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極小翻意による非単調推論

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現在の非単調推論の定式化においては、特別な推論規則や公理を知識に付け加えることで信念を表現し、その結果として非単調性が現れる。これに対して我々は信念を直接表現して、知識が付加されたときにどう信念を変化させるかを示す戦略を用いることで非単調推論の定式化を行った。

この定式化においては、信念は一階述語論理で記述され、知識が追加された場合に信念と矛盾する知識が例外として扱われるように信念が極小に変更される。本稿では、まず知識と信念の関係について論じ、つぎに極小翻意のための証明論とモデル論について述べ、ある種の非単調推論の例を示す。

Nonmonotonic Reasoning by Minimal Belief Revision

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This paper presents a formalism of *nonmonotonic reasoning.* In our formalism, initial belief is represented directly. Unlike the current formalisms such as default logic or circumscription we can express the belief without any extra inference rules or special axiom. The only constraint for a belief is that it must entail knowledge. Then, we define a belief revision strategy called *minimal belief revision.* Minimal belief revision minimizes the difference between the previous belief and the new belief so that what was true in the previous belief remains true in the new belief as far as possible.

This paper discusses why belief revision must occur when belief does not entail added knowledge, presents a proof theory and model theory for minimal belief revision and shows that minimal belief revision performs some kind of nonmonotonic reasoning.

1 Introduction

In real life, we are sometimes forced to draw some conclusion even if there is not enough information. For one solution to those situations, we use our belief (or hypothesis) to complement unknown information. However, since the results from a belief are not logically true, they must be defeated if they are found to be false. Such reasoning is called *nonmonotonic reasoning* and has been formalized by various researchers [McCarthy80, McDermott80, and Reiter80].

Roughly speaking, current formalisms such as circumscription [McCarthy80], nonmonotonic logic [McDermott80] and default logic [Reiter80] add special axioms or define extra inference rules to incorporate the idea that an unknown fact is assumed to be false unless it is explicitly known to be true. The special axioms or extra inference rules can be regarded as producing methods of plausible belief from the current knowledge. If more knowledge is added, a different belief is produced by those axioms or extra inference rules.

This paper presents another formalism of nonmonotonic reasoning. Our approach of formalizing nonmonotonic reasoning is different from those formalisms in the following points.

(1) A belief is represented directly as a plausible hypothesis without any extra inference rules or special axioms. If no conclusion is derived from incomplete knowledge, a belief is used to complement unknown information. The only constraint for a belief is that it must entail knowledge; in other words, it is a detailed hypothetical description of knowledge to supplement a lack of knowledge.

(2) A belief is directly revised when more information is added. Since a belief has a hypothetical character, it is not always true. Therefore, if a belief does not entail added information, it must be changed to satisfy the above constraint. This process of change is called *belief revision.*

This paper investigates a special strategy of belief revision called *minimal belief revision* and shows that this strategy performs some kinds of nonmonotonic reasoning. The idea of minimal belief revision is that default rules are first defined as belief, and if any counter-example is found, the belief is changed so that the counter-example is treated as an exception to maintain consistency.

For example, suppose that belief for flying birds is expressed by the following:

$$\forall x(bird(x) \supset fly(x)).$$

The above belief expresses directly that every bird flies. Even if we only know $bird(A)$, we conclude $fly(A)$ as a consequence of the above belief. However, if we find $\neg fly(A)$ in addition to $bird(A)$, then the belief must be changed to keep consistency. However, we do not want to throw away the above belief completely, but we still want to believe that any bird other than A flies. The minimal belief revision strategy performs such revision and changes the above belief into the following:

$$\forall x(x \neq A \equiv \forall x(bird(x) \supset fly(x))).$$

This belief revision is nonmonotonic, because from the previous belief, we can derive $fly(A)$ if we know $bird(A)$, whereas from the new belief, we can no longer derive $fly(A)$.

This paper first discusses the relationship between knowledge and belief, then shows the proof theory and model theory of minimal belief revision and some examples of nonmonotonic reasoning with it.

2 Monotonic Knowledge and Nonmonotonic Belief

This section shows why belief revision must occur when belief does not entail added knowledge. Let κ and β be a set of knowledge and a set of beliefs respectively. [Hintikka62] requires knowledge and belief to satisfy the following relation:

$$\kappa \subseteq \beta,$$

which means that if an agent knows p, he also believes p. In this paper, knowledge and beliefs are represented as logical formulas. Let knowledge and belief be formulas of K and B. The above requirement can be expressed as follows:

$$B \models K,$$

which means that B entails K [1].

