

人工市場における市場の複雑さの発展の分析

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あらまし 本研究は、効率的市場仮説を、市場参加者個人の予測方式の複雑さと市場価格の挙動の複雑さの関係という新しい視点から捕らえ直し、人工市場モデルを用いたシミュレーション結果から、仮説の検討を行った。その結果、今までの効率的市場仮説と異なる以下の2点を見つけた。(a) 市場の複雑さは個人の複雑さとともに変化する。(b) 市場参加者が予測次元を上げていっても、市場価格の挙動の構造は消えずに、最終的には構造は存在するが完全に把握することのできない状態になった。

キーワード 人工市場, 複雑さ, 効率的市場仮説, 金融市場, 相関次元分析

Analysis of Development of Market Complexity in an Artificial Market

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Abstract In this study we rethought efficient market hypothesis from a viewpoint of complexity of market participants' prediction methods and market price's dynamics, and examined the hypothesis using simulation results of our artificial market model. As a result, we found the two difference from the hypothesis. (a) Complexity of markets was not fixed, but changed with complexity of agents. (b) When agents increased the complexity of their prediction methods, structure of dynamic patterns of market price didn't disappear, but it can't be described by equation of any dimensions.

Key words Artificial Market, Complexity, Efficient Market Hypothesis, Financial Market, Correlation Dimension Analysis.

1. Introduction

Are you surprised if the performance of financial specialists' forecasts is the same as that of randomly generated forecasts?

In the field of economics, the theory of financial markets called the efficient market hypothesis was proposed in the 70s, and it has caused many arguments till today. By this hypothesis, the movement of the price of financial markets is a random walk, and cannot be predicted. Therefore, the performance of all the forecasts is the same. Theories of financial engineering which developed greatly today are based on this hypothesis, and they assume financial prices as the stochastic process.

Although many statistical verification of the hypothesis was performed using actual data, since the hypothesis included a market participant's expectation formation, it has not been verified directly. In recent years, however, the artificial market approach which builds a virtual market model and performs a simulation into a computer appeared, and researches in this approach try to verify the hypothesis directly [1-3].

This study rethinks the efficient market hypothesis from the new viewpoint of the relation between the complexity of market participants' prediction formulas and the complexity of the movement of a market price. And this study examines the hypothesis from the simulation result using the artificial market model.

2. The efficient market hypothesis seen from complexity

The main points of the efficient market hypothesis are summarized as follows.

- Each market participant of a financial market takes in very quickly and exactly all the information related to the movement of a market price, and uses it for price expectation.

- The market price that determined by the dealings between such market participants is reflecting properly all the relevant information that is available at present.

- Therefore, there is no room for a certain person to find out the new relation between a mar-

ket price and the available information, and to become advantageous from other persons. That is, the movement of a market price becomes a random walk driven only by new information, and nobody can predicted it.

When the above-mentioned main points are recaptured from the viewpoint of complexity, the efficient market hypothesis contains the following things implicitly.

- In order to take in suitable information, each market participant is going to complicate his prediction formula by learning, and is going to hold the structure of the determination formula of the market price.

- The structure of a price determination formula is fixed and independent of the learning of market participants. Finally the structure is detected by the market participants, and it will disappear. That is, the efficient market hypothesis needs the two premises: (a) the independence of the complexity of the movement of a market price from the complexity of each market participant's prediction formula and (b) the existence of motivation of leaning by each market participant.

On the other hand, by the artificial market simulation, de la Maza [4] found that when the dimension of market participants' prediction formula went up from 0 to 1, the movement of a market price also changes from a random walk to linearity. That is, he showed the possibility the complexity of market participants and the complexity of a market are not independent.

Then, what is the motivation to which each market participant complicates his prediction formula? Joshi et.al. [7] think that it is because the situation similar to the prisoner's dilemma game has occurred. In their artificial market model, taking in the technique of the moving average of a technical analysis to a prediction method, and raising the dimension of a prediction formula from 0 to 1 corresponds to the default strategy of the prisoner's dilemma game. On the other hand, not using a technical analysis for prediction corresponds to the cooperation strategy. From the simulation result, the two following conditions for becoming a prisoner's dilemma situation were seen.

