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ウェーブレット分解によるマルチスケール画像復元

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あらまし　複数枚の観測画像に強い相関がある場合、各画像を独立に復元するより、相互相関を積極的に利用したマルチチャンネル復元方法が有利であることが報告されている。本研究はウェーブレット分解後のスケール間における相関に着目し、一つの観測画像をウェーブレット分解し、各スケールにマルチチャンネルのウィーナフィルタの適用を試みる。スケール間の相関を利用しやすくするため、ウェーブレット分解としては方向性を持たない冗長な à trous 分解を用いた。計算機実験より、提案法は従来のウィーナフィルよりよい結果が得られることを確認した。

キーワード　マルチチャンネルウィーナフィルタ, à trous 分解, スケール間相関、画像復元

A Multiscale Wiener Approach for Image Restoration

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Abstract Multichannel restoration approaches have been shown to outperform independent single-channel ones when multiple observations with significant correlation are available. In this paper, we present a wavelet-based multiscale Wiener restoration approach that takes into account the highly correlated cross-scale edge structure in wavelet domain to obtain superior results to the conventional Wiener approach. A specific redundant filter bank algorithm known as the à trous algorithm is used in dyadic wavelet transform so that the decomposed scales can be associated directly with the multichannel Wiener restoration filter. Experiments show the proposed multiscale Wiener restoration approach is capable of outperforming the conventional Wiener approach.

Key words multichannel Wiener filter, the à trous algorithm, cross-scale correlation, image restoration

1. Introduction

Multiscale approach to signal and image processing has evoked significant attention largely motivated by the great activities in the field of wavelets. Wavelet domain is suitable for natural image modeling in that its bases, wavelets, are well localized both in time/space and frequency domain. Several signal processing operators, such as denoising, pattern matching, compression and estimation, are shown to benefit from the multiscale representation [1] [2].

There are also a few attempts in the field of multiscale image restoration. When multiple observations with significant correlation are available, multichannel restoration approaches were proposed to take the advantage of such correlation [9] [10]. Motivated by the highly correlated edge structure in wavelet-decomposed image scales, Banham *et al* firstly proposed to associate wavelet scales with the multichannel Wiener restoration in [6]. Zervakis *et al* further investigated the vector-matrix formulation in [7]. Their approach was based on the widely used orthogonal wavelet proposed by Mallat [3]. It was shown that such a multichannel approach replaces the global stationary assumption in conventional image restoration with weaker, thus more practical ones within each scale. The critical problem of their approach is that no satisfactory estimation method for cross-scale correlation is found, and only the theoretic results which estimates such statistics from the original images were presented in their papers.

In this paper, we use the particular filter bank version of wavelet known as the à trous algorithm [4] [5]. The à trous algorithm is redundant, that is, no decimation/expansion is performed in decomposition/reconstruction. This greatly simplifies the formulation of the multiscale restoration filter because without decimation all the operators in the filter bank now commute with the restoration operator under the block-circulant approximation. The à trous algorithm is also different from Mallat's one in that it creates only one high frequency component in one scale that includes edges in all directions. This not only decreases the computational load in wavelet decomposition and reconstruction, more importantly also avoids the possibility of attenuating the edge correlation between scales when they are decomposed into different directional components. Experimental results show that such a multiscale Wiener restoration approach can outperform the conventional one.

This paper is organized as follows. The restoration problem definition and a brief review of Wiener filter are given in Section 2. In Section 3, we give the detail of our multiscale restoration approach. The experiments and results are discussed in Section 4. We conclude this paper in Section 5.

2. Image Degradation Model and Wiener Filter

The general model for a degraded image can be formulated as

$$g = Df + n \tag{1}$$

where a lexicographic ordering of the original image f, the observed image g, and the observation noise n, is used. The observation noise is assumed as additive white Gaussian. The blur operator D is represented as a block-Toeplitz matrix because the blur is space invariant.