We also define that knowledge is monotonic. Following Hintikka, we regard knowledge as a subset of truth, and therefore, knowledge increases monotonically. However, since we regard a belief as a set of plausible hypotheses, a belief must be nonmonotonic if we keep consistency of belief. For example, suppose that the current knowledge contains neither α nor $\neg\alpha$ and the belief contains α. Then, if $\neg\alpha$ is added to knowledge, the belief must contain it because of the

[1]Note that circumscription and default logic satisfy this requirement, because both produce a belief including an initial axiom.

above requirement. However, the simple addition of $\neg\alpha$ to the belief leads to a contradiction. Therefore, in this case, α in the belief must be retracted in order to maintain consistency. Therefore, when new knowledge is added, a belief must be revised so that it contains new knowledge and is satisfiable. This process is called *belief revision.*

A strategy on how to change belief is needed since there are many ways of changing the belief. The next section concentrates on one particular belief revision strategy called *minimal belief revision.* Minimal belief revision is a strategy by which differences between the previous belief and revised belief are minimized.

3 Minimal Belief Revision

The main idea of minimal belief revision is that we compute the differences between a model of previous belief and a model of new knowledge, and choose pairs of models where the difference between models is minimal. As a result, what was true in the previous belief remains true in the new belief if it is not contradictory to the added knowledge.

We explain the idea with the following example. Let knowledge K be identical to $\mathbf{T}$, which denotes a true proposition, and belief B be identical to $p \wedge q$. Then suppose that α, which is equivalent to $\neg p \vee \neg q$, is added to knowledge K; we must revise belief because $B \not\models K \wedge \alpha$. Let the new belief be NB. NB must entail $K \wedge \alpha$ by the above requirement, that is, a set of models of NB must be a subset of the following set:

$$\{\{\neg p, q\}, \{p, \neg q\}, \{\neg p, \neg q\}\},$$

which is a set of all models of $K \wedge \alpha$ (a model is represented as a set of the propositional constants and negation of propositional constants that are true in the model).

First, we compute the difference between each model of B and each model of $K \wedge \alpha$. In this case, we compute the differences of $(\{p,q\},\{\neg p,q\})$, $(\{p,q\},\{p,\neg q\})$ and $(\{p,q\},\{\neg p,\neg q\})$ which are $\{p\}$,$\{q\}$ and $\{p,q\}$ respectively (the difference set is represented as propositional constants which have different truth values for each model in the pair). Then we select pairs whose differences are minimal in terms of set inclusion, that is, $(\{p,q\},\{\neg p,q\})$ and $(\{p,q\},\{p,\neg q\})$. Then the new belief, NB, by minimal belief revision is the disjunction of all those models of knowledge in the above pairs, that is:

$$(\neg p \wedge q) \vee (p \wedge \neg q).$$

While new knowledge has a model of $\{\neg p, \neg q\}$, NB does not have it. This is because what was true in the previous belief remains true as far as possible by minimal belief revision. Thus, either p or q (not both) remains true in the new belief.

We generalize this idea to knowledge and belief expressed in a first-order language. In the following subsections, minimal belief revision is defined in a second-order language. In a second-order language, we can use predicate variables and function variables in addition to object variables. Predicate variables vary over predicates and function variables vary over functions. In addition, we use predicate constants such as $\mathbf{T}$ for true, $\mathbf{F}$ for false and $=$ for equality, and logical connectives such as $\oplus$ for exclusive-or and $\equiv$ for equivalence.

We also give the model theory of minimal belief revision for a second-order language. A ***structure***, M, for a second-order language consists of a domain D, which is a non-empty set, and an interpretation function such that every n-ary function constant, F_n, is mapped onto a function from D^n to D (written $M[F_n]$), and every n-ary predicate constant, P_n, is mapped into a subset of D^n (written $M[P_n]$). N-ary function variables range over any function from D^n to D, and n-ary predicate variables range over any subset of D^n. $< t_1, ..., t_n >_M$ denotes an interpreted tuple where $t_1, ..., t_n$ are terms. If $P_n(t_1, ..., t_n)$ is true in M, this fact is expressed as $< t_1, ..., t_n >_M \in M[P_n]$. A ***model*** of a second-order sentence is any structure, M, such that every formula in the set is true in M.