Condition 1 If one raises his prediction dimension, his prediction becomes more accurate and the profit of his dealings result increases. Thus, the motivation of the default strategy exists.

Condition 2 However, when everybody raised the dimension, the movement of the market price became more complicated, and the prediction accuracy has fallen rather than the time of everybody's not using the technical analysis.

Thus, since everybody raised the dimension of his prediction formula in pursuit of profits, the prediction accuracy becomes worse than before.

In the following sections, by the artificial market simulation, we analyze the complexity of a market and the prisoner's dilemma situation when a prediction dimension becomes larger.

3. Artificial market model

The artificial market is a virtual financial market with 50 virtual dealers (agents) in a computer. One financial capital and one non-risk capital exist in this artificial market. Each agent expects the movement of the financial price, and he changes the position of the financial and non-risk capital so that the utility of his expected profit may become the maximum. In the artificial market, one term consists of four step of expectation, an order, price determination, and learning, and time progresses discretely by repeating these four steps.

3.1 Expectation

Each agent expects the change value of the financial price of this term using the weighted sum of the change value of past financial price. That is, in this study, since fundamentals information does not exist in a market, the agents expects the change value of the financial price only by the technical analysis.

The expectation formula of each agent is auto the regressive integral moving average model ARIMA($n, 1, 0$), where n means the number of the terms of the price changes used for expectation. The larger n is, the larger the dimension of an expectation formula is. Thus in this study, n is regarded as the complexity of each agent's expectation.

The expectation formula is as follows, when P_t is the financial price of this term which is not yet determined and \tilde{y}_t is the expectation the change of

financial price ($P_t - P_{t-1}$).

$$\begin{aligned}\tilde{y}_t &= \sum_{i=1}^n b_i y_{t-i} + e_t \\ &= \mathbf{x}_t' \mathbf{b}_t + e_t\end{aligned}\quad (1)$$

Here, e_t is the normal distribution whose average is 0 and standard deviation is 0.1, \mathbf{b}_t is a vector with the coefficient of the prediction formula^(註1), $(b_1, \dots, b_n)'$, and \mathbf{x}_t is a vector of the explanation variables of the prediction formula, i.e., the past price changes^(註2), $(y_{t-1}, \dots, y_{t-n})'$.

3.2 Order

It is assumed that each agent has the utility function of expected profit with risk avoidance. Then the optimum quantity of the position of the financial capital with the maximum utility, q_t^* , is proportional to the expected change value y_t of the formula (1).

$$q_t^* = ay_t, \quad (2)$$

where a is a coefficient. Each agent's amount of orders o_t is the difference between the optimum position q_t^* and the current position q_{t-1} .

$$o_t = q_t^* - q_{t-1} \quad (3)$$

If the market price P_t is lower (higher) than his expected price ($P_{t-1} + y_t$), each agent order to buy (sell). The amount of order is o_t .

If $o_t > 0$,

$$\begin{cases} \text{Buy } o_t & (P_t \leq P_{t-1} + y_t) \\ \text{No action} & (P_t > P_{t-1} + y_t) \end{cases}$$

If $o_t < 0$,

$$\begin{cases} \text{No action} & (P_t < P_{t-1} + y_t) \\ \text{Sell } o_t & (P_t \geq P_{t-1} + y_t) \end{cases}$$

3.3 Price determination

All the orders of 50 agents in the market are accumulated, and the market price of this term is determined as the value where the demand and supply are balanced. Dealings are transacted between the buyer who gave the price higher than a market price, and the seller of a lower price.

(註1) : The initial value of the coefficients \mathbf{b}_0 is given with the uniform random numbers from -1 to 1.

(註2) : At the start, the initial values of price \mathbf{x}_0 are generated by the normal distribution whose average is 0 and standard deviation is 1.