By assuming D and n are known or have been estimated, there are numerous approaches to solve the ill-posed equation (1). The direct stochastic regularization leads to the choice of an approach that estimates the restored image $\hat{f}$ according to

$$\min E\|f - \hat{f}\|^2. \tag{2}$$

Wiener filter, which is the linear estimate satisfying (2), is then given by

$$\hat{f} = R_{ff}D^T(DR_{ff}D^T + R_{nn})^{-1}g \tag{3}$$

where R_{ff} is the autocorrelation of original image which must be estimated. Assuming each of the matrices in (3) are block-circulant, it can be efficiently solved in the discrete Fourier transform(DFT) domain. Note that the circulant property of autocorrelation matrix R_{ff} assumes the global stationarity of the image. We approximate the power spectrum S_{ff}, i.e. the DFT of R_{ff}, by the periodogram of the observation g

$$S_{ff} \approx GG^* \tag{4}$$

where G is the DFT of g, and * stands for the complex conjugate.

When N observations, $g_i (i = 1, 2, ..., N)$, with significant cross-channel correlation are available,

$$g_i = D_i f_i + n_i, \quad i = 1, 2, ..., N \tag{5}$$

the multichannel Wiener filter which takes the advantage of cross-channel correlation structure performs better than the single-channel one independently applied on each channel. With the notations below

$$g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}, f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix}, n = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \tag{6}$$

$$D = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 \\ \vdots & & \cdots & 0 \\ 0 & 0 & \cdots & D_N \end{bmatrix} \tag{7}$$

$$R_{ff} = \begin{bmatrix} R_{f_1 f_1} & R_{f_1 f_2} & & R_{f_1 f_N} \\ R_{f_2 f_1} & R_{f_2 f_2} & & R_{f_2 f_N} \\ \vdots & & \cdots & \vdots \\ R_{f_N f_1} & R_{f_n f_2} & & R_{f_N f_N} \end{bmatrix} \tag{8}$$

the multichannel Wiener filter has just the same formulation as equation (3). We only consider the simple case without cross-channel blur. Because of the general stationary assumption for each channel, the correlation matrix (8) consists of block-circulant sub-matrices, thus the restored images (3) can be efficiently calculated in DFT domain using the algorithms proposed in [9] and [10].

The multichannel Wiener filter is mostly used in video image processing, where successive image sequence are regarded as multichannel inputs, and in color image processing, where the RGB bands are treated as a 3-channel image. In the next section, we propose to use the wavelet-decomposed scales as channels in the multichannel Wiener filter.

3. Multiscale Wiener Restoration

We address the multiscale Wiener restoration approach in this section. The degraded image is decomposed into wavelet domain using the *à* trous algorithm [4] [5]. The *à* trous algorithm is undecimated and creates one set of wavelet coefficients at each scale. The decomposed scales are directly associated to the multichannel Wiener restoration filter to yield a nonstationary restoration approach.

3.1 Wavelet transform and the *à* trous algorithm

Given a continuous signal $s(t)$ and the wavelet function $\psi(t)$, the Discrete Wavelet Transform(DWT) is defined as

$$w(2^j, n) = \frac{1}{\sqrt{2^j}} \int \psi^*(\frac{t-n}{2^j}) s(t) dt. \tag{9}$$

Some algorithms have been proposed to discretize (9). Mallat proposed the widely used decomposition algorithm [3], which is decimated and creates three sets of wavelet coefficients of different directions for one scale in two dimension. The decimation operation makes Mallat's decomposition translation-variant, and is found to cause some unpleasant artifacts in image denoising [8]. We propose to use the *à* trous(with holes) algorithm, which is undecimated and creates only one set of wavelet coefficients in one scale. Fig. 1 depicts the filter bank structure of the *à* trous algorithm, where $H_0(z_1, z_2)$ is the 2-D low-pass filter and $H_1(z_1, z_2)$ is the 2-D high-pass filter. In stead of the decimation operation, the *à* trous algorithm inserts zeros(holes) between successive filter coefficients(e.g. from $H_0(z_1, z_2)$ to $H_0(z_1^2, z_2^2)$). We will use the notation $H_i^j (i = 0, 1$ and $j = 0, \cdots, J-1)$ to denote the spatial matrix of the low/high-pass filter used in scale $j+1$.