3.1 Proof Theory

Let $B(P)$, $K(P)$ and $\alpha(P)$ be first-order sentences whose predicate constants are among those of $P = p_1, ..., p_n$, and $B(P) \models K(P)$. $B(P)$ is the current belief and $K(P)$ is the current knowledge and $\alpha(P)$ is the added knowledge.

We define a minimal revised belief, $NB(P)$, with respect to $(K(P), B(P))$ and $\alpha(P)$ as follows.

$$NB(P) \stackrel{\text{def}}{=} \exists P_B(B(P_B) \wedge K(P) \wedge \alpha(P) \wedge \neg\exists P'_B \exists P'(B(P'_B) \wedge K(P') \wedge \alpha(P') \wedge (P'_B, P') \prec (P_B, P)))\ ^2,$$

where

(1) P_B is a tuple of predicate variables $p_{B1}, ..., p_{Bn}$ which have the same arities of $p_1, ..., p_n$ respectively, and P'_B is also a tuple of predicate variables $p'_{B1}, ..., p'_{Bn}$ which satisfy the same condition as

[2]Since this definition of minimal revised belief is expressed in second-order language, we may need to restrict language so that the new belief can be defined in first-order language. A technique similar to [Lifschitz85] could be used.

P_B, P' is a tuple of predicate variables $p'_1, ..., p'_n$ which also satisfy the same condition as P_B,

(2) and $B(P_B)$ is a sentence obtained by substituting predicate variables of P_B for any occurrence of corresponding predicate constants in $B(P)$, and $B(P'_B)$ and $K(P')$ are sentences obtained in a similar way,

(3) and $(P'_B, P') \prec (P_B, P)$ is an abbreviation of:
$(P'_B, P') \preceq (P_B, P) \wedge \neg((P_B, P) \preceq (P'_B, P'))$,
where $(P'_B, P') \preceq (P_B, P)$ is an abbreviation of:
$\forall \mathbf{x}((p'_{B1}(\mathbf{x}) \oplus p'_1(\mathbf{x})) \supset (p_{B1}(\mathbf{x}) \oplus p_1(\mathbf{x}))) \wedge ... \wedge$
$\forall \mathbf{x}((p'_{Bn}(\mathbf{x}) \oplus p'_n(\mathbf{x})) \supset (p_{Bn}(\mathbf{x}) \oplus p_n(\mathbf{x})))$.

$(P'_B, P') \prec (P_B, P)$ means informally that the difference of extensions for P'_B and P' is less than the difference of extensions for P_B and P. $NB(P)$ expresses informally that there is some tuple of extensions for P_B which changes minimally into a tuple of extensions for P.

3.2 Model Theory

Let $B(P), NB(P), K(P)$ and $\alpha(P)$ be the same sentences in the above proof theory, and let M_B and M'_B be models of $B(P)$, and let M_{NB} and M'_{NB} be models of $K(P) \wedge \alpha(P)$. We define a partial order relation, $\preceq$, over pairs of models. $(M'_B, M'_{NB}) \preceq (M_B, M_{NB})$ is defined as the following.

(1) M_B, M'_B, M_{NB} and M'_{NB} have the same domain.

(2) Every constant and function receives the same interpretation in M_B, M'_B, M_{NB} and M'_{NB}.

(3) The following statement is true. (We write $< \mathbf{x} >$ as an interpreted term in M_B, M'_B, M_{NB} and M'_{NB}, because it receives the same interpretation in all of those models.)
$\forall \mathbf{x}((< \mathbf{x} > \in M'_B[p_1] \oplus < \mathbf{x} > \in M'_{NB}[p_1]) \supset$
$(< \mathbf{x} > \in M_B[p_1] \oplus < \mathbf{x} > \in M_{NB}[p_1])) \wedge ... \wedge$
$\forall \mathbf{x}((< \mathbf{x} > \in M'_B[p_n] \oplus < \mathbf{x} > \in M'_{NB}[p_n]) \supset$
$(< \mathbf{x} > \in M_B[p_n] \oplus < \mathbf{x} > \in M_{NB}[p_n]))$.

This ordering means that the difference of extensions of each p_i in M'_B and M'_{NB} is not more than the difference of extensions of each p_i in M_B and M_{NB}.