3.4 Learning

Each agent updates the coefficients \mathbf{b}_t of the prediction formula (1) using the successive least-squares method with the information on the change y_t of the newly determined market price^(註3). The least-squares method is as follows [5].

$$\mathbf{b}_{t+1} = \mathbf{b}_t + \frac{(\mathbf{X}'_t \mathbf{X}_t)^{-1} \mathbf{x}'_t (y_t - \mathbf{x}'_t \mathbf{b}_t)}{f_t}, \quad (4)$$

where \mathbf{X}_t is a learning matrix which starts by $\mathbf{X}_0 = 100 \times \mathbf{I}$ (\mathbf{I} is a unit matrix), and is updated by the following formula.

$$(\mathbf{X}'_t \mathbf{X}_t)^{-1} = (\mathbf{X}'_{t-1} \mathbf{X}_{t-1})^{-1} - \frac{(\mathbf{X}'_{t-1} \mathbf{X}_{t-1})^{-1} \mathbf{x}'_t \mathbf{x}'_t (\mathbf{X}'_{t-1} \mathbf{X}_{t-1})^{-1}}{f_t} \quad (5)$$

$$f_t = 1 + \mathbf{x}'_t (\mathbf{X}'_{t-1} \mathbf{X}_{t-1})^{-1} \mathbf{x}_t \quad (6)$$

4. Simulation result

In the next section, we examine the complexity of the market and the prisoner's dilemma-situation when the prediction dimension became large using the artificial market model.

4.1 Development of the complexity of a market

In order to examine the independence of the complexity of the movement of a market price from the complexity of each market participant's prediction formula, we carried out the correlation dimension analysis^(註4). All 50 agents have the prediction formulas of the same n dimension. We carried out the simulation with 4000 terms 100 times. Changed the embedding dimensions, the correlation dimensions was calculated using the price data of 3885 terms at the second half while learning were stabilized to some extent (Fig.1).

As a result, when a prediction dimension was 0, the the correlation dimension curve was convex downward like the theoretical value of a random walk (fig. 1a). That is, there is no structure in the dynamics of the market price. However, when the prediction dimension increase a little,

(註3) : When $n = 0$, the prediction value is a random number and learning is not performed.

(註4) : The procedure of the correlation dimension analysis was described in [9, 10].

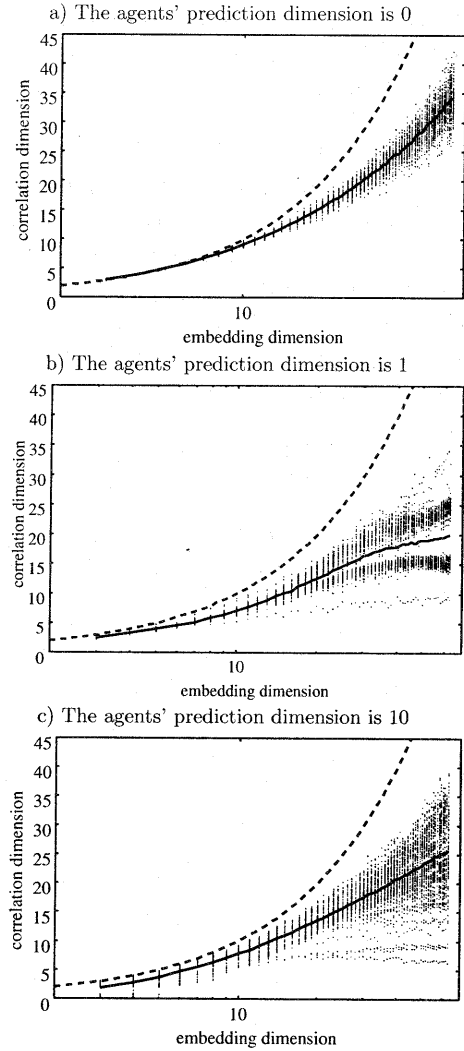


図1 Correlation dimensions : X-axis is the logarithm of embedding dimensions. A solid line is an average of the correlation dimension of 100 paths. A dotted line is the theoretical value of a random walk.

the correlation dimension curve was convex upward and saturated (fig. 1b). Thus, the structure that could be described by an equation of a finite dimension appeared in the dynamics of the market price. Furthermore, when the prediction dimension was raised, the correlation dimension curve became a straight line (fig. 1c). Thus, the correlation dimension curve was neither convex downward like a

random walk nor saturated. That is, there was a structure in the dynamics of the market price, but it could not be described by an equation of any finite dimension.

