferred from $GD^0(E)\cup W$ *wrt* JUST(GD(E)), *i.e.*, $GD^0(E)\cup W\vdash_{JUST(GD(E))}\phi$ if $\phi\in E$.

【PROOF】 By lemma2.3, it is sufficient to show that ϕ is circumscriptively inferred from $GD^0(E)\cup W$ *wrt* JUST(GD(E)) if ϕ has a default proof in DT, because ϕ is contained by the consistent extension E for (D, W).

For the existence of a default proof of ϕ, it could be assumed to be the sequence $D_0, D_1, \ldots, D_n$.

(1) When n=0, it is trivial that $GD^0(E)\cup W\vdash_{JUST(GD(E))}\phi$ from $W\vdash\phi$;

(2) For n>0, $D_0, D_1, \ldots, D_n$ could be assumed to be:

$D_n=\varnothing$, *by the property3 of default proof;*

$$D_{n-1}=\left\{\frac{\alpha_1^{(n-1)}(x): M\beta_1^{(n-1)}(x)}{\gamma_1^{(n-1)}(x)},\ldots,\frac{\alpha_{m_{n-1}}^{(n-1)}(x): M\beta_{m_{n-1}}^{(n-1)}(x)}{\gamma_{m_{n-1}}^{(n-1)}(x)}\right\}$$

$$D_{n-2}=\left\{\frac{\alpha_1^{(n-2)}(x): M\beta_1^{(n-2)}(x)}{\gamma_1^{(n-2)}(x)},\ldots,\frac{\alpha_{m_{n-2}}^{(n-2)}(x): M\beta_{m_{n-2}}^{(n-2)}(x)}{\gamma_{m_{n-2}}^{(n-2)}(x)}\right\}$$

$$\cdots\cdots$$

$$D_i=\left\{\frac{\alpha_1^{(i)}(x): M\beta_1^{(i)}(x)}{\gamma_1^{(i)}(x)},\ldots,\frac{\alpha_{m_i}^{(i)}(x): M\beta_{m_i}^{(i)}(x)}{\gamma_{m_i}^{(i)}(x)}\right\}$$

$$\cdots\cdots$$

$$D_0=\left\{\frac{\alpha_1^{(0)}(x): M\beta_1^{(0)}(x)}{\gamma_1^{(0)}(x)},\ldots,\frac{\alpha_{m_0}^{(0)}(x): M\beta_{m_0}^{(0)}(x)}{\gamma_{m_0}^{(0)}(x)}\right\}$$

By the properties (1) and (2) of default proof, we have:

$$W\vdash\left\{\alpha_1^{(n-1)}(x),\ldots,\alpha_{m_{n-1}}^{(n-1)}(x)\right\}$$

$$W\cup\left\{\gamma_1^{(n-1)}(x),\ldots,\gamma_{m_{n-1}}^{(n-1)}(x)\right\}\vdash\left\{\alpha_1^{(n-2)}(x),\ldots,\alpha_{m_{n-2}}^{(n-2)}(x)\right\}$$

$$\cdots\cdots$$

$$W\cup\left\{\gamma_1^{(i)}(x),\ldots,\gamma_{m_i}^{(i)}(x)\right\}\vdash\left\{\alpha_1^{(i-1)}(x),\ldots,\alpha_{m_{i-1}}^{(i-1)}(x)\right\}$$

$$\cdots\cdots$$

$$W\cup\left\{\gamma_1^{(0)}(x),\ldots,\gamma_{m_0}^{(0)}(x)\right\}\vdash\{\phi\}\qquad(*)$$