A *minimally different pair*, (M_B, M_{NB}), with respect to $B(P)$ and $K(P) \wedge \alpha(P)$ is defined as the pair of models for $B(P)$ and $K(P) \wedge \alpha(P)$ respectively such that there is no pair, (M'_B, M'_{NB}), such that $(M'_B, M'_{NB}) \preceq (M_B, M_{NB})$ and not $(M_B, M_{NB}) \preceq (M'_B, M'_{NB})$.

A *minimal revised model*, M_{NB}, with respect to $B(P)$ and $K(P) \wedge \alpha(P)$ is defined as the model such that there exists M_B such that (M_B, M_{NB}) is a minimally different pair with respect to $B(P)$ and $K(P) \wedge \alpha(P)$.

The relation between the proof theory and the model theory is as follows.

Proposition 1. *Let $B(P)$, $NB(P)$, $K(P)$ and $\alpha(P)$ be the same sentences in the proof theory. M_{NB} is a model of $NB(P)$ iff M_{NB} is a minimal revised model with respect to $B(P)$ and $K(P) \wedge \alpha(P)$.*

This proposition means that any result derived from $NB(P)$ is true in all minimal revised models.

As shown by the following proposition, if added knowledge is consistent with the belief, a set of models of the new belief is a maximum restricted set of models of belief so that any model in the new belief entails added knowledge.

Proposition 2. *Let $B(P)$, $NB(P)$, $K(P)$ and $\alpha(P)$ be the same sentences in the proof theory. If $\alpha(P)$ is consistent with $B(P)$, $NB(P) \equiv B(P) \wedge \alpha(P)$.*

The proofs of the above propositions are found in the appendix.

4 Examples

The previous formula of a minimal revised belief, $NB(P)$, can be translated into the following form:
$\exists P_B(B(P_B) \wedge K(P) \wedge \alpha(P) \wedge$
$\quad \forall P'_B \forall P'((B(P'_B) \wedge K(P') \wedge \alpha(P') \wedge$
$\quad\quad (P'_B, P') \preceq (P_B, P)) \supset (P'_B, P') \equiv (P_B, P)))$,
where $(P'_B, P') \equiv (P_B, P)$ is an abbreviation of:
$\forall \mathbf{x}((p'_{B1}(\mathbf{x}) \oplus p'_1(\mathbf{x})) \equiv (p_{B1}(\mathbf{x}) \oplus p_1(\mathbf{x}))) \wedge ... \wedge$
$\forall \mathbf{x}((p'_{Bn}(\mathbf{x}) \oplus p'_n(\mathbf{x})) \equiv (p_{Bn}(\mathbf{x}) \oplus p_n(\mathbf{x})))$.
The above formula is used in the following examples.

The first example shows a propositional case.

Example 1:

$P = p, q$
$B(P) \equiv p \wedge q$,
$K(P) \equiv \mathbf{T}$,
$\alpha(P) \equiv \neg p \vee \neg q$
$B(P)$ and $K(P)$ are the current belief and the current knowledge respectively and $\alpha(P)$ is the added knowledge. Then the minimal revised belief, $NB(P)$, is defined as follows.
$NB((p, q)) \equiv$
$\exists p_B \exists q_B($
$\quad p_B \wedge q_B \wedge (\neg p \vee \neg q) \wedge$
$\quad \forall p'_B \forall q'_B \forall p' \forall q'(($

$p'_B \wedge q'_B \wedge (\neg p' \vee \neg q') \wedge$
$((p'_B \oplus p') \supset (p_B \oplus p)) \wedge$
$((q'_B \oplus q') \supset (q_B \oplus q))) \supset$
$((p'_B \oplus p') \equiv (p_B \oplus p)) \wedge$
$((q'_B \oplus q') \equiv (q_B \oplus q)))).$

In this example, each propositional variable varies over **F** and **T**. Each tuple of the truth-value assignment for (p'_B, q'_B, p', q') makes the conditional part of the second conjunct false except (**T**,**T**,**T**,**F**), (**T**,**T**,**F**,**T**) and (**T**,**T**,**F**,**F**). Then the above formula is reduced to:

$\exists p_B \exists q_B ($
$p_B \wedge q_B \wedge (\neg p \vee \neg q) \wedge$
$((q_B \oplus q) \supset (p_B \equiv p)) \wedge$
$((p_B \oplus p) \supset (q_B \equiv q))).$

Then, each tuple of the truth-value assignment for (p_B, q_B) makes the above formula false, except (**T**,**T**). Then the above formula is reduced to:

$$(\neg p \vee \neg q) \wedge (p \vee q),$$

which is equivalent to the result in the previous section.