In practical computation, given the image data $c_0(m, n) = s(m, n)$ the smooth coefficient c_j of scale j is computed from scale $j-1$ as

$$c_j = H_0^{j-1} c_{j-1}. \tag{10}$$

The high-pass filters are designed so that the wavelet coefficients w_j are just the difference between two successive smooth scales

$$w_j = c_{j-1} - c_j. \tag{11}$$

Note that we get only one coefficient set in one scale here even in two dimension. The algorithm then allows us to reconstruct the original data as

$$c_0 = c_J + \sum_{j=1}^{J} w_j \tag{12}$$

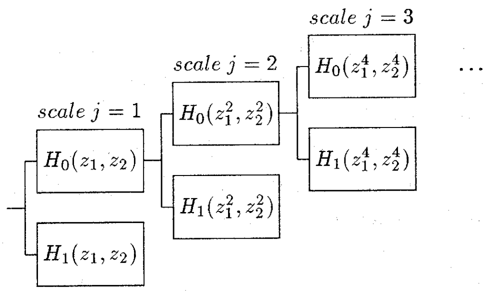
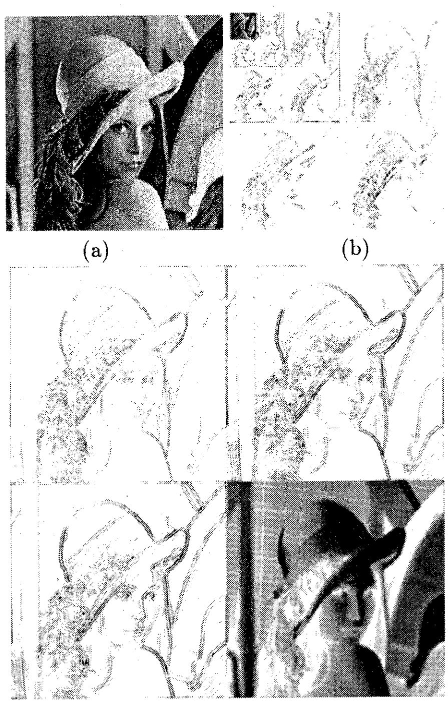


Fig. 1 The 2-D *à trous* decomposition filter bank(H_0:the low-pass filter, H_1:the high-pass filter).



(c)

Fig. 2 (a) Image *Lena* (b) three-level Mallat's decomposition (c) three-level *à trous* decomposition.

where J is the final scale.

Fig. 2 shows the *Lena* image after 3 level decomposition using both Mallat's and the *à* trous algorithms. The low-pass filter used in the *à* trous algorithm is

$$\begin{pmatrix} 1/4 & 1/2 & 1/4 \end{pmatrix} \otimes \begin{pmatrix} 1/4 \\ 1/2 \\ 1/4 \end{pmatrix} \tag{13}$$

where $\otimes$ is the Kronecker operator. This filter is also used in all the experiments hereafter. Fig. 2 shows that the edge structure are highly correlated across scales.

3.2 Multiscale restoration approach

Using the *à* trous algorithm depicted in Fig.1, wavelet coefficients of image s in scale j can be expressed in matrix-vector formulation as

$$\begin{aligned} w_1^s &= H_1^0 s, \\ w_j^s &= H_1^{j-1} H_0^{j-2} \cdots H_0^0 s, \quad j = 1, \cdots, J \\ c_J^s &= H_0^{J-1} \cdots H_0^0 s. \end{aligned}$$

Replacing s with the observed image g in (1), we get

$$\begin{aligned} w_1^g &= H_1^0 (Df + n), \\ w_j^g &= H_1^{j-1} H_0^{j-2} \cdots H_0^0 (Df + n), \\ c_J^g &= H_0^{J-1} \cdots H_0^0 (Df + n). \end{aligned}$$

All the operators H_i^j and D are circulant matrices under space-invariant assumption, so they commute with each other and we obtain

$$w_j^g = D w_j^f + w_j^n, \quad j = 1, \cdots, J \tag{14}$$

$$c_J^g = D c_J^f + c_J^n. \tag{15}$$

By approximating that the white Gaussian noise is still white Gaussian after decomposition, the above equations reflect the multichannel degradation model in (5). So we can decompose the observed image to wavelet domain, perform multiscale restoration, and then reconstruct the result to the spatial domain.