For $D_{n-1}, \ldots, D_i, \ldots, D_0$, the union of $D_{n-1}, \ldots, D_i, \ldots, D_0$ is the set of generation defaults *wrt* the extension containing ϕ. Then there are:

$$\alpha_1^{(n-1)}(x)\wedge\neg{}^n\beta_1^{(n-1)}(x)\supset\gamma_1^{(n-1)}(x)$$

$$\cdots\cdots$$

$$\alpha_{m_{n-1}}^{(n-1)}(x)\wedge\neg{}^n\beta_{m_{n-1}}^{(n-1)}(x)\supset\gamma_{m_{n-1}}^{(n-1)}(x)$$

$$\cdots\cdots$$

$$\alpha_1^{(i)}(x)\wedge\neg{}^n\beta_1^{(i)}(x)\supset\gamma_1^{(i)}(x)$$

$$\cdots\cdots$$

$$\alpha_{m_i}^{(i)}(x)\wedge\neg{}^n\beta_{m_i}^{(i)}(x)\supset\gamma_{m_i}^{(i)}(x)$$

$$\cdots\cdots$$

$$\alpha_1^{(0)}(x)\wedge\neg{}^n\beta_1^{(0)}(x)\supset\gamma_1^{(0)}(x)$$

$$\cdots\cdots$$

$$\alpha_{m_0}^{(0)}(x)\wedge\neg{}^n\beta_{m_0}^{(0)}(x)\supset\gamma_{m_0}^{(0)}(x)\qquad(**)$$

in $GD^0(E)$ *wrt* GD(E).

Let W_i be a set of W with all formulas $\neg\beta_j^{(i)}(x)\supset{}^n\beta_j^{(i)}(x)$, for any j, $1\leqq j\leqq m_i$.

A pair (W, i) is used to stand for

$$W_i\cup\{\gamma_1^{i}(x),\ldots,\gamma_{m_i}^{i}(x)\}\vdash\{\alpha_1^{(i-1)}(x),\ldots,\alpha_{m_{i-1}}^{(i-1)}(x)\}$$

in *, for any i, $n\leqq i\leqq 0$, except for $\{\gamma_1^{n}(x),\ldots,\gamma_{m_n}^{n}(x)\}=\varnothing$ and $\{\alpha_1^{-1}(x),\ldots,\alpha_{m_{i-1}}^{-1}(x)\}=\{\phi\}$, where i is the superscript of γ's in the left hand side of $\vdash$.

A pair (i, j) is used to stand for

$$\alpha_j^{i}(x)\wedge\neg{}^n\beta_j^{i}(x)\supset\gamma_j^{i}(x)$$

in **, where i and j are the superscript and subscript of α respectively.

According to lemma2.2, for any $\neg{}^n\beta(x)$, it can be derived from the result of circumscribing every predicate symbol ${}^n\beta$ in W_i if $\beta(x)$ is consistent. Then we can conclude with follows:

Circumscribing ${}^n\beta_1^{(n-1)}, \ldots, {}^n\beta_{m_{n-1}}^{(n-1)}$ in the union of W_1 and **, then $\{\gamma_1^{(n-1)}(x), \ldots, \gamma_{m_{n-1}}^{(n-1)}(x)\}$ can be derived from Circum($W_1\cup\{**\}$; $\{{}^n\beta_1^{(n-1)}, \ldots, {}^n\beta_{m_{n-1}}^{(n-1)}\}$; $\{{}^n\beta_1^{(n-2)}, \ldots, {}^n\beta_{m_{n-2}}^{(n-2)}, \ldots, {}^n\beta_1^{(0)}, \ldots, {}^n\beta_{m_0}^{(0)}\}$) because of ($W_1$, n−1), (1, n−1), . . . , (m_{n-1}, n−1) and the consistency of $W\cup{}^n\beta_1^{(n-1)}(x), \ldots, {}^n\beta_{m_{n-1}}^{(n-1)}(x)$ by property (4) of default proof.

By the same way, $\gamma_1^{i}(x), \ldots, \gamma_{m_i}^{i}(x)$ can be derived from Circum($W_{n-i}\cup\{**\}$; $\{{}^n\beta_1^{(i)}, \ldots, {}^n\beta_{m_i}^{(i)}\}$; $\{{}^n\beta_1^{(i-1))},\ldots,{}^n\beta_{m_{i-1}}^{(i-1))},\ldots,{}^n\beta_1^{(0)},\ldots,{}^n\beta_{m_0}^{(0)}\}$), and *etc*..

Thus Circum($W_n\cup\{**\}$; $\{{}^n\beta_1^{(0)}, \ldots, {}^n\beta_{m_0}^{(0)}\}$) infers ϕ.