The second example shows inference from the belief.

Example 2:

$P = b, f$
$B(P) \equiv \forall x(b(x) \supset f(x)),$
$K(P) \equiv \mathbf{T},$
$\alpha(P) \equiv b(A).$

$B(P)$ expresses that every bird flies, and $\alpha(P)$ expresses that A is a bird. In this case, $\alpha(P)$ is consistent with $B(P)$, therefore, the new belief, $NB(P)$ is $B(P)$ $\wedge$ $\alpha(P)$ by proposition 2. Belief, that A flies, can be derived from the new belief. This example shows that if the added knowledge is consistent with the current belief, we can infer normal results from the current belief and the added knowledge.

The next example shows treatment of the counterexample to the belief which was discussed in the introduction.

Example 3:

$P = b, f$
$B(P) \equiv \forall x(b(x) \supset f(x)) \wedge b(A),$
$K(P) \equiv b(A),$
$\alpha(P) \equiv \neg f(A).$

$B(P)$ expresses that every bird flies and A is a bird, and $\alpha(P)$ expresses that A does not fly. In fact, $B(P)$ is the new belief of example 2. Then the new belief is defined as follows.

$NB((b, f)) \equiv$
$\exists b_B \exists f_B ($
$\forall x(b_B(x) \supset f_B(x)) \wedge b_B(A) \wedge b(A) \wedge \neg f(A) \wedge$
$\forall b'_B \forall f'_B \forall b' \forall f'(($
$\forall x(b'_B(x) \supset f'_B(x)) \wedge b'_B(A) \wedge b'(A) \wedge \neg f'(A) \wedge$
$\forall x((b'_B(x) \oplus b'(x)) \supset (b_B(x) \oplus b(x))) \wedge$
$\forall x((f'_B(x) \oplus f'(x)) \supset (f_B(x) \oplus f(x)))) \supset$
$(\forall x((b'_B(x) \oplus b'(x)) \equiv (b_B(x) \oplus b(x))) \wedge$
$\forall x((f'_B(x) \oplus f'(x)) \equiv (f_B(x) \oplus f(x))))).$

Let $b'_B(x)$ and $f'_B(x)$ be identical to $b_B(x)$ and $f_B(x)$ respectively and let $b'(x)$ be identical to $b_B(x)$ and let $f'(x)$ be identical to $f_B(x) \wedge x \neq A$.

Then the left-handside of the last conjunct of $NB((b, f))$ becomes as follows:

$\forall x(b_B(x) \supset f_B(x)) \wedge b_B(A) \wedge b_B(A) \wedge$
$\neg(f_B(A) \wedge A \neq A) \wedge$
$\forall x((b_B(x) \oplus b_B(x)) \supset (b_B(x) \oplus b(x))) \wedge$
$\forall x((f_B(x) \oplus (f_B(x) \wedge x \neq A)) \supset (f_B(x) \oplus f(x)))$

We can easily see that all conjuncts except the last are true, assuming $\forall x(b_B(x) \supset f_B(x))$ and $b_B(A)$. Concerning the last conjunct, it is reduced to:

(1) when $x = A$,
$(f_B(A) \oplus \mathbf{F}) \supset (f_B(A) \oplus f(A))$
which is true assuming $f_B(A)$ and $\neg f(A)$;

(2) when $x \neq A$,
$(f_B(x) \oplus f_B(x)) \supset (f_B(x) \oplus f(x))$
which is true.

Therefore, the left-handside of the last conjunct of $NB((b, f))$ is true, assuming the other conjuncts of $NB((b, f))$. Thus, we can derive the following from $NB((b, f))$.

$\exists b_B \exists f_B ($
$\forall x(b_B(x) \supset f_B(x)) \wedge b_B(A) \wedge b(A) \wedge \neg f(A) \wedge$
$\forall x((b_B(x) \oplus b_B(x)) \equiv (b_B(x) \oplus b(x))) \wedge$
$\forall x((f_B(x) \oplus (f_B(x) \wedge x \neq A)) \equiv (f_B(x) \oplus f(x)))).$

The second conjunct from the last is equivalent to $\forall x(b_B(x) \equiv b(x))$ and the last conjunct is equivalent to $\neg f(A) \wedge \forall x(x \neq A \supset (f_B(x) \equiv f(x)))$.