According to Nakajima [9, 10], as a result of analyzing Tokyo Stock Exchange Stock Price Index data, the logarithm of a correlation dimension went up linearly like this simulation result in fig. 1c. That is, when each agent's prediction dimension increases, like the price data in the real-world, the dynamics of the price in the artificial market can be described roughly by an equation of some dimensions. And the more precise description is also attained by increasing the number of dimension. However, the movement of price data can not be described completely by an equation of any finite dimensions. That is, the number of the variables related to the movement cannot be specified completely.

4.2 Merit of complicating a prediction formula

We investigated the merit of complicating the prediction formula. The dimensions of 25 agents' prediction formulas was set to n , and the dimension of the prediction formula of the other 25 agents was $n + 1$. Each simulation had 4000 terms which consisted of the four steps in section 3. The averages of forecast errors were calculated both about the agent group with n dimensions and about the group of $n + 1$ dimensions. The forecast errors were the difference between each agent's prediction value and a market price. The initial value of random numbers was changed and 100 simulations was carried out^(注5). Figure 2 shows the difference between the forecasts errors of the group with $n + 1$ dimensions and those of the group of n dimensions. While the number of dimensions in the prediction formula is small, the merit of complicating prediction formulas is large. The agent who can predict correctly can increase his profit. Thus, when the number of dimensions is small, the conditions 1 of the prisoner's dilemma situation in the section 2. are hold. However, when the number of dimensions becomes

(注5) : Since the calculation of averages were impossible when the market price had diverged, we carried out simulations until we could get 100 simulations whose paths did not diverge.

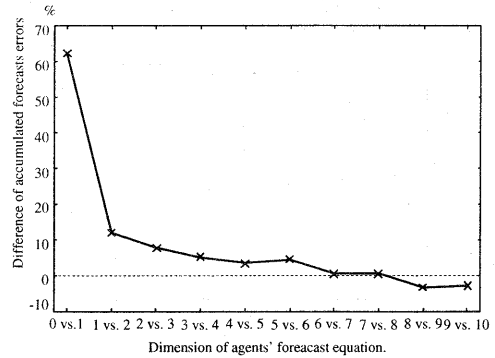


图2 比较 forecast errors: Y-axis is a difference of forecast errors (forecast errors of the group of n dimensions are 100). Positive (negative) values mean that forecast errors of the group of $n + 1$ dimensions are small (large).

large, the merit of complicating prediction formulas disappears.

4.3 The demerit in the whole market

We examined whether the prediction of prices becomes harder in the whole market as increase of the dimension of prediction formulas. In this simulation, 50 prediction formulas of all agents were the same n dimension. We carried out the simulation with 4000 terms 100 times^(注6). After having accumulated the forecasts errors in 4000 terms and taking an average of 50 agents in 100 simulations. (Fig.3). As a result, when the number of dimensions in the prediction formula was small, the forecast error became large, as the number of dimensions increased. That is, the conditions 2 of the prisoner's dilemma situation in the section 2. were hold. However, it has converged to the fixed value when the number of dimensions was larger than three.

5. New efficient market hypothesis

The simulation results are summarized as follows.

- When each market participant's prediction dimension is 0, the movement of a market price resembles a random walk. If the prediction dimension increases, the structure which can be described by an equation of a finite dimension appears in the

(注6) : The path to diverge was not seen when all agents' prediction formula was the same dimension.

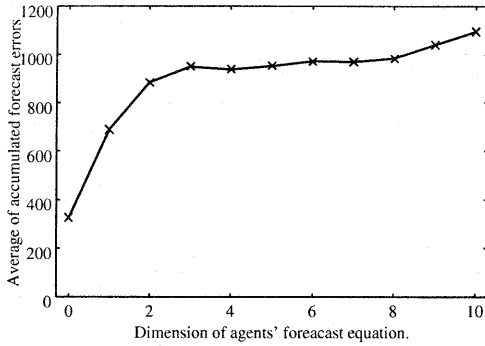


图3 Forecast errors

movement of price.

- Therefore, if each agent increases his prediction dimension, since the prediction dimension approaches to the dimension of the price determination formula and his prediction becomes more accurate. Thus, the merit of complicating prediction formulas exists. However, if everybody increases their prediction dimension, prediction accuracy becomes smaller than before. That is, it will become the prisoner's dilemma situation.

- If everybody continues to increase the prediction dimension in the prisoner's dilemma situation, the movement of a market price come to have the structure that can not be described completely by an equation of any finite dimensions.