According to [4], we have

$$\text{Circum}(W\cup\{**\}; \{{}^n\beta_1^{(n-1)},\ldots,{}^n\beta_{m_{n-1}}^{(n-1)}\}>\ldots>\{{}^n\beta_1^{(1)},\ldots,{}^n\beta_{m_1}^{(1)}\}>\{{}^n\beta_1^{(0)},\ldots,{}^n\beta_{m_0}^{(0)}\})$$
$$=\bigwedge_{1\leq i\leq n}\text{Circum}(W_i\cup\{**\}; \{{}^n\beta_1^{(n-i)}, \ldots, {}^n\beta_{m_{n-i}}^{(n-i)}\}; \{{}^n\beta_1^{(n-i-1))},\ldots,{}^n\beta_{m_{n-i-1}}^{(n-i-1))},\ldots,{}^n\beta_1^{(0)},\ldots,{}^n\beta_{m_0}^{(0)}\}).$$

Then ϕ is derived from Circum($W\cup\{**\}$; $\{{}^n\beta_1^{(n-1)},\ldots,{}^n\beta_{m_{n-1}}^{(n-1)}\}>\ldots>\{{}^n\beta_1^{(1)},\ldots,{}^n\beta_{m_1}^{(1)}\}>\{{}^n\beta_1^{(0)},\ldots,{}^n\beta_{m_0}^{(0)}\}$). Hence ϕ is derivable from the prioritized circumscription of ${}^n\beta_1^{(n-1)}$, . . . ,

$^n\beta m_{n-1}{}^{(n-1)}, \ldots, {}^n\beta_1{}^{(1)}, \ldots, {}^n\beta m_1{}^{(1)}, {}^n\beta_1{}^{(0)}, \ldots, {}^n\beta m_0{}^{(0)}$ in W∪{**} *wrt* the priority > among $^n\beta$'s. That is, ϕ is circumscriptively inferred from $GD^0(E) \cup W$.

QED

As shown in the above proposition, a *wff* ϕ believable in a default theory (D, W) is circumscriptively inferred from $GD^0(E) \cup W$. By Corollary1.1, we have also know that a *wff* believable in a default theory is satisfied by a model of DT.

4. Minimal Entailment and Default Reasoning

Theorem3 [3]

Let ϕ be a theory in first-order language and S be a set of predicate symbols occurring in T. A *wff* ϕ circumscriptively derivable from T *wrt* S is true in all of minimal models of T in S, *i.e.*, $T \models_S \phi$ if $T \vdash_S \phi$.

However theorem3 cannot always be conversed because for a theory, not every one of its models has a submodel minimal in some predicate symbols.

Up to now, we have observed the relationship between believability in a default theory and circumscriptive inference on the transformed abnormal theory in last section, and the relationship between circumscriptive inference and minimal entailment just above. Then we can simply figure out what between believability in default reasoning and minimal entailment in model-theoretical sense. Summarize theorem2 and theorem3, we conclude with the following theorem.

Theorem4

Let DT = (D, W) be a closed default theory of clausal form and E a consistent extension of (D, W). A closed *wff* ϕ believable in (D, W) is minimally entailed by $GD^0(E) \cup W$ *wrt* JUST(GD(E)), *i.e.*, $GD^0(E) \cup W \models_{TUST(GD(E))} \phi$ if $DT \vdash^B \phi$.

5. Conclusions

In this paper, we have introduced a predicate symbol $^n\beta$ for the justification $\beta(x)$ of each default in a default theory (D, W). Then a default can be rewritten as two *wffs* and a default theory can be transformed into a theory in first-order language. It has been shown that the reasoning on a default theory could be simulated by circumscribing additional predicate symbols $^n\beta$'s in the transformed abnormal theory of the default theory. The reason we propose this transformation of defaults is that, in our opinion, two ways of non-monotonic reasoning, both default logic and circumscription, proposed based upon seemingly entirely contrary observations of incomplete knowledge in the real world, are both appropriate to represent incomplete knowledge in a large class, *i.e.*, knowledge with an infinite number of exceptions. Then we would like to expect default logic not only to be a tool flexible to cope with incomplete knowledge but also to possess a model-theoretical explanation for its default reasoning through connecting it with circumscription which has a solid semantic foundation of minimal entailment.

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