Therefore, the above formula is reduced to:

$\exists b_B \exists f_B (\forall x(b(x) \supset f_B(x)) \wedge b(A) \wedge \neg f(A) \wedge$
$\forall x(x \neq A \supset (f_B(x) \equiv f(x)))).$

Since we can derive $\forall x(f_B(x) \supset (x \neq A \supset f(x)))$ from the last conjunct, we can derive the following from the above formula:

$\forall x(b(x) \supset (x \neq A \supset f(x))) \wedge b(A) \wedge \neg f(A),$

which is equivalent to:

$\forall x(x \neq A \equiv (b(x) \supset f(x))).$

While $f(A)$ was true in the previous belief, we can no longer derive $f(A)$ from this new belief. Thus, this example shows nonmonotonicity of minimal belief revision. And, from the new belief, we can still show that every bird except A flies. This is an effect of minimal belief revision.

5 Related Research

5.1 Formalisms of Nonmonotonic Reasoning

The current formalisms of nonmonotonic reasoning try to define extra inference rules or axioms to produce belief. Default logic [Reiter80] uses special inference rules called *defaults* and circumscription [McCarthy80] adds special axioms to the knowledge. However, if we wish to represent a belief that every bird flies, then we cannot express this belief directly but must modify it to match special mechanisms of the above formalisms. In default logic, we must present the above belief by using extra inference such as:

$$\frac{bird(x) : \mathbf{M}fly(x)}{fly(x)}.$$

In circumscription, we must introduce special predicate ab to express the above belief as:

$$\forall x((bird(x) \land \neg ab(X)) \supset fly(x)),$$

and minimize ab. However, in our formalism, belief can be represented directly as:

$$\forall x(bird(x) \supset fly(x)).$$

We use a kind of minimization technique adopted in circumscription to formalize minimal belief revision. However, while circumscription minimizes predicates to produce a belief, minimal belief revision minimizes the difference between the previous belief and the new belief.

5.2 Truth Maintenance System

In a sense, the formalism in this paper can be regarded as a generalization of the truth maintenance system [Doyle79], because the TMS uses hypothetical contexts which correspond to models of belief in our formalism and performs belief revision. However, while the current TMS can only manipulate propositions (or ground sentences), our formalism can manipulate any arbitrary sentences. Moreover, TMS uses only one context at one time, whereas we can use multiple contexts at one time because a sentence for belief expresses a set of models.

5.3 Database Updates

In the database community, there have been several reports on research on semantics of updates. For example, [Fagin83 and Kuper84] define minimal updates of syntactic formulas in databases. However, they do not give a model theoretical analysis. Moreover, any previous contents in the database can be updated in their formalism, whereas in our formalism, hypothetical belief is distinguished from true knowledge and only belief can be changed by belief revision.

6 Conclusion

This paper presents a formalism of nonmonotonic reasoning by direct representation of belief and belief revision. Belief is defined as a detailed description of knowledge so that it entails knowledge. Belief revision occurs when belief does not entail added knowledge. This paper concentrates on a particular belief revision strategy called *minimal belief revision.* Minimal belief revision treats the counter-example for the previous belief as an exception in order to maintain consistency. It also keeps what was true in the previous belief as far as possible. This paper presents the proof theory and the model theory for minimal belief revision. However, since the proof theory is presented in the second-order language, it is not computable in general. We must investigate some useful subset of first-order sentences to make minimal belief revision computable.

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Appendix: Proofs of Propositions

Proposition 1. *Let $B(P)$, $NB(P)$, $K(P)$ and $\alpha(P)$ be the same sentence in the proof theory. M_{NB} is a model of $NB(P)$ iff M_{NB} is a minimal revised model with respect to $B(P)$ and $K(P) \land \alpha(P)$.*

Proof:

($\Rightarrow$) Suppose that M_{NB} is a model of $NB(P)$, but for every model, M_B, of $B(P)$, (M_B, M_{NB}) is not a minimally different pair with respect to $B(P)$ and $K(P) \land \alpha(P)$.

In other words, for every pair (M_B, M_{NB}), there is a pair of models (M'_B, M'_{NB}) for $B(P)$ and $K(P) \land \alpha(P)$ such that $(M'_B, M'_{NB}) \preceq (M_B, M_{NB})$ and not $(M_B, M_{NB}) \preceq (M'_B, M'_{NB})$.