The structure of the movement of a market price changed as market participants changed their prediction formulas. That is, the complexity of market participants and the complexity of a market are not independent unlike the efficient market hypothesis. The simulation results also suggests that the structure of the dynamics of price data did not disappear when market participants continue to complicate their prediction formulas. In the final state, however each market participant increases his prediction dimension, he can not predict the market price completely.

In such the state where there is no "correct answer" of learning, it is thought that a procedure of learning by each market participant becomes the key factor to the movement of a market price in addition to a result of learning. As Kichiji [8] said, the efficiency of learning by a market participant, the

difference in the cognitive framework, the interaction between market participants, and the method of informational choice, etc. become important.

Another key point is the mechanism of market price determination. In this study we assumed that the market price were determined discretely as an equilibrium price. Alternatively we can assume that the market price is determined continuously as a transaction price of each dealing. The mechanism of market price determination is the mechanism how to accumulate the individual complexity on the complexity of a market. Therefore, it has large influence on the relation between the complexity of market participants' prediction formulas and the complexity of the movement of a market price. It is interesting to examine whether the same simulation can be acquired when the mechanism of market price determination changes.

6. Mechanism of preventing divergence.

As other relation between the dynamics of market prices and agents' complexity, we propose the mechanism of preventing rate divergence.

We carried out two types of computer simulation. In the first type of simulations, the numbers of all 50 agents' prediction dimension were the same as n . In the second type, the number of 25 agents' prediction dimension were n and that of the other 25 agents' prediction dimension were $n + 1$. We varied n from 0 to 9 and carried out the two types of simulations with 4000 terms 100 times at each prediction dimension n .

As a result, there was paths which market prices diverged^(注7) only in the simulations where the agents have different prediction dimensions. There was no path diverged when all the agents have the same prediction dimension.

We found that the convergence of agents' prediction formula prevented rate divergence. In the simulation paths diverged, agents have different coefficients of prediction formula and there were always both demand and supply in markets. Thus, the

(注7) : We regarded a simulation path as diverged when the value of simulation rate exceeded the value that could be represented by double precision type.

trend could continue until the rate paths diverged. By contrast, in the simulation paths that did not diverge, when values of the common n coefficients converged there was only demand or supply in the market. Thus the trend stopped.

This mechanism is supported by the fact that the simulation path did not diverge when all the agents have the same prediction dimension. In this case, the values of coefficients were easier to converge because the numbers of them were the same.

Moreover, other simulation results supported the mechanism. Figure 4 shows percentages of simulation paths diverged when the numbers of prediction dimensions are n and $n + 1$. When $n = 0$, there were 25 random-forecasting agents in the market. Thus the values of coefficients were difficult to converge and the percentage became the highest. The simulation results provided another support. As the number of coefficients, n , increased, the convergence became more difficult. Therefore more simulation paths diverged.

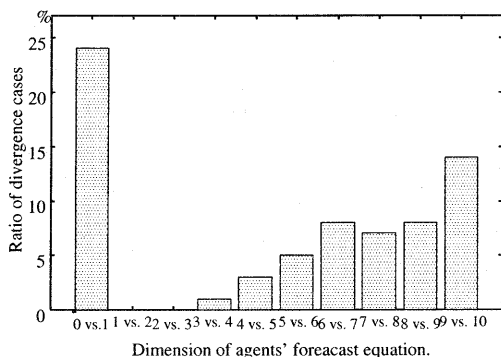


图 4 Percentages of Simulation Paths Diverged.

This mechanism supported the results of our preceding analysis of contrary opinion phenomena using our other artificial market [6].

7. Conclusion

This study examined an efficient market hypothesis using artificial market approach. As a result, the following two points different from an efficient market hypothesis were found.

- While the prediction dimension of agents is

small, the structure which can be described to the movement of a market price exists, and the motivation of increasing the prediction dimension exists.

- Even if the market participant increases the prediction dimension, the structure of the movement of a market price does not disappear. Finally, however each market participant increases his prediction dimension, he can not predict the market price completely.

As future works, we want to investigate the influence of (a) the procedure of learning by a market participant and (b) the mechanism of the price determination on the relation between between the complexity of market participants' prediction formulas and the complexity of the movement of a market price.

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