The use of cross-scale correlation results in a reduction of the global stationary assumption imposed on the observed image in conventional Wiener restoration. This can easily be understood in the correlation structure equation (8), since only the sub-matrices are assumed to be block-circulant, not the whole matrix R_{ff} itself. A more theoretical explanation can be made in the DFT domain. Wavelet filters are nothing but band filters in the DFT domain. While stationary approach restores each frequency independently, the multiscale approach tries to explores and utilize the cross-scale(frequency) correlation, thus obviously reflects a nonstationary one.

Our approach differs from the previous one in the following two points. Since the complicated decomposition of the blur operator D in wavelet domain is unnecessary now, this formulation under the undecimated *à* trous algorithm is more general and much simpler than that in [6] and [7] based on the decimated orthogonal wavelet. Also the *à* trous algorithm creates only one high frequency component in one scale, which may avoids the possibility of attenuating the edge correlation between scales when they are decomposed into different directional components. While Banham *et al* addressed the difficulties in estimating the cross-scale correlation in their paper, our experiments show that the usual periodogram estimate from the observed image can improve the restoration.

4. Experiments

In this section, we demonstrate the effectiveness of the proposed multiscale restoration algorithm through experiments.

Three images *Squares* and *Goldhill*(Fig. 3), and *Lena*(Fig. 4.(a)), all of size 256 × 256, are used as sample images in experiments. These images possess different characteristics. The blur operator used is the uniform blur of size 7 × 7. The test images are also contaminated with white Gaussian noise at 40,30 and 20 dB in blurred signal-to-noise ratio(BSNR). The restoration results are evaluated in terms of improved signal-to-noise ratio(ISNR) which is defined as

$$ISNR \doteq 10\log_{10}\frac{\| f-g \|^2}{\| f-\hat{f} \|^2}. \qquad (16)$$

Table 1 denotes the results of the two restoration approaches tested in our experiments: *Wiener* denotes the conventional single channel Wiener restoration in spatial domain, *MultiScale Wiener* denotes the proposed multiscale approach in wavelet domain. In all cases, the proposed multiscale approach shows improvement from 0.2dB to 0.9dB over the conventional spatial domain one. The images of *Lena* at BSNR= 30dB are shown in Fig. 4. for a visual inspection. The visual difference in the results of *Wiener* and *MultiScale Wiener* can be seen clearly.

Fig. 3 Sample images, from left to right:*Squares* and *Goldhill.*

Table 1 Results in ISNR(dB) for the three sample images

BSNR	Wiener	MulitScaleWiener
	Lena	
20dB	1.99	2.68
30dB	2.66	3.46
40dB	3.80	4.60
	Squares	
20dB	3.32	3.59
30dB	3.84	4.64
40dB	5.53	6.48
	Goldhill	
20dB	1.75	2.38
30dB	2.46	3.14
40dB	3.60	4.20

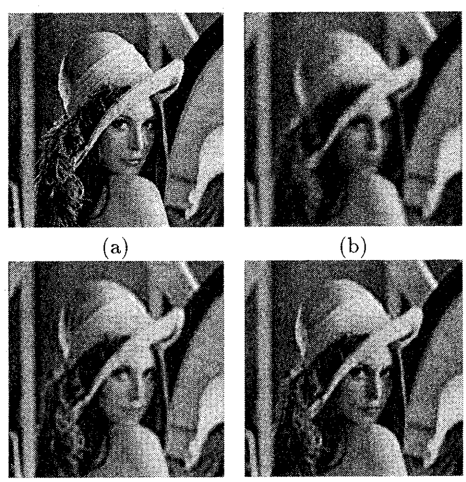


Fig. 4 Images of *Lena* (a)the original (b)the degraded (c)the spatial Wiener result (d)the multiscale Wiener result.

5. Conclusion

In this paper, we presented a multiscale Wiener approach for image restoration in wavelet domain. By taking into account the significant cross-scale correlation in wavelet domain, this approach shows improvement over the conventional Wiener filter in spatial domain.

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