Therefore

$\forall \mathbf{x}((<\mathbf{x}>\in M'_B[p_1] \oplus <\mathbf{x}>\in M'_{NB}[p_1]) \supset$

$(<\mathbf{x}>\in M_B[p_1]\oplus <\mathbf{x}>\in M_{NB}[p_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M'_B[p_n]\oplus <\mathbf{x}>\in M'_{NB}[p_n])\supset$
$(<\mathbf{x}>\in M_B[p_n]\oplus <\mathbf{x}>\in M_{NB}[p_n]))$,

and

$\neg($
$\forall\mathbf{x}((<\mathbf{x}>\in M_B[p_1]\oplus <\mathbf{x}>\in M_{NB}[p_1])\supset$
$(<\mathbf{x}>\in M'_B[p_1]\oplus <\mathbf{x}>\in M'_{NB}[p_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M_B[p_n]\oplus <\mathbf{x}>\in M_{NB}[p_n])\supset$
$(<\mathbf{x}>\in M'_B[p_n]\oplus <\mathbf{x}>\in M'_{NB}[p_n])))$.

Let $M_{NB}[p_{Bi}] = M_B[p_i](1\leq i\leq n)$, where p_{Bi} is a predicate constant which is not in P. Then since $M_B\models B(P)$, $M_{NB}\models B(P_B)$, where $P_B=(p_{B1},...,p_{Bn})$. Similarly, let $M_{NB}[p'_{Bi}] = M'_B[p_i]$ and $M_{NB}[p'_i] = M'_{NB}[p_i]$, then $M_{NB}\models B(P'_B)$ where $P'_B=(p'_{B1},...,p'_{Bn})$, and $M_{NB}\models K(P')\wedge\alpha(P)$ where $P'=(p'_1,...,p'_n)$.

And since

$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p'_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p'_1])\supset$
$(<\mathbf{x}>\in M_{NB}[p_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p'_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p'_n])\supset$
$(<\mathbf{x}>\in M_{NB}[p_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p_n]))$,

and

$\neg($
$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p_1])\supset$
$(<\mathbf{x}>\in M_{NB}[p'_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p'_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p_n])\supset$
$(<\mathbf{x}>\in M_{NB}[p'_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p'_n])))$,

by substituting $M_{NB}[p_{Bi}]$ for $M_B[p_i](1\leq i\leq n)$, $M_{NB}[p'_{Bi}]$ for $M'_B[p_i]$, and $M_{NB}[p'_i]$ for $M'_{NB}[p_i]$ respectively in the above statement, for any tuples of extensions, P_B, satisfying $B(P_B)$, there exist P'_B and P' satisfying $B(P'_B)$ and $K(P')\wedge\alpha(P')$ respectively such that $M_{NB}\models(P'_B,P')\prec(P_B,P)$.

It contradicts the fact that there exists P_B satisfying $B(P_B)$, such that

$M_{NB}\models\neg\exists P'_B\exists P'(B(P'_B)\wedge K(P')\wedge\alpha(P')\wedge(P'_B,P')\prec(P_B,P)))$

($\Leftarrow$) Suppose that M_{NB} is a minimal revised model and $M_{NB}\models\forall P_B((B(P_B)\wedge K(P)\wedge\alpha(P))\supset\exists P'_B\exists P'(B(P'_B)\wedge K(P')\wedge\alpha(P')\wedge(P'_B,P')\prec(P_B,P)))$.

In other words, for any P_B satisfying $B(P_B)$, we can take P'_B and P' such that $B(P'_B)\wedge K(P')\wedge\alpha(P')\wedge(P'_B,P')\prec(P_B,P)$.

Then since $M_{NB}\models(P'_B,P')\prec(P_B,P)$,

$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p'_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p'_1])\supset$
$(<\mathbf{x}>\in M_{NB}[p_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p'_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p'_n])\supset$
$(<\mathbf{x}>\in M_{NB}[p_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p_n]))$,

and

$\neg($
$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p_1])\supset$
$(<\mathbf{x}>\in M_{NB}[p'_{B1}]\oplus <\mathbf{x}>\in M_{NB}[p'_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M_{NB}[p_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p_n])\supset$
$(<\mathbf{x}>\in M_{NB}[p'_{Bn}]\oplus <\mathbf{x}>\in M_{NB}[p'_n])))$.

We take M_B, M'_B and M'_{NB} such that they have the same domain as M_{NB}, and every constant and function receives the same interpretation in M_B, M'_B and M'_{NB} as in M_{NB}, and the interpretations of predicates of P in M_B, M'_B and M'_{NB} receive the interpretations of predicates of P_B, P'_B and P' in M_{NB} respectively.
Since $M_{NB}\models B(P_B)\wedge B(P'_B)\wedge K(P')\wedge\alpha(P')$, $M_B\models B(P)$, $M'_B\models B(P)$ and $M'_{NB}\models K(P)\wedge\alpha(P)$.

And since

$\forall\mathbf{x}((<\mathbf{x}>\in M'_B[p_1]\oplus <\mathbf{x}>\in M'_{NB}[p_1])\supset$
$(<\mathbf{x}>\in M_B[p_1]\oplus <\mathbf{x}>\in M_{NB}[p_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M'_B[p_n]\oplus <\mathbf{x}>\in M'_{NB}[p_n])\supset$
$(<\mathbf{x}>\in M_B[p_n]\oplus <\mathbf{x}>\in M_{NB}[p_n]))$,

and

$\neg($
$\forall\mathbf{x}((<\mathbf{x}>\in M_B[p_1]\oplus <\mathbf{x}>\in M_{NB}[p_1])\supset$
$(<\mathbf{x}>\in M'_B[p_1]\oplus <\mathbf{x}>\in M'_{NB}[p_1]))\wedge ...\wedge$
$\forall\mathbf{x}((<\mathbf{x}>\in M_B[p_n]\oplus <\mathbf{x}>\in M_{NB}[p_n])\supset$
$(<\mathbf{x}>\in M'_B[p_n]\oplus <\mathbf{x}>\in M'_{NB}[p_n])))$,

by substituting $M_B[p_i](1\leq i\leq n)$ for $M_{NB}[p_{Bi}]$, $M'_B[p_i]$ for $M_{NB}[p'_{Bi}]$, and $M'_{NB}[p_i]$ for $M_{NB}[p'_i]$ respectively in the above statement, for M_{NB} and any models M_B satisfying $P(B)$, there exists a pair of models (M'_B,M'_{NB}), satisfying $P(B)$ and $K(P)\wedge\alpha(P)$ respectively such that $(M'_B,M'_{NB})\preceq(M_B,M_{NB})$ and not $(M_B,M_{NB})\preceq(M'_B,M'_{NB})$.

It contradicts the fact that M_{NB} is a minimal revised model. **QED**

Proposition 2. *Let $B(P)$, $NB(P)$, $K(P)$ and $\alpha(P)$ be the same sentences in the proof theory. If $\alpha(P)$ is consistent with $B(P)$, $NB(P)\equiv B(P)\wedge\alpha(P)$.*

Proof:

Let M be any model of $B(P)\wedge\alpha(P)$. Since $B(P)\models K(P)$, M is a model of $K(P)\wedge\alpha(P)$. Let us consider a pair of models for $B(P)$ and $K(P)\wedge\alpha(P)$, (M,M). It is a minimally different pair because for any pair of models of $B(P)$ and $K(P)\wedge\alpha(P)$, (M',M''), $(M,M)\preceq(M',M'')$, that is there is no pair of models (M',M'') such that $(M',M'')\preceq(M,M)$ and not $(M,M)\preceq(M',M'')$.

If M is a model of $K(P)\wedge\alpha(P)$, but not $B(P)\wedge\alpha(P)$, we can show that it is not a minimal revised model. Suppose that it is a minimal revised model. Then there is a model of $B(P)$, M', such that (M,M') is a minimally different pair. Since M is not a model of $B(P)$, M and M' are not identical. Let M'' be a model of $B(P)\wedge\alpha(P)$. Then $(M'',M'')\preceq(M,M')$ and not $(M,M')\preceq(M'',M'')$ because M and M' are not identical. It contradicts the fact that (M,M') is a minimally different pair. Thus, M is not a minimal revised model.

Therefore, a set of all models of $B(P) \wedge \alpha(P)$ is equivalent to a set of all minimal revised models, that is, a set of all models of $NB(P)$. In other words, $NB(P) \equiv B(P) \wedge \alpha(P)$. **QED**

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